Title:
Computer-Aided Detection of Exact Reflection and Axisymmetry in B-rep CAD Models

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Introduction:
Geometric symmetry (hereinafter symmetry) is often introduced into mechanical parts or assemblies as it is beneficial in terms of function [13], structural analysis [15], manufacturing [11], assembling [3], reducing complexity, or aesthetics. For example, symmetrically designed parts are less prone to assembly errors and require less assembly time [3]. Further, symmetry is used in manufacturing to define the parting planes in the stamping and molding processes [11]. In Computer-Aided Engineering (CAE), symmetry is often exploited to reduce the size of the 3D model, consequently reducing the computational effort of the analysis [15]. Moreover, in technical drawing, symmetrical parts may be drawn half in section and half in outside view, reducing the no. of required views [14].

During Computer-Aided Design (CAD), there is often the need to check if symmetrically designed 3D CAD models indeed exhibit the intended type of symmetry. However, the symmetry information is seldom directly stored in the native CAD models and never in the neutral exchange file formats. An exceptional case when the symmetry information is stored in the native CAD model is when the geometric shape has been created by, for instance, a mirroring feature. One way to retrieve the symmetry information is the visual recognition by the human. However, visual recognition may be too difficult and time-intensive for complex geometric shapes or a large number of 3D CAD models. In addition, exact symmetry cannot be obtained by human visual recognition in any CAD model [11]. Hence, computer-aided *symmetry detection* (SD) is preferred, which deals with the automatic identification of the planes and axes of symmetry in 2D or 3D digital objects. The present study proposes an approach for detecting exact reflection and axisymmetry (see Fig. 1) in 3D CAD models using its Boundary Representation (B-rep) as input.

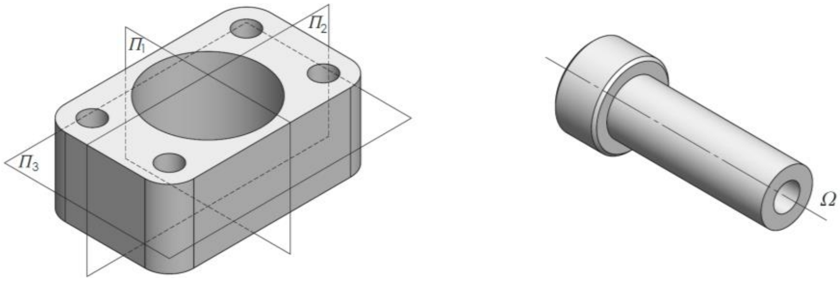

Fig. 1: A mechanical part exhibiting reflection symmetry with three planes of symmetry Π_1, Π_2, & Π_3 (left), and an axisymmetric part with one axis of symmetry Ω (right).

<u>Related work:</u>

An object is symmetrical if it is invariant under geometric transformations such as *reflection*, *rotation*, *translation*, or *combinations* of these [10]. Computer-aided SD can be classified according to different criteria: in terms of input data – *discrete* [5] vs *continuous* [10]-[12], in terms of scale – *global* [5]-[7][10]-[12] vs *partial* (or *quasi-symmetry*) [11][12][15] vs *local* [10][11], in terms of accuracy – *exact* [7][10]-[12] vs *approximate* [5][11], in terms of distance metrics – *extrinsic* [10]-[12] vs *intrinsic* [6], and in terms of transformation type – *reflection* [10]-[12], *rotational* (*axisymmetry* [10]-[12] and *cyclic* [15] symmetry), *dihedral* [1] (combination of reflection and rotational symmetry), etc. This paper focuses on exact symmetry because the SD accuracy of mechanical parts needs to be at least within the manufacturing accuracy (10^{-6} m) [11]. Further, the objective of this research is to detect global reflection and axisymmetry (see Figure 1), which are the two most common types of symmetry in mechanical engineering [8]–[11].

