Title:
G^1 Hermite Interpolating with Discrete Log-aesthetic Curves and Surfaces

Authors:
Kazumichi Yagi, yagi.kazumichi.14@shizuoka.ac.jp, Shizuoka University
Sho Suzuki, sho.suzuki.14a@shizuoka.ac.jp, Shizuoka University
Shin Usuki, usuki@shizuoka.ac.jp, Shizuoka University
Kenjiro T. Miura, miura.kenjiro@shizuoka.ac.jp, Shizuoka University

Keywords:
log-aesthetic curve, log-aesthetic surface, discretization , G^1 Hermite interpolation

DOI: 10.14733/cadconfP.2019.207-211

Introduction:

Recently, aesthetic design which takes account of designability has become popular. In the aesthetic design, the creation of high quality curve and surface models is demanded. However, on current CAD systems, the operator must move control points by trial and error to obtain high-quality curves and surfaces. This incurs high costs and requires a great deal of expertise. Therefore, an efficient method to generate fair curves and surfaces is desirable to achieve high quality that will satisfy aesthetic requirements of customers.

Aesthetic curves were proposed by Harada et al. as curve whose logarithmic distribution diagram of curvature (LDDC) can be approximated by straight line. Miura et al. [2] derived analytical solution of the curves whose logarithmic curvature graph (LCG)- an analytical version of the LDDC is strictly given by a straight line and proposed these lines as general equations of aesthetic curves. For a given curve, we assume the arc length of the curve, the radius of curvature and slope of LCG are denoted by s and ρ and α, respectively. When $\alpha \neq 0$, one of the general equations of aesthetic curves is given by the following equation.

$$\rho^\alpha = cs + d \tag{1}$$

where, c and d are constraints. We call curves satisfying the above equations as log-aesthetic curves. Fig.1 illustrates log-aesthetic curves for various α values.

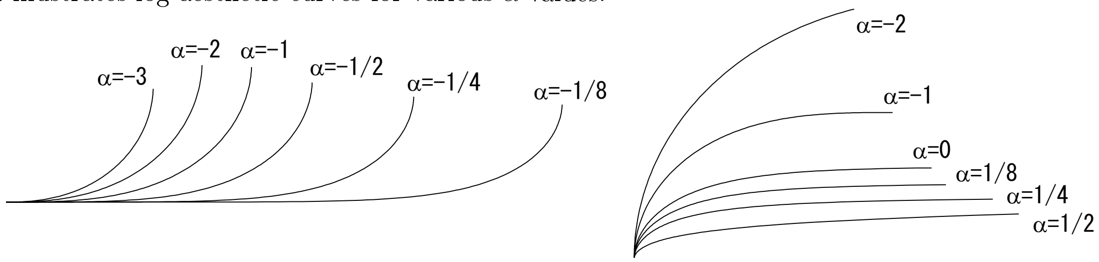

Fig. 1: Log-aesthetic curves with various α's.

Proceedings of CAD'19, Singapore, June 24-26, 2019, 207-211

As a formulation of log-aesthetic surfaces, some surface formulas besides the minimum variation log-aesthetic surface have been proposed that generate free-form surfaces by sweeping the log-aesthetic curve [1, 5].Harada et al proposed the log-aesthetic curved surface [1]. It is defined as a sweeping surface using two profile curves, which are composed of log-aesthetic curves, and one guide line composed of a non-log-aesthetic curve. Saito et al. proposed the complete log-aesthetic surface [5]. It is defined as a pure sweeping surface with two log-aesthetic curves. This formulation also uses the log-aesthetic curve as the guide line and guarantees that all parametric curves are log-aesthetic. Suzuki et al proposed a new formulation of the minimum variation log-aesthetic surface(MVLAS) for scale-invariance and Parameterization-independence [6]. However, it takes time to generate these curves and surfaces.

In this research, in order to solve the problem, focusing on discrete curves that can be expected for high speed in generation, we propose discretization of log-aesthetic plane curves based on point sequence interpolation by discrete clothoid curves, and G^1 Hermite interpolating method that generates curves from end points and the tangential direction there. In addition, we extend the method used in the curves to the surfaces.

