Title:
A Procedure for Identifying Planes and Axes of Symmetry Candidates in B-rep CAD models

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Introduction:
Symmetry is a geometrical property beneficial in many applications in mechanical engineering [6][7][9][11][12]. In mechanical design during solid modeling, 3D CAD models are often shaped symmetrically for different reasons: to simplify the 3D CAD modeling process, to reduce the complexity of assemblies & the number of unique parts [4], or to minimize assembly errors & assembly time [11]. The symmetry information (i.e., the planes & axes of symmetry) is most often not explicitly stored in the native 3D CAD models unless the final shape of the model has been created using mirroring or pattern operations. The neutral exchange file formats currently also do not support storing any symmetry information. Therefore, the existence of symmetry in the 3D CAD models is usually checked & recognized visually by mechanical engineers. This may be a tedious & time-consuming task, especially if the 3D CAD model's shape is geometrically complex or consists of a large number of topological entities. Thus, *Computer-Aided Symmetry Detection* (CASD) is preferred, which supports mechanical engineers in detecting the planes & axes of symmetry in the 3D CAD models. A common approach of many CASD techniques [1][6][11] is to create a set of planes of symmetry candidates (POSCs) & axes of symmetry candidates (AOSCs), which are then evaluated to identify the actual planes & axes of symmetry among them. The candidates are usually identified from the input model's geometry (e.g., point clouds, surfaces, mesh triangles, etc.) by for instance pair matching [11]. Consequently, a significant number of candidates may be generated without their practical need in detecting symmetries, thus making the CASD computationally demanding [5]. Hence, this study is exclusively focused on the stage of CASD that deals with the identification of POSC & AOSC in B-rep CAD models. For that purpose, a procedure for reducing the number of POSC & AOSC for exact global & partial axi- & reflection symmetric 3D CAD models with Boundary Representation (B-rep) is proposed.

Related work:
The existing studies obtained the POSCs & AOSCs from the B-rep's topological elements (e.g., loops, faces, etc.) & their underlying geometrical properties [6][11] or from the principal axes of inertia [1]. *Li et al.* [6] obtained the POSCs & AOSCs from one, two, or three adjacent faces using the intrinsic parameters of the underlying analytic surfaces & their intersections (vertices, edges, & loops). Then, a two-level propagation process over the B-rep was used to determine the global or local planes or axes of symmetry. The drawback of the study is that a combinatorial analysis was used to obtain the combinations of surfaces, their adjacencies, & intersections for identifying the candidates. Consequently, if the 3D CAD model contains some non-predicted combinations of analytical surfaces or any numerical surfaces (e.g., B-spline), the corresponding candidates may remain undetected. *Buric et al.* proposed the use of three POSCs & three AOSCs aligned with the principal axes of inertia &

passing through the center of gravity (COG). The approach did not apply to partially symmetric 3D CAD models or those exhibiting exact reflection symmetries that are misaligned with the principal axes. *Tate et al.* identified the POSCs by pairing identical loops of the same type through their geometric properties (e.g., surface area, number of edges, etc.), while the AOSCs were identified from single loops. Then, duplicate POSCs & AOSCs were eliminated by comparison of their location & orientation. The obstacle of this study is that the proposed similarity criterion was not adequate as two non-identical topological elements (in this case loops) can have the same geometrical properties (e.g., loop area, number of edges, etc.). Hence, the present paper investigates the use of similarity measures for identifying similar topological elements & to pair them to generate POSCs.

In general, similarity has been studied in mechanical design to support designers in generating new designs [3], or in manufacturing to extract existing product information such as cost estimations in machining [2]. Moreover, recognizing similarities in 3D CAD models may be beneficial for the reuse of existing design solutions [14]. Thereby, a given input CAD model (new design) is used to retrieve similar CAD models from the database (existing designs). Further, similarity recognition may be exploited for the clustering of CAD models [13]. This study is, however, focused on common similarity measures from statistics which are used to compare the similarity between two finite data sets. For instance, the *Cosine Similarity* (CS) computes the cosine of the angle between two vectors **A** & **B**:

$$\mathrm{CS} = \cos\,\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{\|\mathbf{A}\|\|\mathbf{B}\|} = \frac{|X \cap Y|}{\sqrt{|X|\cdot|Y|}}\,. \tag{1.1}$$