Generally, the approaches related to SD in 3D objects can be divided into *geometry-based* and *view-based*. The *geometry-based* approach uses the geometrical information of 3D objects as input. For that purpose, different types of 3D objects are used such as CAD models [10]–[12], cable-strut structures [1], voxel models [4], NURBS models [2], point clouds [6], etc. The geometry-based approach address the detection of approximate [4][6] as well as exact symmetry [1][7][8][11][12]. Some of them use the local surface information [11][12] (e.g., surface normal, Gaussian curvature, etc.) and while others do not use it [1][6]. The common strategy of geometry-based techniques is to first identify a large number of candidates for the plane of symmetry (POS) or the axis of symmetry (AOS) for the given input model. The candidates are then evaluated with respect to the input geometry to determine if some of them also represent the true POS/AOS. The POS/AOS candidates are obtained by principal component analysis [6], pair matching [12], from the intrinsic surface properties [11], etc. In the *view-based* approach, the 3D object is converted into a 2D representation such as an image [5] or a projected view [9]. The view-based approach addresses the detection of approximate symmetry and is therefore inappropriate for its implementation in CAD models, where usually the goal is to detect exact symmetry.

The SD in 3D CAD models has been studied from two aspects: *feature* and *B-rep*. The first aspect uses design features, Boolean operations, and the feature (history) tree for the detection of exact reflection and axisymmetry [7][8]. However, this aspect is restricted to native CAD models and may be sensitive to the designer's bad modeling habits (e.g., redundant feature modeling, modeling of symmetric shapes using non-symmetric features, etc.). The second aspect uses geometry and topology information of the B-rep [10]-[12][15] as input, which enables the SD of native CAD models as well as neutral exchange file formats. For detecting global reflection and axisymmetry in B-rep CAD models, the study [12] used loop properties (e.g., loop area, centroid, normal, etc.) to identify identical loop pairs. The POS/AOS candidates were calculated as the resultant vector of two unit normal vectors from identical loop pairs and were ranked according to cumulative loop area and compared to extract the final POS/AOS. Another research [11] proposed a divide-and-conquer approach for detecting exact and partial global reflection and axisymmetry in B-rep CAD models, using faces as input. First, in the divide phase, the candidates for the POS/AOS were obtained through the local symmetry properties of the faces and their intersections. Then, in the conquer phase, the local symmetry properties were propagated to the global level by matching coincident local candidates into global POS/AOS. To reduce the meshing complexity in CAE, the study in [15] proposed an approach for detecting cyclic regions in quasi-axisymmetric B-rep CAD models using a manually assigned AOS as input. Further, in [10], a graph-based approach was used to extract multi-scale (i.e., at different geometric scales) symmetric regions and extract symmetry relations among these regions. The proposed approach addressed exact reflection, rotational, and translational symmetry. Generally, the proposed SD approaches related to B-rep CAD models have two main drawbacks: (1) they are computationally complex, mainly due to the high number of POS/AOS candidates, and are therefore not suitable for practical application, and (2) they are restricted to analytical geometry, i.e., up to five basic types of analytical surfaces: plane, cylinder, cone, sphere, and torus (Fig. 2). The present paper introduces an approach that addresses both drawbacks. When it comes to state-of-the-art CAD systems, to our knowledge, only one of them offers a tool for SD. The Symmetry check tool [16] examines the existence of reflection symmetry in a part or assembly and identifies symmetrical, asymmetrical, and unique faces by different coloring. However, the tool has several drawbacks: only reflection symmetry can be checked, the POS candidate needs to be manually assigned by the user, and it is not possible to check more than one POS candidate simultaneously.

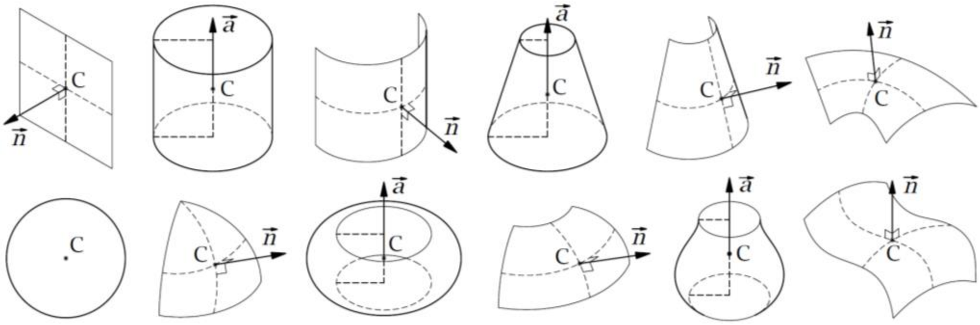

Fig. 2: Different types of surfaces (from top left to bottom right): (a) Plane (b) Cylinder, (c) Partial cylinder, (d) Cone, (e) Partial cone, (f) Blend surface, (g) Sphere, (h) Partial sphere, (i) Torus, (j) Partial torus, (k) Surface of revolution, and (l) B-spline surface.