Related work:

In plane case, Schneider et al proposed a algorithm to construct an interpolating closed discrete clothoid spline (DCS) purely based on its characteristic differential equation[4]. In the algorithm, firstly, initial polygon are specified and interpolation points are inserted between the polygon so as to interpolate between initial point sequences so that curvature change becomes monotonous. Furthermore, they extended the algorithm of planar clothoid splines to closed surfaces of arbitrary topology[4].

Euler-Lagrange equation:

The Euler-Lagrange equation in the variational problem is a partial differential equation that characterizes a functional, In this research, we transform the discrete curve and surface by this Euler-Lagrange equation. From the variational principle, the log-aesthetic curve satisfies $\rho^\alpha = cs + d$, so it is reformulated as a curve that minimizes the energy between two points in the space with arc length s on the horizontal axis and $\sigma = \rho^\alpha$ on the vertical axis[3]. Then, the functional of the log-aesthetic curve is given by Eq.2[7].

$$K_{LAC} = \int \sigma_s^2 ds \tag{2}$$

Therefore, the Euler-Lagrange equation is expressed as Eq.3.

$$\frac{d}{ds}\left(\frac{\partial \sigma_s^2}{\partial \sigma_s}\right) = \frac{d}{ds}(2\sigma_s) = 2\sigma_{ss} = 0 \tag{3}$$

Moreover

$$\frac{d}{ds}(\kappa^{-\alpha}) = -\alpha\kappa^{-(1+\alpha)}\kappa_s \tag{4}$$

$$\begin{aligned}\frac{d^2}{ds^2}(\kappa^{-\alpha}) &= \frac{d}{ds}(\alpha\kappa^{-(1+\alpha)}\kappa_s) \\ &= \alpha(1+\alpha)\kappa^{-(2+\alpha)}\kappa_s^2 - \alpha\kappa^{-(1+\alpha)}\kappa_{ss} \end{aligned} \tag{5}$$

Hence we obtain the following expression.

$$\begin{aligned}(1+\alpha)\kappa^{-(2+\alpha)}\kappa_s^2 - \kappa^{-(1+\alpha)}\kappa_{ss} &= 0 \\ (1+\alpha)\kappa_s^2 - \kappa\kappa_{ss} &= 0\end{aligned} \tag{6}$$

Proceedings of CAD'19, Singapore, June 24-26, 2019, 207-211

When $e_i = 1, 2(min, max)$ as a unit principal direction vector and $\sigma = (\rho^i)^{\alpha_i}, \sigma_i^i = \frac{d\sigma^i}{de_i}$, the functional K_{LAS} of MVLAS is defined as Eq.7 by extending the functional (Eq.2) of the log-aesthetic curve to surfaces with respect to the principal curvature [6].

$$\begin{aligned} K_{LAS} &= \int \sum_{i=1}^{2} (\sigma_i^i)^2 dA \\ &= \int \sum_{i=1}^{2} (\alpha_i (\kappa^i)^{-(1+\alpha_i)} \kappa_i^i)^2 dA \end{aligned} \tag{7}$$

On the other hand, the unit principal direction vector e_i is given by Eq.8 when the eigen vector that corresponding to e_i is (ξ_i, η_i).

$$\hat{e}_i = \frac{\partial S}{\partial u}\xi_i + \frac{\partial S}{\partial v}\eta_i \tag{8}$$

From Eq.8, the principal direction differential $\frac{d\kappa_i}{de_i}$ of the principal curvature becomes as shown in Eq.9, paying attention to the fact that the principal direction is a unit vector.

$$\frac{d\kappa^i}{de_i} = \frac{1}{\sqrt{E}}\frac{d\kappa^i}{du}\xi_i + \frac{1}{\sqrt{G}}\frac{d\kappa^i}{dv}\eta_i \tag{9}$$

Moreover, $\xi_1 = \eta_2$ are 1, $\xi_2 = \eta_1$ are 0, when put $E = G = 1$ and consider curvature line coordinates s, t. Hence Eq.7 will be following equation.