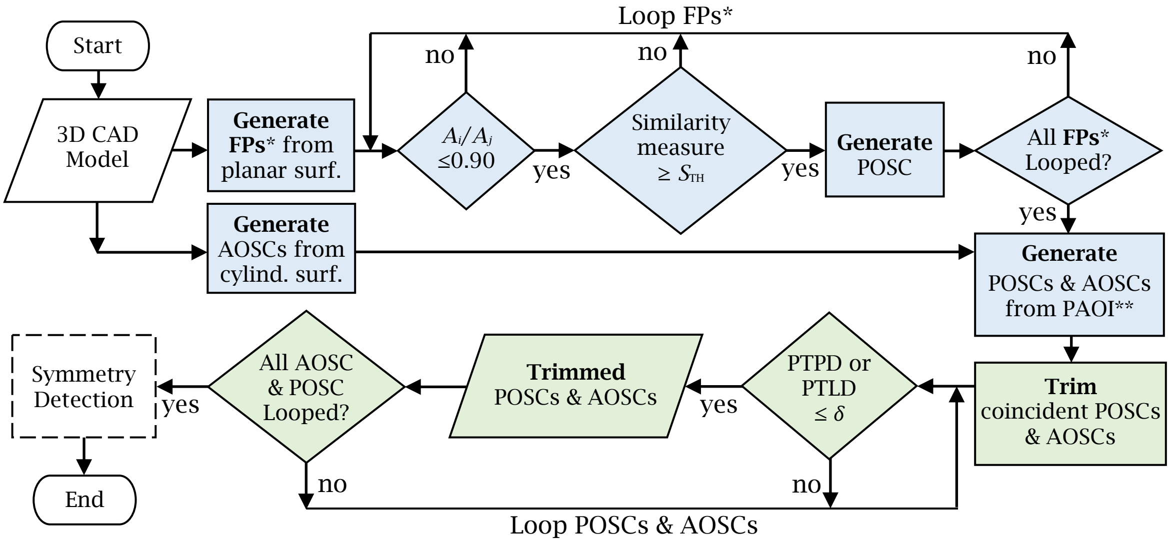
CS was utilized to compute the similarity between two Opitz code vectors (the CAD model features were presented by alphanumerical digits) [14]. Another similarity measure, the *Jaccard index* (JI) is defined as the size of the *intersection* divided by the size of the *union* of two finite data sets X & Y:

$$\mathrm{JI} = \frac{|X \cap Y|}{|X \cup Y|} = \frac{|X \cap Y|}{|X| + |Y| - |X \cap Y|}\,, \tag{1.2}$$

where $|X|$ & $|Y|$ represent the cardinalities of the sets. The JI has been used for clustering purposes [10], to measure the similarity between machines/parts & group them. Alternative similarity measures related to the Jaccard index are the *Sørensen–Dice coefficient* (SDC), which is defined as twice the size of the intersection divided by the sum of their cardinalities. The *Szymkiewicz–Simpson coefficient* (SSC) or *Overlap coefficient* is described as the ratio between the size of the intersection & the smaller cardinality of two data sets. The *Braun-Blanquet coefficient* (BBC) represents the size of the intersection divided by the larger cardinality of two data sets. The similarity measures range between 0 (non-similar) & 1 (absolutely similar). In this study, the mentioned similarity measures are explored in terms of their possibilities & applicability for detecting similar face pairs in B-rep CAD models.

