Title:
Modeling, Evaluation, and Optimization for Product Adaptation Considering Uncertainties

Author:
Deyi Xue, dxue@ucalgary.ca, University of Calgary

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Introduction:
Product adaptation aims at satisfying new functional requirements through changes of configurations and parameter values of the manufactured products during their operation stages. This approach can extend the product lifespans for customers and reduce waste to protect the environment [2].

Design problems for product adaptation are primarily classified into two categories: design for adaptation of existing products and design of new adaptable products. In our previous work, a method to identify the adapted design from the existing product was developed, and this method was used for converting an existing Toyota Camry with an internal combustion engine (ICE) into an electric vehicle (EV) [4]. This method was improved for the design of a new adaptable ICE vehicle that could be easily converted into an EV, considering potential changes in environmental policies [5]. Both configurations and parameters of products were considered in these design methods. In addition, uncertainties in configurations and parameters of products were also studied [1].

The objective of this work is to develop a framework for the adaptation of existing products and the design of new adaptable products, considering uncertainties based on our previous research results. Methods of modeling, evaluation, and optimization have been introduced for this design framework.

Design Framework for Product Adaptation Considering Uncertainties:
The design framework for product adaptation considering uncertainties is shown in Fig. 1. Design processes in this framework are classified into two categories.

- *Design for adaptation of an existing product:* In this type of design, an existing product is first selected as the original product configuration, and the design of this original product configuration is defined. Design adaptation for product adaptation is then conducted to obtain the design of the adapted product configuration and the process of product adaptation based on the new functional requirements. The original product configuration is subsequently changed to the adapted product configuration through the process of product adaptation.
- *Design of a new adaptable product:* In this type of design, both the original product configuration and the potential adapted product configuration are considered in design based on the requirements of both the original functions and the potential new functions. The design of the original configuration is used to manufacture the original product configuration. When new functional requirements arise in the operation stage, since product adaptation has been considered in the design, the product with the original configuration can be easily changed into the product with the adapted configuration.

Multiple solutions are usually created and evaluated to obtain the optimal design in these design processes. Methods and tools of modeling, evaluation, and optimization are used in design for product adaptation considering uncertainties.

- *Modeling:* A product design is modeled by its configurations (i.e., original configuration and adapted configuration), and each of these configurations is modeled by its parameters. In addition,

uncertainties in configurations and parameters are also modeled.

- *Evaluation:* For each of the design configurations, various evaluation measures are selected, and these evaluation measures are defined as numerical functions of design parameters. In addition, variations of evaluation measures due to uncertainties of design parameters are also considered.
- *Optimization:* Because a product design is modeled by its configurations and parameters of these configurations, optimization is also conducted at two levels: configuration optimization to achieve the optimal design configurations and parameter optimization to achieve the optimal parameter values of the configurations. Evaluation measures and variations of evaluation measures are used to build the optimization objective functions.

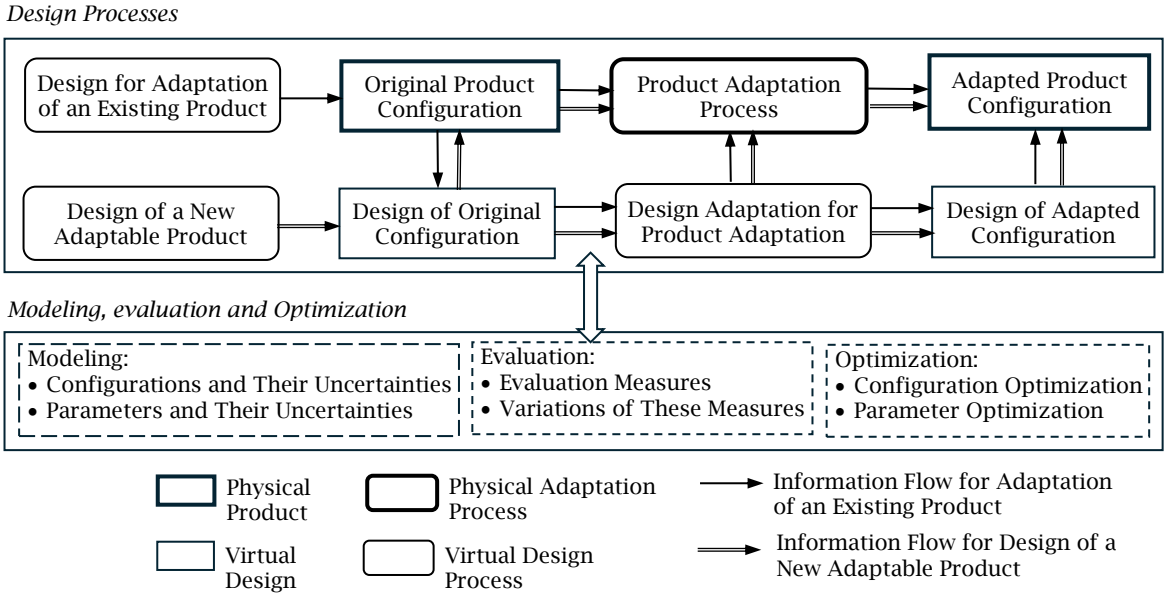

Fig. 1: Design framework for product adaptation considering uncertainties.

