Title:
Generation of Lattice Structures with Convolution Surface

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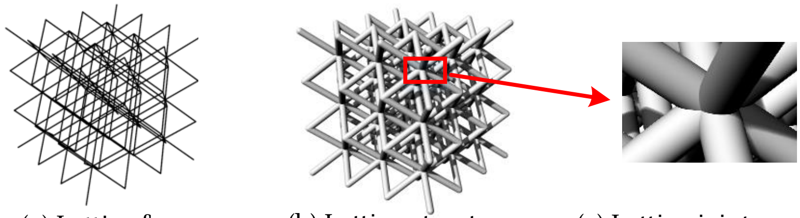
Introduction:
Lattice structure is a type of cellular structure which is made of interconnected struts with a certain periodic arrangement. By varying the topology of lattice unit cell as well as its relative density, lattice structures can achieve a wide range of physical properties for a broad spectrum of applications [1]. To further improve the performance of lattice structures, several design methods [3,7,9-11,13] for lattice structures have been developed recently. Among them, design parameters including cell topology [9], cell shape [3], relative density/porosity distribution [11], base materials [10], lattice orientations [13], and the generation of conformal lattice structures [7] have been investigated to further improve the functional performance of designed lattice structures.



(a) Lattice frame (b) Lattice structures (c) Lattice joints generated by Boolean operation

Fig. 1: Lattice structure and its joint.

Besides these design parameters, lattice joints (shown in Fig. *1*(c)) which connect different lattice struts also have a significant effect on the mechanical properties of designed lattice structures [8]. However, currently there is little research to investigate how to improve the mechanical properties of lattice structures by controlling the shape of lattice joints. Simple solid Boolean operations among lattice struts have been widely used to model the shape of lattice joints, as shown in Fig. *1*(c). The sharp edges of these joints may cause stress concentrations and limit the further improvement of the mechanical performance of lattice structures. In this paper, a new lattice generation method is developed based on convolution surfaces. The developed method can generate smooth and crease free joints with controllable size for a given relative density. In this abstract, the main idea and concepts used in the proposed method will be introduced. Based on the proposed method, lattice structures with convolution joints are generated. The effect of joint size on the effective elastic modulus of generated lattice structures will be discussed. A short conclusion will be drawn at the end of this abstract.

Main Idea:

Basic Concepts

In this subsection, two important concepts used in the proposed lattice generation method are introduced. They are lattice frame and convolution surface. The concept of lattice frame is defined to represent the skeleton of a lattice structure. The relation between a lattice structure and its frame is illustrated in Fig. *1*. To mathematically define the frame of a lattice structure, the unidirectional graph is used which can be expressed as:

$$G_s = (P, E) \tag{2.1}$$

where P is a set of points represent the nodes of lattice frame. E represents the set of lattice struts. The element e_i defined in E can be expressed as:

$$e_i = (p_j, p_l), \quad p_j, p_l \in P, j \neq l \tag{2.2}$$

In this paper, the lattice frame is considered as the input of the proposed method. It can be generated by the previous developed method [9] once the lattice cell topology and unit cell size have been decided. The generated lattice frame will be considered as the skeleton for the convolution surface used in this paper. The concept of convolution surface has been first proposed by Bloomenthal and Shoemake [2]; it is an implicit surface which can be generally defined by the following equations:

$$S = \{p | F(p) - t = 0, p \in R^3\} \tag{2.3}$$

$$F(p) = \int_M g(q) f(p-q) dM = (f \otimes g)(p) \tag{2.4}$$

where S represents the convolution surface. t is the constant iso-value. F is the scalar field defined as the convolution between two functions f and g along the skeleton M. q is the point on the skeleton M. In Eqn. (2.4), g is the geometric function which can be defined as:

$$g(p) = \begin{cases} 1, p \in M \\ 0, p \notin M \end{cases} \tag{2.5}$$

As to function f used in Eqn. (2.4), it can also be called as kernel function. In this research, Gaussian function is chosen as the kernel function for the convolution surface of lattice structures, and its formulation is provided below

$$f(p) = ae^{-\frac{||p||^2}{2\sigma^2}} \tag{2.6}$$

where a and σ are two parameters of the Gaussian kernel function. The value of parameter a is fixed and sets to 1 in this research. Parameter σ controls the size of lattice joint and its value can be changed by designers. Comparing to other lattice generation methods, the convolution surface used in this paper has some unique advantages. Firstly, it can blend the surface at the connection of skeleton without creases and sharp corners. Secondly, the superposition property of convolution surfaces enables the modeling of lattice struts in a parallel manner. Moreover, the size of lattice joint is also easy to be controlled by changing the parameter σ defined in the convolution surface.

