Title:
A Level-set Non-uniform Offset Method for Iso-scallop Toolpath Planning Based on Mesh Surface Conformal Mapping

Authors:
Yue-Feng Li, lucaslyf@foxmail.com, South China University of Technology
Qing-Hui Wang, wqh@scut.edu.cn, South China University of Technology
Wen-Feng Gan, gwf421@zju.edu.cn, ZWSOFT CO., LTD (Guangzhou)

Keywords:
Toolpath Planning, Conformal Mapping, Level-set, Non-uniform Offset, Mesh Surface

DOI: 10.14733/cadconfP.2022.382-387

Introduction:
The most traditional method for free-form surface machining path generation is based on parametric surface [3,7], but it only applies to a single surface patch [4]. Compared with parametric surface, mesh surface is simpler and more robust. More importantly, mesh surface can be applied to composite surface. As a result, the mesh surface representation model has received a lot of attention and is widely used for toolpath planning [5,8].

The conformal mapping algorithm based on mesh surface can map tree-dimensional (3D) mesh surface to two-dimensional (2D) mesh plane, so that toolpath planning can be carried out in 2D domain. Although this method decreases the dimension of toolpath planning from 3D to 2D and considerably reduces computational complexity, it still requires a high number of 2D intersection calculations in traditional 2D point-sequence curves (PS-curves) non-uniform offset algorithm [1,6]. To adapt to different mesh surfaces and toolpath parameters, it is necessary to constantly improve the 2D PS-curves non-uniform offset algorithm.

Level-set is a method for calculating and analyzing interface motion proposed by S. Osher and J. A. Sethian [9]. It is simple, universal, and easy to expand to higher dimension space. At present, it is widely used in many fields, such as shape optimization, material deposition, curvature movement, path planning, medical image processing and so on [10]. In the toolpath planning, level-set function can naturally deal with the topological changes in the process of path offset without requiring other methods to deal with the problem of 2D PS-curves intersection. Therefore, it can be used in conjunction with mesh surface conformal mapping for toolpath planning to replace the traditional 2D PS-curves non-uniform offset method, ensuring that the problem of 2D intersection is completely avoided.

In this work, based on the conformal mapping algorithm, a level-set non-uniform offset method is proposed to replace the 2D PS-curves non-uniform offset algorithm, to avoid the complex intersection problem of 2D PS-curves. Firstly, the 2D mesh plane is generated by using the conformal mapping algorithm, and a series of discrete points are generated by equidistantly dispersing the plane area. To calculate the mapping stretching coefficient, the shortest distance direction from the discrete points to the 2D mesh plane boundary is used to approximate replace the 2D line spacing direction. Then, the approximate stretching mapping coefficient is used to calculate the 2D line spacing value at each discrete point, and the value is used as the normal velocity in the level-set function. In this way, the non-uniform offset based on level set is realized, and iso-scallop toolpath can be planned in 2D domain. Then, the final toolpath can be obtained by inverse mapping the 2D non-uniform offset toolpath back to the 3D mesh surface. Finally, a fast evaluation algorithm is proposed to quickly evaluate the generated toolpath. This method is applicable to most free-form mesh surfaces and has higher robustness.

<u>Main Idea:</u>
In this work, the core problem is the realization of non-uniform offset based on level-set and the calculation of non-uniform offset parameters.
Implementation of non-uniform offsetting based on level-set
The level-set function has the following properties:

$$\varphi(x,t) < 0 \; for \; x \in \Omega$$
$$\varphi(x,t) > 0 \; for \; x \in \overline{\Omega} \quad (1.1)$$
$$\varphi(x,t) = 0 \; for \; x \in \partial\Omega = \Gamma(t)$$

When the interface $\Gamma(t)$ is constantly moving, the motion interface $\Gamma(t)$ can be captured by finding where $\varphi = 0$. The topological change is implied in the change of value φ, so that the intersection problem in the path offset procedure does not need to be considered.