$$\begin{aligned} K_{LAS} &= \int\int \sum_{i=1}^{2} \frac{1}{g_{ii}} (\alpha_i (\kappa^i)^{-(1+\alpha_i)} \kappa_i^i)^2 \sqrt{EG - F^2} dsdt \\ &= \int\int \sum_{i=1}^{2} (\alpha_i (\kappa^i)^{-(1+\alpha_i)} \kappa_i^i)^2 dsdt \\ &= \int\int \sum_{i=1}^{2} (\sigma_i^i)^2 dsdt \end{aligned} \tag{10}$$

where, $F = 0$ because the principal directions are orthogonal to each other. From Eq.10, we obtain the following Euler-Lagrange equation.

$$\sum_{i=1}^{2} \sigma_{ii}^i = 0 \tag{11}$$

$$\sum_{i=1}^{2} \left((1+\alpha_i)(\kappa_i^i)^2 - \kappa^i \kappa_{ii}^i\right) = 0 \tag{12}$$

Schneider et al defined Discrete Clothoid Spline as a curve where the discrete curvature satisfies the following equation[4].

$$\Delta\kappa_i = \kappa_{i-1} - 2\kappa_i + \kappa_{i+1} = 0 \tag{13}$$

κ_i can be updated by following equation.

$$\kappa_i = \frac{\kappa_{i-1} + \kappa_{i+1}}{2} \tag{14}$$

Proceedings of CAD'19, Singapore, June 24-26, 2019, 207-211

Moreover, Schneider et al extended this theory of the curves to surfaces, considering a one-ring at the vertex, and updated the inner vertex of the triangular mesh by Eq.15.

$$\kappa_i = \frac{1}{6}\sum_{l=1}^{6} H_{i,l} \tag{15}$$

Likewise, the curvature of the discrete log-aesthetic surface is updated the vertex of the Quadrilateral mesh from the following expression.

$$\sum_{j=1}^{2} \sigma_i^j = \frac{1}{8}\sum_{j=1}^{2}\sum_{l=1}^{8} \sigma_{i,l}^j \tag{16}$$

Hence when $\alpha_1 = \alpha_2 = -1$

$$\sum_{j=1}^{2} \sigma_i^j = \sum_{j=1}^{2} \rho_j^{-1} = \kappa_1 + \kappa_2 = 2H \tag{17}$$

since Eq.16 is twice the average curvature, the result of the optimization agrees with the case of Schneider.

Processing procedure:

The details of curves and surfaces generation procedure are shown below.

1. In curve case, input the start point, the end point, the tangent vector at there, α, and number of subdivision. In surface case, input the surface that to determine the boundary conditions, α, and number of subdivision.
2. While keeping the boundary conditions to satisfy G^1 continuity, put the initial points.
3. In order to maintain G^1 continuity, optimize the position of the target vertex by minimizing the objective function while fixing two points from the start point and end point respectively. Implement the optimization until the convergence condition is satisfied.
4. Repeat processing for the specified number of subdivision, and output the curve (or mesh) when finished.

Results:

We show the results of G^1 hermite interpolating with log-aesthetic curve and surface in Fig.2, Fig.3 respectively. Fig.2 shows the generated log-aesthetic curve($\alpha = -1.0$) and the curvature distribution($\alpha = -1.0$). The number of subdivision is 4 and processing time is 0.35s. It can be seen from the curvature distribution that the curvature of the curve monotonically changes.Fig.3 shows the generated log-aesthetic surface, mean curvature distribution, and zebra map.The number of subdivision is 3 and processing time is 40s.

Conclusions:

In this research, we propose a G^1 Hermite interpolation method based on a discrete log-aesthetic curve aiming at speeding up curve generation based on point sequence interpolation based on discrete clothoid curve, and generated log-aesthetic plane curves. Also, by extending the method used on the curve to a surface, we propose a G^1 Hermite interpolation method based on a log-aesthetic surface, and generated log-aesthetic surfaces. In the future, we aim to speed up the algorithm and aim to implement it as a CAD plug-in.