Working flow of developed lattice generation method

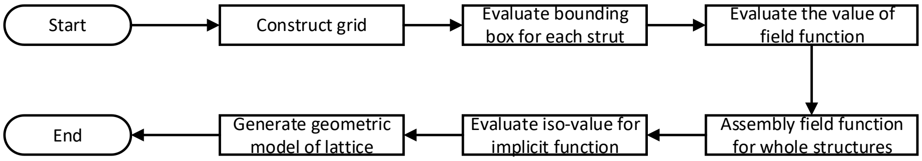


Fig. 2: General working flow of the proposed design method.

The general working flow of the proposed lattice generation method is shown in Fig. 2. The entire process can be divided into six steps. Among these six steps, step 2 and step 3 can run parallelly for each strut, which significantly improves the efficiency of developed algorithm. The detailed techniques used in each step will be carefully discussed in the following.

In the first step, the bounding box of input lattice frame will be generated first. Then, a structured grid will be constructed inside the bounding box of the lattice frame. This grid will be used to describe the convolution field F generated for a lattice structure.

In the second step, sub-domains for all the struts defined in the lattice frame will be generated. The constructed sub-domain should contain all the grid points inside its corresponding lattice strut e_i. A specific algorithm is developed in this paper to generate such sub-domain based on the estimated diameter of each lattice strut.

In the third step, the value of convolution field function F will be evaluated for the grid points in each sub-domain respectively. For the sub-domain D_i, only its correspond strut e_i is considered as the skeleton for the convolution field function F defined in Eqn. (2.4). For a specific grid point p_i , its field function value for strut e_j can be calculated by the following equation:

$$F_j(p_i) = \begin{cases} \int_{e_j} g(q)\, f(p_i - q) de_j, p_i \in D_j \\ 0, p_i \notin D_j \end{cases} \tag{2.7}$$

where g is the geometric function for e_j defined in Eqn. (2.5), f is kernel function defined in Eqn. (2.6). In this step, the parameter σ of kernel function can be changed by designers to further control the size of lattice joint.

In the fourth step, the calculated field functions for all struts can be assembled to obtain the field function for the entire lattice structure. Specifically, for each grid point p_i defined in the bounding box of the lattice structure, its field function value can be calculated based on the following equation

$$F(p_i) = \sum_{j=1}^{j=n} F_j(p_i) \tag{2.8}$$

where $F_j(p_i)$ is the convolution field function defined for the strut e_j and n represents the total number of struts. Based on Eqn. (2.8), the convolution field of the entire lattice can be obtained.

The fifth step is to evaluate the iso-value of convolution field F for a given relative density. To achieve this purpose, the following equations need to be solved.

$$\rho = \frac{v(t)}{v_l} = \frac{\int_V h(t,p) dV}{v_l} \tag{2.9}$$

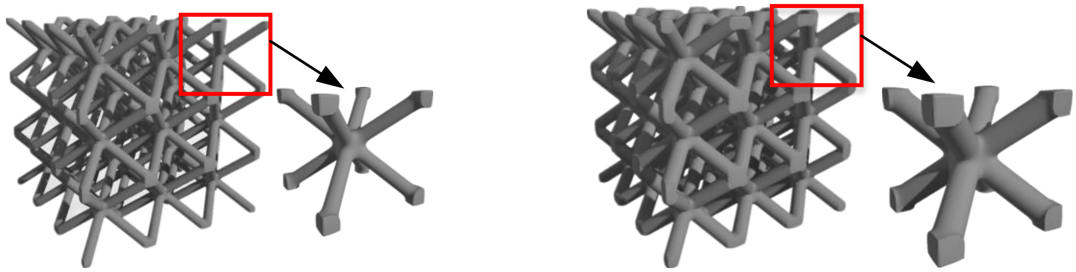
$$h(t,p) = \begin{cases} 1, F(p) - t \geq 0 \\ 0, F(p) - t < 0 \end{cases} \tag{2.10}$$

where ρ is the target relative density of lattice structures, v_l is the volume of the bounding box of lattice frame. $v(t)$ represents the volume of the generated lattice structure and it can be calculated by volumetric integral of function h defined in Eqn. (2.10) over the bounding box of lattice frame. The analytical solution of Eqn. (2.9) is difficult to obtain, since there is no close-form formulation for field function F . A numerical method is developed in this research to solve Eqn. (2.9) for the iso-value to achieve the desired relative density.

In the last step, the iso-surface of convolution field F is generated by the marching cube algorithm [5]. The surface model can be used for visualization as well as manufacturing processes. For those additive manufacturing processes which can directly take the slice image as input, the convolution field can be directly converted to the slice images for manufacturing. It further reduces the time for model conversion.