A typical way for constructing level-set functions is the symbolic distance function, which can be expressed as:

$$\varphi(x, t = 0) = \pm d(x) \quad (1.2)$$

$d(x)$ is from point x in the calculation domain to the closed interface $\Gamma(0)$ minimum distance. When point x is inside interface $\Gamma(0)$, the sign of $d(x)$ is negative, and when it is outside, it is positive.

The starting interface $\Gamma(0)$ is embedded in a higher one-dimensional level-set function φ as a zero level-set. Then, the interface motion problem is turned into an initial value problem, which uses the zero level-set of a time-varying function to calculate the motion of the interface $\Gamma(t)$.

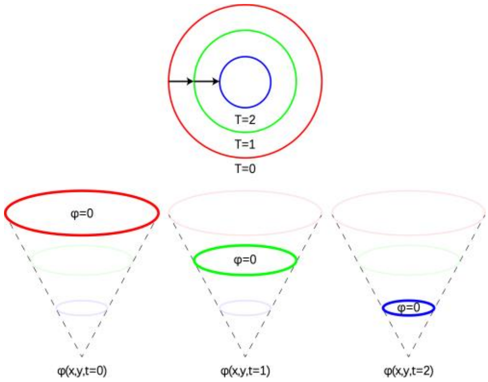

Fig. 1: Level-set function and boundary propagating.

As shown in Figure 1, the zero level-set of the function is always consistent with the motion interface $\Gamma(t)$, which can be expressed as:

$$\varphi(x(t), t) = 0 \quad (1.3)$$

through the chain rule, it can be calculated as:

$$\varphi_t + \nabla\varphi(x(t), t) \cdot x'(t) = 0 \quad (1.4)$$

if v_N is the velocity in the normal direction, then:

$$x'(t) \cdot n = v_N \quad (1.5)$$
$$n = \frac{\nabla\varphi}{|\nabla\varphi|} \quad (1.6)$$

the motion equation of φ is:

$$\frac{\partial\varphi}{\partial t} + v_N|\nabla\varphi| = 0 \quad (1.7)$$

specify conditions:

$$\varphi(x, t = 0) = 0 \quad (1.8)$$

This is the level-set equation introduced by S. Osher and J. A. Sethian [9]. When the normal velocity v_N is only a function related to $\vec{x}$, t and $\nabla\varphi$, the Equation (1.7) is a standard Hamilton-Jacobi equation. By solving the Hamilton-Jacobi equation, the value of $\varphi(x(t), t)$ at any moment can be calculated, and the set of x at $\varphi(x(t), t) = 0$ can be contoured to a new toolpath.

Therefore, the non-uniform offset based on level-set can be achieved by giving different normal velocities v_N to point x in the calculation domain.

Calculation of Non-uniform Offset Parameter

The iso-scallop toolpath can be constructed in a 2D plane by setting the normal velocity v_N of each point in the calculation domain to the line spacing l_{side} that satisfies the scallop height constraint, which can be calculated directly using the equation $l_{side} = \sqrt{8hr}$, h is the preset maximum scallop height and r is the tool radius. Therefore, the normal velocity v_N in Equation (1.7) can be defined as:

$$v_N = l_{side} \tag{1.9}$$

In this paper, the conformal mapping method of mesh surface is used to map a 3D mesh surface into a 2D mesh plane. The 2D mesh will deform during this process, so the 2D line spacing l_{side} cannot be calculated directly. Therefore, the mapping stretch factor σ_θ^t needs to be introduced to calculate the 2D line spacing:

$$l_{side} = L_{side}/\sigma_\theta^t \tag{1.10}$$

where:

$$L_{side} = \sqrt{8hr\rho_{side}/(\rho_{side} \pm r)} \tag{1.11}$$

L_{side} is 3D line spacing, . When the point x is not in the closed interface $\Gamma(0)$, there is no mapping stretch deformation: $\sigma_\theta^t = 1$, and ρ_{side} is infinite, so $l_{side} = \sqrt{8hr}$.