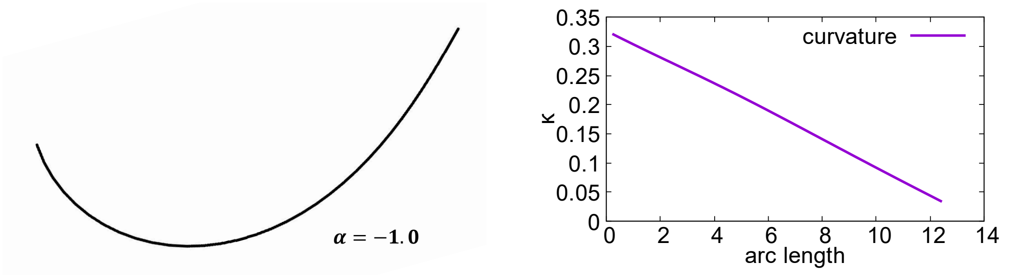

Fig. 2: G^1 Hermite interpolating with discrete log-aesthetic curve Left:generated curves Right:curvature distribution($\alpha = -1.0$).

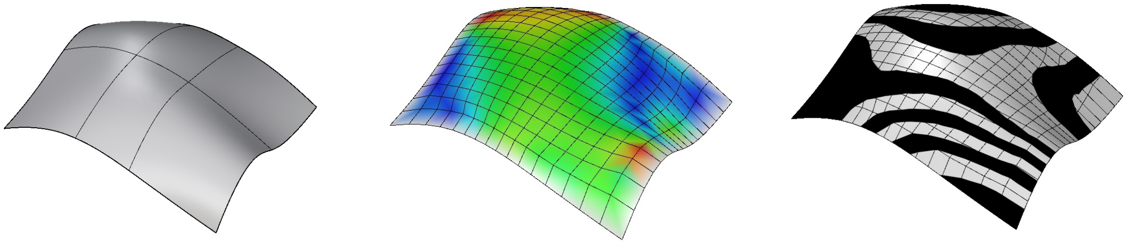

Fig. 3: G^1 Hermite interpolating with discrete log-aesthetic surface Left:input Middle:output(mean curvature distribution) Right:zebra map.

References:

[1] Inoue, J.; Harada, T.: An algorithm for generating log-aesthetic curved surfaces and the development of a curved surfaces generation system using vr, IASDR, The International Association of Societies of Design Research, Seoul, Korea, 2009, pp. 2513-2522.

[2] Miura, K.T.: A general equation of aesthetic curves and its self-affinity, Computer-Aided Design and Applications, 3(1-4), 2006, 457-464. https://doi.org/10.1080%2F16864360.2006.10738484

[3] Miura, K.T.; Shirahata, R.; Agari, S.; Usuki, S.; Gobithaasan, R.U.: Variational formulation of the log-aesthetic surface and development of discrete surface filters, Computer-Aided Design and Applications, Vol.9, No.6., 2012, pp.901-914. https://doi.org/10.3722%2Fcadaps.2012.901-914

[4] Schneider, R.; Kobbelt, L.: Discrete fairing of curves and surfaces based on linear curvature distribution, Curve and Surface Design, Saint-Malo, 1999, pp. 371-380.

[5] Shikano, K.; Saito, T.; Yoshida, N.: Complete log-aesthetic surfaces by logarithmic helical sweep, SIAM Conference on Geometric Design(GD/SPM13), 2013.

[6] Suzuki, S.; Gobithaasan, R.U.; Usuki, S.; Miura, K.T.: A new formulation of the minimum variation log-aesthetic surface for scale-invariance and parameterization-independence, Computer-Aided Design and Applications, 15(1), 2018, 611-666. https://doi.org/10.1080%2F16864360.2018.1441232

[7] Suzuki, S.; Gobithaasan, R.U.; Salvi, P.; Usuki, S.; Miura, K.T.: Minimum variation log-aesthetic surface, Asian Conference on Design and Digital Engineering (ACDDE) 2016, Jeju, Korea, Oct. 25-28, 2016.

Proceedings of CAD'19, Singapore, June 24-26, 2019, 207-211