Modeling for Product Adaptation Considering Uncertainties:

A product design is modeled by its configurations and the parameters of these configurations. Fig. 2 shows changes in models in the two types of design processes.

The generic product design considering all candidates of original configurations and adapted configurations is modeled by an AND-OR tree as shown in Fig. 2(a). Partial design solutions, including components (e.g., a gear) and sub-assemblies (e.g., a pair of gears), are described by design nodes. Design nodes are classified into two categories: *unadaptable nodes* that cannot be changed (e.g., replaced or removed) for product adaptation and *adaptable nodes* that can be changed for product adaptation. A design node is associated with a probability, representing the chance that this design node is actually used in the manufactured product due to uncertainties in design, manufacturing, and operation. A design node is further defined by its parameters (e.g., tooth number of a gear). Uncertainty of a parameter is modeled by its tolerance. Sub-nodes of a super-node are associated with three types of relations: AND relation when all the sub-nodes need to be selected to support the super-node, OR relation when only one sub-node needs to be selected to support the super-node, and OR-A (i.e., OR for adaptation) relation when different sub-nodes are used for different configurations of the product, considering product adaptation. In this work, only the original configuration before product adaptation and the adapted configuration after product adaptation are considered.

Specific original design configuration candidates are created from the generic product design AND-OR tree through tree-based search. Fig. 2(b) shows a specific original design configuration with multiple adapted design configuration candidates created from the AND-OR tree given in Fig. 2(a).

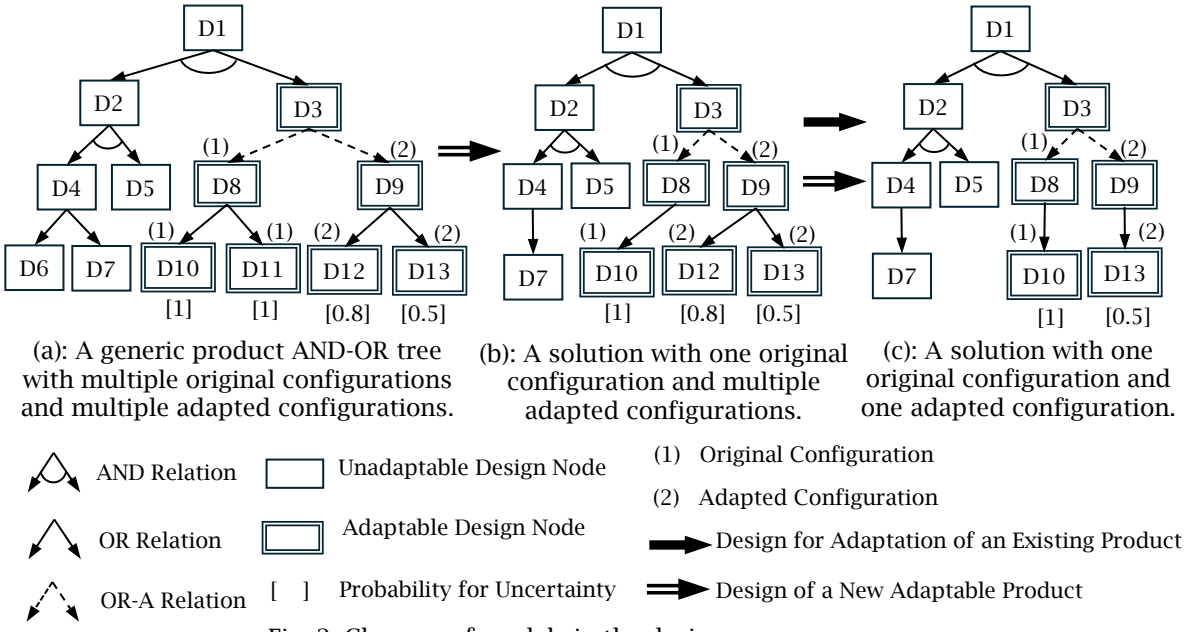

Fig. 2: Changes of models in the design processes.

