Title:
Entropy Assessment of Symmetry in Tessellation Design Based on Information Theory

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Introduction:
The tessellation design of building facade is an important work of architect: utilizing the simple unit and particular symmetrical rules to form a symmetrical pattern which is regular and aesthetic, as shown in Fig. 1. Group theory is a mathematical tool for studying symmetrical patterns. Using a symmetry group to analyze a pattern can generate a variety of different symmetrical patterns by finite unit pattern.

Fig. 1: The symmetry patterns in tessellation of architectural facade design.

These symmetrical rules should be delivered to the constructor as procedural information to improve design efficiency and the constructability in the subsequent construction stage. In the construction stage, if the constructor needs more procedural information which is delivered from design stage, which indicate that this design is difficult for construction. On the contrary, if the constructor needs less information, which indicates that this design is relatively simple for construction. The key is uncertainty. In other words, when the uncertainty is high, it is relatively difficult for construction for constructor, vice versa. Therefore, quantification of the uncertainty that is procedural information in design stage can be used as a measure of the complexity of pattern design and the constructability of design in the construction stage [1].

Main Idea:
1. Symmetry Group and Tessellation:
Mathematicians call the collection of all the symmetry operations or motions that leave a particular geometric object fixed its symmetry group [3]. These identity transformations include five basic

motions: translation, mirror, rotation, sliding mirror and identity transformation. For example, a square symmetry group is the following set:

$$G=\{ I, R_{\pi/2}, R_{\pi}, R3_{\pi/2}, M_a, M_b, M_c, M_d \}$$

Where I represents a invariant transformation with on effect, that is, rotation of 360° or 0° . $R_{\pi/2}$, R_{π} and $R_{3\pi/2}$ respectively represent the rotation of 90° , 180° and 270° . M_a, M_b, M_c and M_d represent the mirror images with a, b, c and d axes, respectively, as shown in Fig. 2.

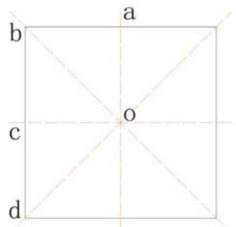

Fig. 2: The symmetry in a square.

According to Syed Jan Abas, a two-dimension repeating pattern is generated by a unit motif along a grid system which is generated by two sets of parallel lines [2]. From left to right, the upper diagram of Fig. 3 describes that the template motif within two sets of parallel lines, that is unit cell, generates the basic pattern unit motif within dash line, according to the five kinds of transformation mentioned above. The different unit cell should form different grid systems, which become the grid of displacement symmetry behind the two-dimensional repeating pattern. Then, unit motif will be copied into complex repeating pattern along the grid system, as shown in the below diagram of
Fig. 3. According to Syed Jan Abas, there are five types of grid system which are oblique parallelograms with unequal edges, rectangles, diamonds without 60° angles, diamonds with 60° angles and square, as shown in

Fig. 3. Different grid systems have their own symmetrical axis and central symmetry points.