To sum up, v_N can be expressed as:

$$\begin{aligned} v_N &= \sqrt{8hr\rho_{side}/(\rho_{side} \pm r)}/\sigma_\theta^t \quad & for\ d(x) \le 0 \\ v_N &= \sqrt{8hr} \quad & for\ d(x) > 0 \end{aligned} \tag{1.12}$$

the mapping stretch coefficient σ_θ^t of anisotropy in triangle Δt can be expressed from Reference [2] as:

$$\sigma_\theta^t = \|w'\| = \sqrt{(\sigma_1^t \cos\ \theta)^2 + (\sigma_2^t \sin\ \theta)^2} \tag{1.13}$$

where θ is the angle between w and w_1, w is the direction of 2D line spacing, w_1 can be calculated from Reference [2]. This paper uses the level-set method to replace the traditional 2D PS-curves offset method, so it is impossible to accurately calculate the direction of 2D line spacing. The simplified algorithm is used to approximate the direction of w, when calculating $d(x)$ in Equation (1.2), taking the direction of the shortest distance from point x in the calculation domain to the closed interface $\Gamma(0)$ as the 2D line spacing direction w, so the value of θ can be calculated quickly. The simulation results show that this simplified method will not have a great impact on the toolpath accuracy.

Now v_N in Equation (1.12) can be determined, and then the algorithm needs to be verified with models.

Case study

This paper uses the MATLAB level-set toolbox written by Ian M. Mitchell to calculate the level-set part and uses a typical half vase model to verify the proposed toolpath generation method.

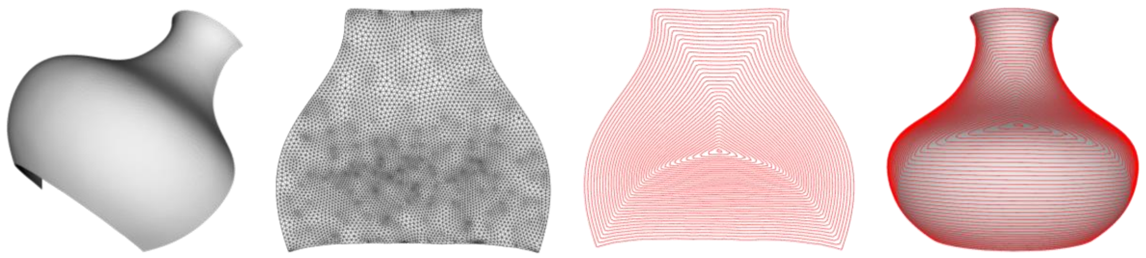

Fig. 2: Toolpath generation for the vase model: (a) 3D mesh surface model, (b) 2D planar mesh, (c) non-uniform offset path on the parametric domain, and (d) iso-scallop toolpath.

Firstly, the conformal mapping algorithm is used to map the 3D mesh in Figure 2(a) to a planar mesh (Figure 2(b)), then its area is discretized into a series of discrete points. The initialization data matrix can be calculated from Equation (1.2) as:

$$Data_0 = \begin{bmatrix} \varphi(x_{(0,0)}, t=0) & \cdots & \varphi(x_{(0,n)}, t=0) \\ \vdots & \ddots & \vdots \\ \varphi(x_{(m,0)}, t=0) & \cdots & \varphi(x_{(m,n)}, t=0) \end{bmatrix} \tag{1.14}$$

Where m is the number of discrete points in the x-axis direction, n is the number of discrete points in the y-axis direction. Through Equation (1.12), let $h = 0.3mm$ and $r = 3mm$, the normal velocity matrix can be calculated as:

$$V_{\text{normal}} = \begin{bmatrix} v_{N(0,0)} & \cdots & v_{N(0,n)} \\ \vdots & \ddots & \vdots \\ v_{N(m,0)} & \cdots & v_{N(m,n)} \end{bmatrix} \tag{1.15}$$

Take $Data_0$ and V_{normal} as input parameters and use level-set toolbox for iterative calculation to generate the non-uniform offset toolpath on the parameter plane as shown in Figure 2(c). Finally, the 2D plane toolpath is inversely mapped to the 3D mesh to generate the final toolpath, as shown in Figure 2(d).