The following rules are used to generate a specific original design configuration with multiple adapted design configurations:

- When a super-node is an unadaptable design node, and its sub-nodes are associated with an OR relation (e.g., Node D4), only one sub-node should be selected.
- When a super-node is an adaptable design node for the original design configuration, and its sub-nodes are associated with an OR relation (e.g., Node D8), only one sub-node should be selected.
- In other cases, all the nodes should be selected.

Specific adapted design configuration candidates are created from the specific original design configuration AND-OR tree through tree-based search. Fig. 2(c) shows a solution with a specific original design configuration and a specific adapted design configuration created from the AND-OR tree given in Fig. 2(b).

The following rules are used to generate a solution with a specific original design configuration and a specific adapted design configuration:

- When a super-node is an adaptable design node for the adapted design configuration, and its sub-nodes are associated with an OR relation (e.g., Node D9), only one sub-node should be selected.
- In other cases, all the nodes should be selected.

From the generic product design AND-OR tree, final design solutions are created, and each solution, C_i ($i = 1, 2, \ldots, n$), is defined by its original configuration $C_i^{(O)}$ and adapted configuration $C_i^{(A)}$ as shown in Table 1. For each configuration, its uncertainty is defined by a probability (i.e., $p_i^{(O)}$ or $p_i^{(O)}$), which is calculated by multiplying the probabilities of all the bottom nodes.

Since a design node is defined by its parameters, and a design configuration is defined by a tree with a collection of design nodes, a design configuration can be defined by the parameters of these design nodes. Parameters for the i-th design solution ($i = 1, 2, \ldots, n$) are defined by $\boldsymbol{X_i}$, and variations of these parameters are defined by $\boldsymbol{X_i^{(L)}} \leq \boldsymbol{X_i} \leq \boldsymbol{X_i^{(U)}}$, where $\boldsymbol{X_D^{(L)}}$ and $\boldsymbol{X_D^{(U)}}$ are the lower and upper boundaries of these parameter values, respectively.

In the design for adaptation of an existing product, only the adapted configuration candidates and their parameters are considered. In the design of a new adaptable product, both the original configuration candidates and the adapted configuration candidates, and the parameters of these configurations are considered.

Table 1: Design solutions created from the generic product design AND-OR tree.

Solution	Original Configuration: $C_i^{(O)}$	$p_i^{(O)}$	Adapted Configuration: $C_i^{(A)}$	$p_i^{(A)}$
C_1	D1,D2,D4,D5,D6,D3,D8,D10	1.0	D1,D2,D4,D5,D6,D3,D9,D12	0.8
C_2	D1,D2,D4,D5,D6,D3,D8,D10	1.0	D1,D2,D4,D5,D6,D3,D9,D13	0.5
C_3	D1,D2,D4,D5,D7,D3,D8,D10	1.0	D1,D2,D4,D5,D7,D3,D9,D12	0.8
C_4	D1,D2,D4,D5,D7,D3,D8,D10	1.0	D1,D2,D4,D5,D7,D3,D9,D13	0.5
C_5	D1,D2,D4,D5,D6,D3,D8,D11	1.0	D1,D2,D4,D5,D6,D3,D9,D12	0.8
C_6	D1,D2,D4,D5,D6,D3,D8,D11	1.0	D1,D2,D4,D5,D6,D3,D9,D13	0.5
C_7	D1,D2,D4,D5,D7,D3,D8,D11	1.0	D1,D2,D4,D5,D7,D3,D9,D12	0.8
C_8	D1,D2,D4,D5,D7,D3,D8,D11	1.0	D1,D2,D4,D5,D7,D3,D9,D13	0.5

Evaluation for Product Adaptation Considering Uncertainties:

A configuration $C_i^{(j)}$ ($i = 1, 2, \ldots, n$; $j = O, A$) in the i-th design solution is defined by its parameters $\boldsymbol{X_i}$. A configuration is evaluated by multiple evaluation measures $E_{i,k}^{(j)}$ ($k = 1, 2, \ldots, m$). Each of these evaluation measures can be calculated from the numerical relation between the parameters and the evaluation measure.