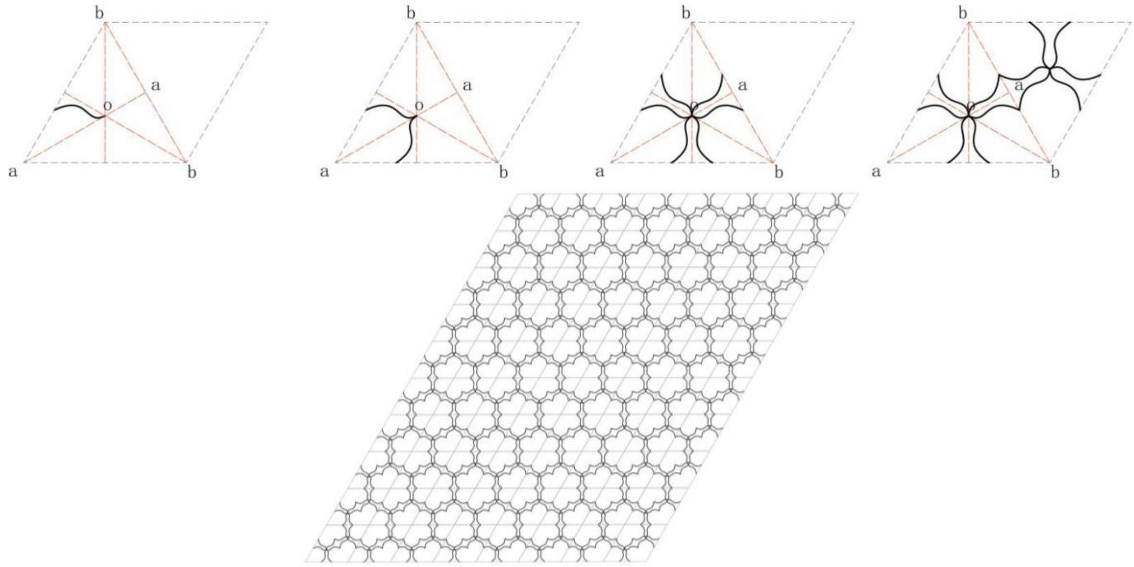

Fig. 3: The generation of unit motif and repeating pattern.

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2. The classification of 17 types of two-dimensional pattern

Based on the five different parallelogram grid systems mentioned above, we can further explore the analysis of possibility of internal symmetry by using the four equidistant transformations on the plane other than the identity transformation. According to the five grid systems, the pattern can be classified by judged that whether this pattern has mirror symmetry, rotation symmetry and sliding mirror, etc. in different situation of each level. The p6m two-dimensional pattern is taken as an example for the judgement of symmetrical classification, as shown in Fig. 4. The p6m pattern is the most common symmetrical type of two-dimensional pattern.

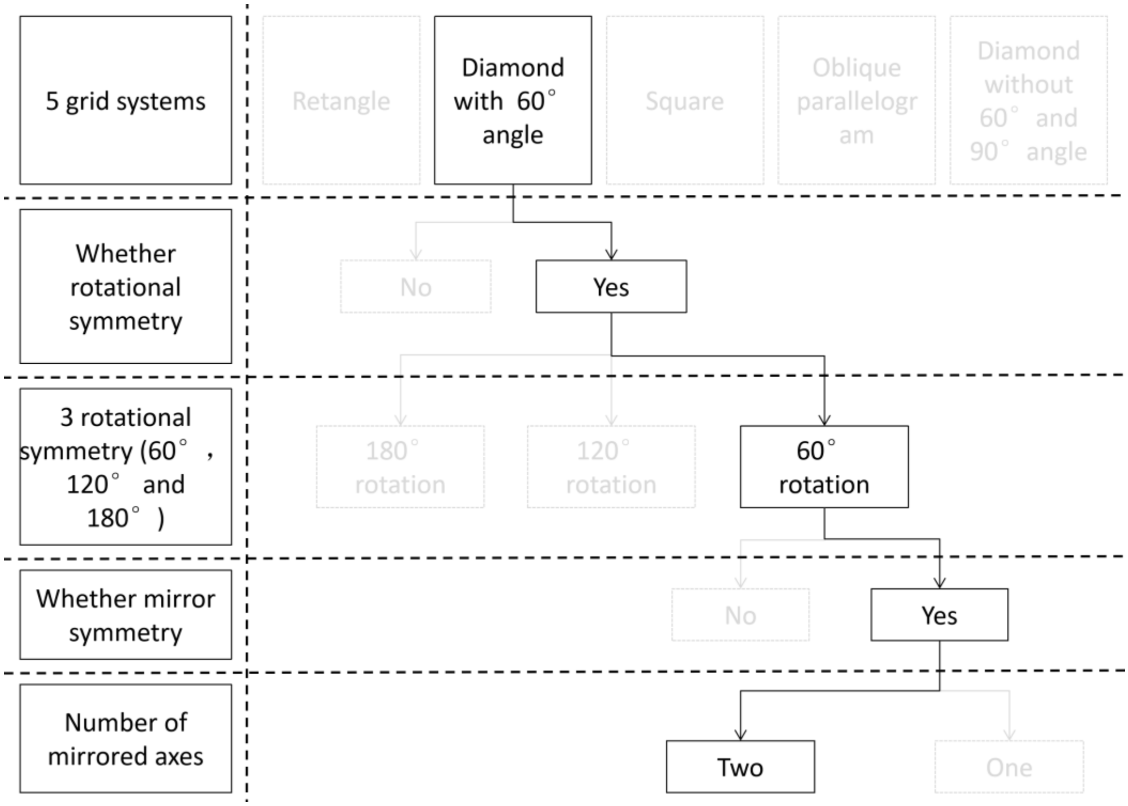

Fig. 4: The analysis of different situation in each level of p6m pattern.