Results and discussion

The developed lattice structure generation method has been implemented based on the commercial CAD software, Siemens NX 12. To validate the proposed method, the lattice frame shown in Fig. *1* (a) with X shape unit cells has been used as the input. The generated lattice structures with convolution surface joints are shown in Fig. 3. To investigate the effects of the joints on the effective elastic modulus of generated lattice structures, the asymptotic homogenization method is used [4]. During the homogenization process, the elastic modulus (1.6 GPa) and Poisson's ratio (0.38) of Form 2 resin [6] is used. Numerical homogenization results for two different types of lattice joints are summarized in Tab. 1. By comparing the effective elastic moduli of lattice structures with different joints, it can be concluded that the lattice structure generated by the proposed method can significantly increase its elastic modulus without increasing its weight. For both relative densities studied in this paper, the proposed generation method can increase the effective elastic modulus of lattice structures more than 20% compared to the lattice with joints generated by conventional Boolean operation.



(a) relative density equal to 0.1 (a) relative density equal to 0.2

Fig. 3: Generated lattice structures with convolution surface joints.

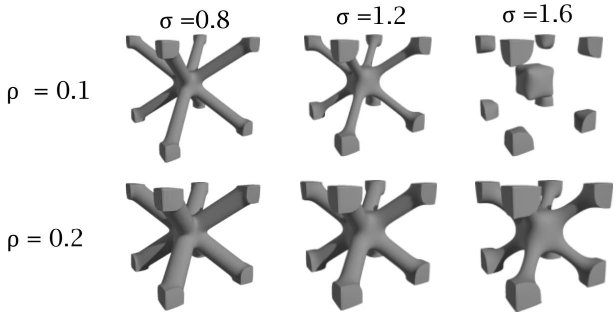


Fig. 4: Effect of σ on the size of lattice joints.

Type of joints	*ρ =0.1*	*ρ =0.2*
Boolean joints	3.0 MPa	16.5 MPa
Convolution joints	3.7 MPa	20.44 MPa

Tab. 1: Effective elastic modulus of "X" shape lattice with different joints and relative density.

Relative density	*σ =0.8*	*σ =1.2*	*σ =1.4*	*σ =1.6*
0.1	3.7 MPa	3.2 MPa	1.4 MPa	0 MPa
0.2	20.44 MPa	23.19 MPa	24.0 MPa	21.7 MPa

Tab. 2: Effective elastic modulus of "X" shape lattice with different joint parameter.

To further investigate the effect of lattice joint's size on its elastic modulus, the parameter σ defined in the kernel function given in Eqn. (2.6) has been changed to obtain the lattice structures with different joints. The effect of σ on the size of lattice joint is shown in Fig. 4. It is obvious that when σ increases, more material is moved from the lattice struts to their joints. For a low relative density lattice, large σ will make struts disconnected (see σ=1.6 in Fig. *4*), since the joint takes too much material and there is not enough material for lattice struts. To further investigate the effects of parameters σ, numerical asymptotic homogenization algorithm was applied to evaluate the effective elastic moduli of generated lattice structures with different σ. The results are summarized in Tab. 2. From this table, it can be concluded that the joint parameter σ also has a significant effect on the elastic modulus of lattice structures. For the X shape lattice with 0.1 relative density, its effective elastic modulus decreases when the parameter of lattice joint increases. However, the X shape lattice with 0.2 relative density shows a different trend. When σ increases, the effective elastic modulus rises first, and then descends. From this result, it can be inferred that there must be an optimal σ for a given relative density of lattice. The value of optimal σ is different for different relative densities. Designers can further tune the value of σ to achieve the desired effective properties for their applications.

Conclusions:

In this paper, a lattice generation method of convolution joints for a specific relative density is proposed. The design process of developed method can be divided into six steps. The algorithms and techniques used in each design step are carefully introduced. Comparing to the lattice structures generated by conventional Boolean operation, the generated convolution lattice joints has three unique merits. Firstly, the generated joints with convolution surfaces are smooth without ceases and sharp corners, which can avoid stress concentrations. Secondly, if an appropriate joint parameter value is selected, the generated lattice structures with convolution joints can achieve better effective elastic modulus comparing to conventional lattice with Boolean joints. Thirdly, the joint parameter can be used by designers to further control the effective properties of lattice structures with given relative density and cell topology. This unique advantage is helpful for designers to achieve the desired effective elastic modulus. An important application of this advantage is that of porous implants when the relative density and cell topology of cells are fixed for osseointegration purpose. Moreover, in those design scenarios where the stiffest structure is needed, an optimization algorithm can be applied to obtain the optimal joint parameter. In general, the developed lattice generation method provides a foundation to further improve the performance of lattice structures by considering the effects of their joints.

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