A Novel Approach for Computer-Aided Symmetry Detection:

The novel SD incorporates four steps: STEP 1 – *Identification* of the POS/AOS candidates, STEP 2 – *Classification* of B-rep faces, STEP 3 – *Evaluation* of classified faces with respect to the POS/AOS candidates, and STEP 4 – *Visualization* of the detected POS/AOS in the 3D modeling space.

The main goal of STEP 1 is to identify the POS/AOS candidates. Mechanical parts usually have up to three reflection POS and one AOS (Fig. 1). If the 3D model exhibits exact symmetry, then the POS/AOS must pass through its center of gravity (COG) [12], which can be exploited for exact SD in CAD models. For uniform density throughout the object, which is most often the case for solid models, the center of mass and center of gravity corresponds, in fact, to the volume centroid. The COG is a standard mass property available in state-of-the-art CAD systems. Although the POS/AOS must pass through the 3D model's COG, their orientation will depend on the geometric shape of the 3D model. Hence, to get the orientation of the POS/AOS candidates, the present approach takes advantage of two other properties: (a) if an object exhibits exact reflection symmetry, then the direction normal to the plane of symmetry is a principal axis, and (b) if an object exhibits exact axisymmetry, then the axis of symmetry is a principal axis. This means that in case of exact symmetry, the POS/AOS will be aligned with the 3D model's principal axes of inertia. The 3D model's principal axis of inertia is also a standard mass property available in state-of-the-art CAD systems. Finally, the candidates for the POS/AOS are defined by the plane and line equation:

$$a\,(x - x_C) + b\,(y - y_C) + c\,(z - z_C) = 0 \tag{2.1}$$

$$(x - x_C)/a = (y - y_C)/b = (z - z_C)/c = 0 \tag{2.2}$$

where the variables a, b, and c represent the components of the principal axis, while x_C, y_C, and z_C are the coordinates of the COG. Since there are three candidates for the POS and AOS there will also be three equations of (2.7) and (2.8).

In STEP 2, each face of the B-rep model is classified based on the type of its underlying surface (see Fig. 1): plane, cylinder, partial cylinder, cone, partial cone, sphere, partial sphere, torus, partial torus, blend surface, surface of revolution, and B-spline surface (Fig. 2). Each face in the model is marked with a unique name (e.g., planes with PL1, PL2, PL3,…, cylinders with CY1, CY2, CY3,…, cones with CO1, CO2, CO3, …, etc.), which enables their tracking and accessibility at any time. In addition, the faces properties such as surface area, perimeter, face centroid, face normal, edge count, loop count, vertex count, etc., are also retrieved to be used later in the evaluation step for pairwise comparison of faces. If the 3D model's COG is not coincident with the origin of the coordinate system, the 3D model will be mathematical translated into it. Finally, all grouped faces and their corresponding properties are stored in the database for the next step.