The proposed Procedure for Identifying Planes and Axes of Symmetry Candidates:

The proposed procedure for identifying the POSCs & AOSCs addresses the mentioned drawbacks of the CASD technique in [1]. It utilizes a combination of three approaches to obtain the candidates: from single faces, similar face pairs, & the principal axes of inertia. The procedure consists of two main phases: *generating* & *trimming* the POSCs & AOSCs (see flowchart in Fig. 1). The initial set of POSCs is *generated* through the pairing of similar faces, but only of the plane surface type, while the initial set of AOSCs is generated from single faces with the underlying cylindrical surface type. In addition, three POSCs & three AOSCs are always generated from the principal axes of inertia to cover possible exact symmetries that are aligned with the principal axes. The procedure for computing the principal axes of inertia is given in the paper [1]. Then face pairs are generated & subjected to quick filtering to reduce the computational effort. For each face pair the ratio between their surface areas needs to be $A_i/A_j \leq 0.90$ ($A_i \leq A_j$) to proceed to the computation of the similarity measure. If the similarity measure is above the threshold value S_{TH}, the location of the POSC is determined from the midpoint between the two face centroids, while its orientation depends on the arrangement between the faces, which may be *parallel*, *coplanar*, or *arbitrarily angled*. If two faces are parallel, the POSC orientation is equal to the normal vector of either one of the faces. If two faces are coplanar, the POSC orientation corresponds to the vector between the face centroids. Finally, if two faces are *arbitrarily angled*, the orientation of the POSC is obtained by subtracting the normal vectors of the two faces. The initially generated POSCs & AOSCs need to be further processed in the *trimming* phase where first duplicates, i.e., coincident candidates, are removed. Then, the point-to-plane distance (PTPD) is computed for each POSC, & the point-to-line distance (PTLD) for each AOSC, to assess their distances from the COG. It is known that if an object is exact global symmetric, its planes and/or axes of symmetry will pass through the COG [11], while in the case of partial symmetric objects, they will be close to the COG [8]. Hence, the PTPD & PTLD are computed & queried to be below an empirically defined tolerance distance $\delta=0.05\cdot D$, where D is the CAD model's minimum bounding box diagonal. Finally, the trimmed POSC & AOSC are used afterward as input for the *Symmetry Detection* to identify the actual POS or AOS, which is not the scope of this study.



*FPs – Face Pairs Generation phase Trimming phase **PAOI – Principal Axes of Inertia

Fig. 1: Flowchart of the proposed procedure for identifying the POSC & AOSC.

The similarity measures JS, CS, SDC, SSC, & BBC have been investigated to explore their applicability for matching similar face pairs. Within the context of this study, faces in the B-rep CAD model can be observed as sets of edges. First, faces are decomposed into edges & designated using a string code (e.g. “oLI10”), as shown in Fig. 2. The first letter of the string code indicates whether the edge belongs to an *outer* “o” or *inner* “i” loop. The next two letters describe the edge's underlying curve type, “LI” for line, “CI” for circle, “EL” for ellipse, “BC” for B-spline curve, & so on. Finally, the last part of the designation represents the length of the curve in millimeters.

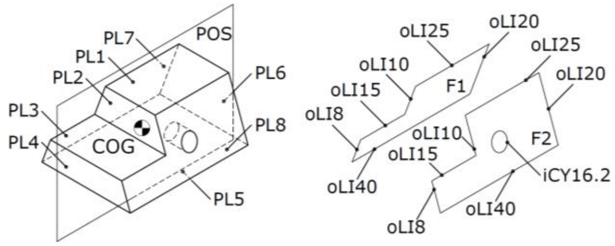


Fig. 2: An example of a part & the designation of its faces & edges.

Tab. 1 shows several test cases of face pairs with varied similarity & their respective similarity measures computed. The numerators in all mentioned similarity measures are the same (the number of common elements in both data sets) & the main difference derives from the denominators. All similarity measures correctly recognize absolute similar (Tab. 1, a) & non-similar face pairs (Tab. 1, f). The JI is the most conservative similarity measure among the tested as the computed scores are the lowest. By its definition, the JI always stronger penalizes differences between two sets of edges, even if one is a proper subset of the other (Tab. 1, e). The BBC penalizes slightly less than JL the differences in the size of the sets as the number of common edges is divided by the larger size between the two sets of edges. The SDC essentially doubles (i.e., "weights") the intersection in the numerator & divides it with the sum of the cardinalities from both sets of edges. Consequently, this produces less penalization than JI & BBC, indicating higher similarity. The difference between the CS & SDC negligible because their denominators ($\sqrt{(|X| \cdot |Y|)}$ vs $0.5(|X|+|Y|)$) will result in nearly identical scores, as long as the number of edges in both faces does not differ significantly (one order of magnitude). The SSC is the least conservative similarity measure, which in certain cases may result in a false positive identical face pair. This happens if all edges in the one face are also found in the other face (Tab. 1, d), then SSC=1 regardless of how many additional edges are in the other face. Based on the computed scores for the given test cases, it can be concluded that the JI & BBC seem to be not adequate to measure the similarity between two faces due to the considerable penalization leading to an underestimation of similarity, while the SSC in some cases may overestimate the similarity. Hence, the CS & SDC, which both produce nearly identical scores, appear to be the most convenient similarity measures between two faces. To consider a face pair similar, the condition $\mathrm{CS} \geq S_{\mathrm{TH}}$ or $\mathrm{SDC} \geq S_{\mathrm{TH}}$ needs to be fulfilled, whereby based on the computed examples S_{TH} can be set to $S_{\mathrm{TH}}=0.75$.