To sum up, the steps of toolpath generation can be summarized as follows:

- Step1: use conformal mapping to map 3D mesh surface M_{3D} to 2D mesh plane M_{2D}. Then extract boundary $\Gamma(0)$ from M_{2D}.
- Step2: calculate the symbolic distance function $\varphi(x, t = 0)$ (Equation (1.2)) based on boundary $\Gamma(0)$ to obtain the initial data matrix $Data_0$ of level-set function. At the same time, calculate the normal velocity matrix V_{normal} from Equation (1.15).
- Step3: input $Data_0$ and V_{normal} into the level-set toolbox to iteratively generate the toolpath $Path_{2\text{D}}$ in the plane of the parameter domain.
- Step4: using the inverse mapping algorithm, $Path_{2\text{D}}$ is inversely mapped to 3D space to generate the final toolpath $Path_{3\text{D}}$.

Then quickly evaluate the toolpath. The evaluation method can be described as follows:

- Definition: m is the index of single toolpath, n is the point index on the current single toolpath. M is the number of toolpaths, N is the number of points in each single toolpath.
- Start: $m = 0$, $n = 0$.
- Step1: calculate the normal curvature radius ρ, feed direction V_{feed}, normal direction V_{nor} and the opposite direction of line spacing $V_{rside} = V_{feed} \times V_{nor}$ at point p_{mn}.
- Step2: construct plane P with V_{rside} and V_{nor}.
- Step3: point p_{mn}' is calculated by intersection of plane P and adjacent toolpath. The distance from point p_{mn} to point p_{mn}' is line spacing L_{side}.
- Step4: calculate $h = \frac{\rho_{side} \pm r}{8 \cdot r \cdot \rho_{side}} \cdot {L_{side}}^2$ from Equation (1.11).
- Step5: if $n < N$, $n = n + 1$, jump to step 1. Otherwise, skip to step6.
- Step6: if $m < M$, $m = m + 1$, jump to step 1. Otherwise, end the calculation.

The Scallop height distribution is shown in Figure 3, and the specific evaluation results are shown in Table 1.

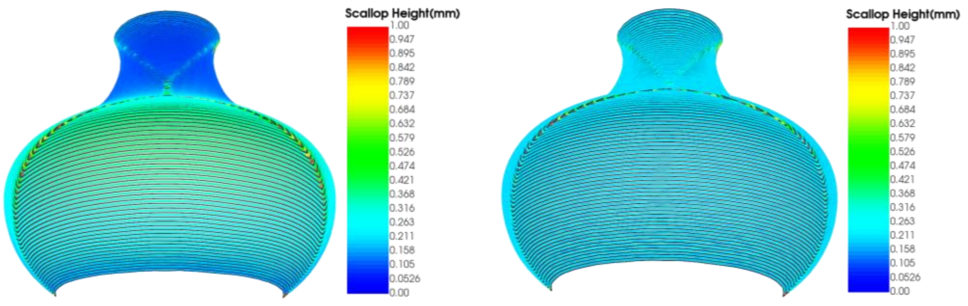

Fig. 3: Scallop height distribution of vase model: (a) uniform offset, (b) non-uniform offset.

Code	h	r	$\bar{h}$

a	0.2	2	0.221099
b	0.2	2	0.204804

Tab. 1: Results of toolpaths.

Where $\bar{h}$ is the average scallop height, it can be seen from the scallop height distribution diagram that the level-set non-uniform offset method proposed in this paper can generate iso-scallop height toolpath with uniform scallop height distribution, so it is effective.