In STEP 3, the classified faces are evaluated with respect to the POS/AOS candidates. Two evaluation procedures are used for that: one for *reflection* and the other for *axisymmetry*. The *evaluation procedure* for *reflection symmetry* relies on pairwise comparison, where all faces from the same group are mutually compared based on their properties to identify symmetric face pairs. For that purpose, the *centroid vector* and the *unit normal vector* or the *unit axis vector* are created for each face (see Figure 2). The *centroid vector* is defined by the initial point (the COG) and the terminal point (the face centroid). The *unit normal vector* or *unit axis vector* are extracted at the face centroid. The face centroid *C* is the geometric center of the underlying surface (Fig. 2). Two faces are reflective symmetric if they fulfill the following criteria: *equality*, *equidistance*, and *direction*. The fulfillment of the *equality* criterion means that two faces have the same values of the following properties: surface area, perimeter, number of edges, loops, and no. of vertices. The *equidistance* criterion is satisfied if two face centroids are equally distanced from the POS, i.e., the magnitudes of their centroid vectors are equal. In addition, the resultant vector of the two centroid vectors is calculated, and the component normal to the POS candidate must be zero. The *direction* criterion is met when two corresponding face unit normal vectors have opposite directions with respect to the POS candidate. All faces which do not belong to some symmetry pair must satisfy the condition that their centroid is coincident with the POS, which is queried with Equation (2.1). Finally, a 3D model exhibits reflection symmetry with respect to one of the POS candidates if the sum of symmetric face pairs and individual faces whose centroids are coincident with the POS candidate is equal to the total number of faces *n* in the 3D model. In the *evaluation procedure* for *axisymmetry,* a face is axisymmetric with respect to the AOS candidate if it fulfills two following criteria: *coincidence* and *direction*. The *coincident* criterion is fulfilled if the face centroid lies on the AOS candidate, which is queried with Equation (2.2). The direction criterion is satisfied if the unit normal vector or the unit axis vector of the face has the same direction as the AOS candidate. Finally, the 3D model exhibits axisymmetry if the sum of all faces that fulfill the two criteria must be equal to the total number of faces in the 3D model. Finally, in the last STEP 4, if the analyzed 3D model exhibits symmetry, the corresponding POS or AOS will be visualized in the 3D modeling space, thus providing the symmetry information to the user.

Validation:

For validation purposes, the proposed SD approach was implemented into the commercial CAD system Solidworks 2020 using its Application Programming Interface (API). The reflection symmetry has been tested on 100 CAD models, while the axisymmetry has been tested on an additional 50 (Fig. 3). The type of mechanical parts subjected to testing were milled and turned parts. The share of the surfaces in the tested parts was as follows: 43.4% planes, 29.5% cylinders, 9.5% cones, 4.9% spheres, 5,9% tori, 1.5% surface of revolution, 2.9% blend surfaces, and 2.4% B-spline surfaces.

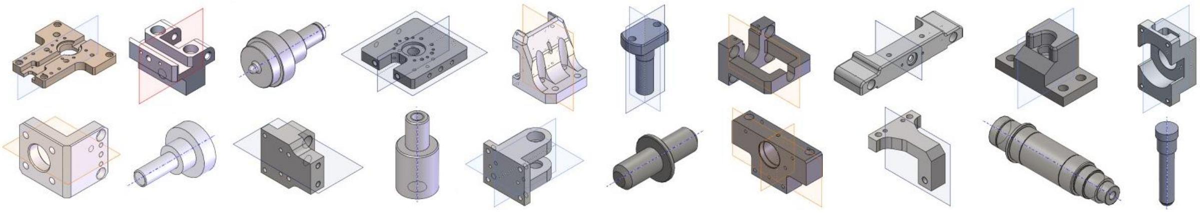

Fig. 3: An example of the tested 3D CAD models with the detected plane(s) or axis of symmetry.

The scope of the testing was to validate the accuracy and computational complexity of the proposed SD approach. The testing was conducted twice – on native CAD models and neutral file format (STEP). Fig. 3 illustrates the detected POS/AOS in some of the test parts. The test results show that the SD accuracy of the POS is 97%. In only 3% of test cases, symmetries have not been recognized in mechanical parts exhibiting multiple reflective symmetries (usually more than three), which is caused if the POS is misaligned with the principal axes of inertia. On the other side, the SD accuracy for axisymmetry is 100%. The empirical computational complexity was evaluated by testing all CAD models on the same hardware, and the symmetry detection running time (in seconds) was measured for each CAD model. Based on that, the empirically computational complexity was evaluated to O(*n*), where *n* is the number of input faces.

Conclusion and Future Work:

This paper proposes an approach for computer-aided SD of exact global reflection and axisymmetry in B-rep CAD models. Our proposed SD approach is not restricted to only 5 analytical surfaces (plane, cylinder, cone, sphere, & torus) like in the prior research, but it also considers other numerical surfaces such as blend surface, B-spline surface, and surface of revolution. During validation, the proposed SD approach confirmed that it enables accurate and computationally efficient detection of the POS/AOS. The future research will be focused on the following points: (i) extending the testing to a larger number of CAD models and other types of mechanical parts (e.g., sheet metal, forged, cast, etc.), (ii) extending the number of the candidates for the POS as in some cases mechanical parts can have more than three POS, and (iii) extending the approach to cyclic and partial symmetry.

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