3. Communication model & Procedural information

Claude Elwood Shannon proposed a mathematical communication model to measure the amount of information in a communication system which became the basis of information theory [4]. The process of architecture design communication can also be explained using Shannon`s model: As shown in Fig. 5, an architect uses drawings, models and sketches to express the design concept, that is, the process of encoding the information of design concept into messages. The constructor decodes the received message into design concept [5]. The same is true for the tessellation design. The design is encoded to symmetrical patterns by designer utilizing symmetry and delivered to constructor. Constructor decodes the pattern for construction. In this process, we can quantify information encoded. These quantified results should serve as an assessment of the complexity of the tessellation design and as assessment of constructability for subsequent construction stage.

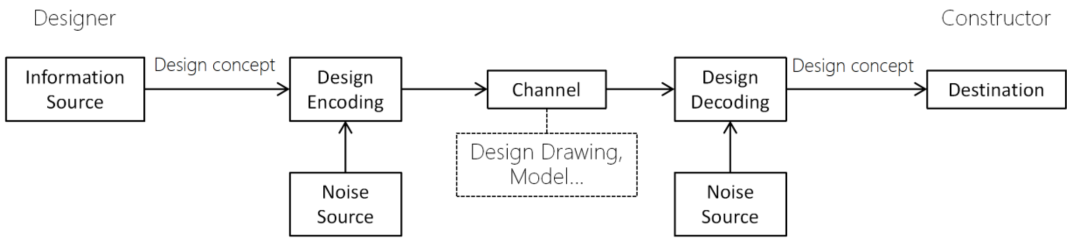

Fig. 5: The mathematical model of communication between designer and constructor.

4. Quantification and Entropy

To quantify the information, Shannon further suggested that information can be measured by calculating the uncertainty, or the unlikelihood, of the recognized situations with a logarithm function over probability [4].

$$H(X) = \sum p(X_i) \log \frac{1}{p(X_i)}$$

Where, $p(X_i)$ depends on all the possibilities that can be selected in the event.

As mentioned above, these 17 symmetrical types are generated based on the five different types of grid system and four different equidistant transformations. Also take the p6m pattern as an example to quantify the amount of information for each step of symmetry type in Fig. 4:

- Step1: select one of the five grid systems. There are five possibilities in these events.
- Step2: there are two possibilities in the selection of whether the design pattern has rotation symmetry in this level, according to different grid system.
- Step3: if it has rotation symmetry, there are three possibilities for rotation which include 180° rotation, 120° rotation and 60° rotation.
- Step4: if the designer select the 60° rotation, there are two possibilities about that whether the design pattern has mirror symmetry in this level.

Therefore, the information amount of symmetry design of p6m pattern is calculated as follow:

$$H = \frac{1}{5}\log 5 + \frac{1}{2}\log 2 + \frac{1}{3}\log 3 + \frac{1}{2}\log 2 = 0.5997$$

It is noted that, both sides of communication need a common knowledge as the basic, which is mentioned by Shannon in his article. The common knowledge is not included in the communication cost[4]. Under the premise of understanding the 17 types of symmetry pattern, the information amount can be quantified by the method mentioned above when the designer designs a certain symmetry pattern.

According to the information theory, information is the uncovering of uncertainty. By making a analogy between design process and communication process, design is actually a process of uncovering uncertainty constantly. As mentioned above, from the selection of five types of grid system to the judgement of whether the pattern has rotation symmetry, to whether the pattern has mirror symmetry, these uncertainties are reduced as the information is revealed in each level until all the uncertainties are uncovered. That is the finalization of the design.

In this design process, we can quantify the information which needs to be revealed and obtain the amount of information as a standardized and repetitive assessment indicator of the symmetry patterns design and be delivered to the construction stage as procedural information. In the subsequent construction stage, the amount of information should serve as a reference indicator in the communication between designer and constructor to improve design efficiency and constructability in subsequent construction stage.

Demonstration:
The tessellation of the architecture façade in South Fujian China is designed according to two-dimensional symmetrical patterns. The generation and quantification of four symmetrical patterns are described in this article as examples that demonstrate how the quantification of entropy facilitates an understanding of the constructability of design**Error! Reference source not found.**.

Conclusion:
Tessellation design is a common method used in architectural facade design. A variety of different symmetry patterns are generated from a simple confined unit pattern, which based on a analysis of patterns by symmetry group. Based on information theory, we should quantify the entropy of symmetry in the design to evaluate the amount of information of different symmetry patterns in the design process. The amount of information should serve as a reference indicator for designer to evaluate the complexity of two-dimension symmetry pattern design. This will guide the subsequent construction.

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