Face pairs		A_i / A_j	CS	JI	SDC	SSC	BBC
a)		1	1	1	1	1	1
b)		0.96	0.80	0.67	0.80	0.80	0.80
c)		0.97	0.95	0.90	0.95	0.95	0.95
d)		0.98	0.91	0.83	0.91	1	0.83
e)		1	0.75	0.60	0.75	0.75	0.75
f)		1	0	0	0	0	0

Tab. 1: Example of face pairs & the computed similarity measures.

<u>Testing:</u>
The proposed procedure has been implemented in Solidworks 2020 using its Application Programming Interface & tested on 150 CAD models (Fig. 3 & 4). The CAD models subjected to testing were exact global (including multiple reflectional symmetric) & partially symmetric. Test results show that the procedure identified in approx. 95% of test cases the correct POSCs & AOSCs, among which were also the actual planes & axes of symmetry. In only 5% of test cases, when the reflectional symmetric CAD models do not have any faces of the plane surface type (Figure 4), the procedure failed to detect the respective POSCs (the typical CAD models where this happens are for instance gears, flanges, etc.). Compared to Li et al. [6], the present study produces ≈105 times fewer candidates per CAD model & respectively ≈138 times fewer than the study by Tate et al. [11].

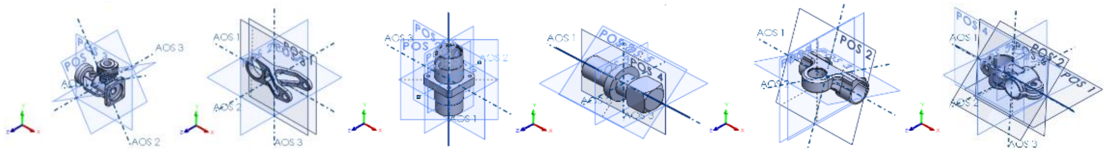


Fig. 3 Examples of CAD models subjected to testing & the obtained POSCs & AOSCs.

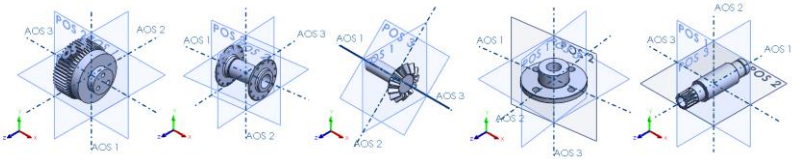


Fig. 4 Examples of CAD models where the corresponding POSCs were not properly detected.

Conclusion:

This paper addresses the identification of the planes & axes of symmetry candidates in B-rep CAD models. The procedure consists of two phases: *generating* & *trimming* candidates. The AOSCs were *generated* from single faces (cylindrical surfaces type), while the POSCs were generated from similar face pairs (planar surface type), which were identified by means of a similarity measure (Cosine similarity). The proposed procedure has been tested on 150 CAD models & the results showed that it identified the right candidates were detected in 95% of test cases. Compared to past studies, the proposed procedure results in fewer candidates, which may be an important factor to consider for reducing the time complexity of CASD. In the future, the proposed procedure shall be improved to detect the corresponding candidates in CAD models without plane surfaces and subjected to further testing.

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