$$E_{i,k}^{(j)} = e_{i,k}^{(j)}(\boldsymbol{X_i}) \tag{1}$$

In the design for adaptation of an existing product, only the adapted configuration is evaluated. In the design of a new adaptable product, both the original configuration and the adapted configuration are evaluated. Suppose the evaluation measures for the two configurations are $E_{i,k}^{(O)}$ and $E_{i,k}^{(A)}$, the evaluation measure considering both configurations is defined by:

$$E_{i,k} = \frac{\alpha^{(O)} p_i^{(O)} E_{i,k}^{(O)} + \alpha^{(A)} p_i^{(A)} E_{i,k}^{(A)}}{\alpha^{(O)} p_i^{(O)} + \alpha^{(A)} p_i^{(A)}} = \frac{\alpha^{(O)} p_i^{(O)} e_{i,k}^{(O)}(\boldsymbol{X_i}) + \alpha^{(A)} p_i^{(A)} e_{i,k}^{(A)}(\boldsymbol{X_i})}{\alpha^{(O)} p_i^{(O)} + \alpha^{(A)} p_i^{(A)}} \tag{2}$$

where $\alpha^{(O)}$ and $\alpha^{(A)}$ are the weighting factors for the original configuration and the adapted configuration, and $p_i^{(O)}$ and $p_i^{(A)}$ are the probabilities of these configurations considering uncertainties. The durations of these two configurations in the product lifespan can be used as the weighting factors.

When multiple evaluation measures are defined by different units (e.g., efficiency, cost), these evaluation measures $E_{i,k}$ are first converted into comparable evaluation indices $I_{i,k}$ between 0 and 1, representing satisfaction levels in these evaluation aspects.

$$I_{i,k} = i_k(E_{i,k}) \tag{3}$$

The overall evaluation index, I, considering all evaluation aspects is defined by:

$$I_i = \frac{\sum_{k=1}^{m}(w_k I_{i,k})}{\sum_{k=1}^{m} w_k} \tag{4}$$

where w_k represents the weighting factor of the k-th evaluation index.

When parameter uncertainties are defined as $\Delta \boldsymbol{X_i} = \boldsymbol{X_i^{(U)}} - \boldsymbol{X_i^{(L)}}$, the variation of the overall evaluation index for the i-th design is achieved as ΔI_i.

Optimization for Product Adaptation Considering Uncertainties:

Optimization is conducted to obtain the final design solution from all feasible candidates. Since a design is modeled by its configurations and parameters, optimization is also conducted at two levels: configuration optimization and parameter optimization. Since both the evaluation measures and variations of evaluation measures are considered to build the optimization objective function, this optimization-based design is also called a robust design.

- *Parameter optimization*

  For each configuration solution C_i defined by its parameters $\boldsymbol{X_i}$, suppose its overall evaluation index

is calculated by $I_i = I_i(\boldsymbol{X_i})$, and the variation of the overall evaluation index is calculated by $\Delta I_i = \Delta I_i(\boldsymbol{X_i})$, parameter optimization is defined by either Eq. (5) or Eq. (6):

$$\max_{w.r.t.\boldsymbol{X_i}} Z_i(\boldsymbol{X_i}) = \frac{I_i(\boldsymbol{X_i})}{\Delta I_i(\boldsymbol{X_i})} \tag{5}$$

$$\max_{w.r.t.\boldsymbol{X_i}} Z_i(\boldsymbol{X_i}) = \beta_1 I_i(\boldsymbol{X_i}) + \beta_2 \frac{1}{\Delta I_i(\boldsymbol{X_i})} \tag{6}$$

where $Z_i(\boldsymbol{X_i})$ is the robustness, and β_1 and β_2 are coefficients to integrate the two optimization objective functions into a single one. Numerical search [3] is employed for parameter optimization.

- *Configuration optimization*
  Among all the configuration solutions C_i ($i = 1, 2, \ldots, n$), the optimal one is achieved through configuration optimization defined by:

$$\max_{w.r.t.C_i} Z_i(\boldsymbol{X_i^*}) \tag{7}$$

where $\boldsymbol{X_i^*}$ is the vector with the optimal parameter values for the i-th configuration solution C_i, and $Z_i(\boldsymbol{X_i^*})$ is the corresponding optimal robustness for the C_i. When the number of configuration solutions is large, genetic programming [3] is employed for the configuration optimization.

Conclusions:

A design framework has been introduced in this research for the adaptation of existing products and the design of new adaptable products, considering uncertainties. Characteristics of this framework are summarized as follows.

- The modeling scheme is effective in describing configurations, parameters, and uncertainties in the configuration and parameters of the original product designs and the adapted product designs.
- The evaluation scheme is effective in obtaining evaluation measures and variations of these evaluation measures, considering configurations, parameters, and uncertainties in configurations and parameters of the original product designs and the adapted product designs.
- The optimization scheme is effective in obtaining the optimal design solution defined by its original configuration and adapted configuration, and parameters of these configurations through the multi-level optimization considering both evaluation measures and variations of evaluation measures.

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ORCID:

Deyi Xue, https://orcid.org/0000-0001-5285-8867

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