Conclusions:

In the previous research of our team, the iso-scallop toolpath planning is realized in the 2D parameter domain by using the mesh surface conformal parameterization technology, which greatly simplifies the relevant geometric calculation. However, it is necessary to continuously improve the 2D non-uniform offset algorithm to apply to different mesh surfaces, toolpath parameters and tool parameters. Based on the previous research, this paper proposes a non-uniform offset algorithm based on level-set, which can completely avoid the intersection problems in the process of 2D non-uniform offset and greatly improve the robustness of toolpath generation algorithm. More importantly, the algorithm is very simple, and the accuracy is controllable. Theoretically, by increasing the density of discrete points, the toolpath accuracy can be continuously improved, but this will increase the computational cost. How to select the appropriate discrete point density under the requirement of toolpath accuracy is one of the subsequent problems to be solved.

In theory, this method can be used for the multi-boundary model (with internal holes) and an iso-scallop toolpath can be generated. However, since the current toolpath evaluation algorithm cannot be applied to the multi-boundary model, this paper does not give this kind of model test cases. The toolpath evaluation algorithm is also one of the problems that need to be optimized later. All the above issues are currently being solved by the author team.

Acknowledgments:

This work was supported by the Key-Area Research and Development Program of Guangdong Province, China [grant number 2021B0101190002].

References:

[1] B.K. Choi; S.C. Park.: A pair-wise offset algorithm for 2D point-sequence curve, Computer-Aided Design, 31(12), 1999, 735-745 https://doi.org/10.1016/S0010-4485(99)00060-3

[2] Chen-Yang Xu; Jing-Rong Li; Qing-Hui Wang; Guang-Hua Hu.: Contour parallel tool path planning based on conformal parameterisation utilising mapping stretch factors, International Journal of Production Research, 57(1), 2019, 1-15. https://doi.org/10.1080/00207543.2018.1456699

[3] Han Z; D. C. H. Yang; Jui-Jen Chuang.: Isophote-based ruled surface approximation of free-form surfaces and its application in NC machining, International Journal of Production Research, 39(9), 2001, 1911-1930. https://doi.org/10.1080/00207540110024663

[4] Lasemi, A.; D. Y. Xue; P. H. Gu.: Recent development in CNC machining of freeform surfaces: a state-of-the-art review, Computer-Aided Design, 42(7), 2010, 641–654. https://doi.org/10.1016/j.cad.2010.04.002

[5] Lee; S.-G.; H.-C. Kim; M.-Y. Yang.: Mesh-based Tool Path Generation for Constant Scallop-height Machining, International Journal of Advanced Manufacturing Technology 37 (1–2), 2008, 15–22. https://doi.org/10.1007/s00170-007-0943-x

[6] Lin Z; Fu J; He Y; et al.: A robust 2D point-sequence curve offset algorithm with multiple islands for contour-parallel tool path, Computer-Aided Design, 45(3), 2013, 657-670. https://doi.org/10.1016/j.cad.2012.09.002

[7] Li; H.; H.-Y. Feng.: Efficient Five-axis Machining of Free-form Surfaces with Constant Scallop Height Tool Paths, International Journal of Production Research 42 (12), 2004, 2403–2417. https://doi.org/10.1080/00207540310001652905

[8] Lu, J.; R. Cheatham; C. G. Jensen; Y. Chen; B. Bowman.: A Three-dimensional Configuration-space Method for 5-axis Tessellated Surface Machining, International Journal of Computer Integrated Manufacturing, 21 (5), 2008, 550–568. https://doi.org/10.1080/09511920701263313
[9] S. Osher; J. A. Sethian.: Fronts propagating with curvature-dependent speed: Algorithms based on Hamilton-Jacobi formulations, Journal of Computational Physics, 79 (1), 1988, 12-49. https://doi.org/10.1016/0021-9991(88)90002-2
[10] S. Osher; R. P. Fedkiw.: Level Set Methods: An Overview and Some Recent Results, Journal of Computational Physics, 169(2), 2001, 463-502. https://doi.org/10.1006/jcph.2000.6636