

Title:

**Calculation and Compensation of Five-Axis Machine Tool Fixture Error**

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Introduction:

In the traditional machining process, when a fixture is fixed after positioning, the workpiece dimension must be measured after the first machining to determine whether it is within the set tolerance. If the machined workpiece exceeds the tolerance, then the error problem must be solved by repairing or adjusting the fixture and locator, which sometimes interrupts the work flow. In recent years, in response to automated production, the five-axis machine tool has become a key equipment of the manufacturing industry. Therefore, this study uses the five-axis machine tool to improve complex fixture repairs or adjustment operations. The mechanical part machining process of the five-axis machine tool can be directly corrected by the calculation, analysis, and compensation of fixture errors in order to obtain accurate mechanical part machining results.

Many studies have identified different potential fixture errors in different ways; thus, the error amount can be known. However, there is no appropriate processing method to reduce the amount/magnitude of fixture errors. Although Wan et al. [7] and Khodaygan [2-3] adjusted the locators to reduce the errors, each adjustment required recalculation, thus complicating the process. Rong et al. [4], Sánchez et al. [5], and Fallah and Arezoo [1] proposed modifying the value of the tool path to reduce errors; however, when there were numerous tool paths to be modified, there were many application problems. Furthermore, while many studies have analyzed and compensated the fixtures of two-axis turning machines or three-axis milling machines, the fixture error compensation of the five-axis machine tool has not yet been studied. Therefore, this study combines a machine tool probing system with the cosine theorem and homogeneous transformation matrix (HTM) operations to measure and calculate the errors resulting from fixture locators. The five-axis machine tool controller is set according to the calculated values for error compensation, and the tilted work plane (TWP) command of the five-axis machine tool enables the workpiece to automatically implement rotation and offset operations according to the preset value for fixture error compensation, in order to compensate the fixture error and increase the geometric dimension accuracy of the workpiece manufactured by the machine tool. The method designed in this study can effectively compensate the fixture error of fixture manufacturing and installation without adjusting the fixture, modifying the tool path, or making very high precision fixtures. Unlike existing fixture error compensation methods aimed at workpieces with specific shapes and regularly-arranged fixture locators, the method designed in this study is applicable to the machining of workpieces in complex shapes; thus, it has more extensive areas of application.

Fixture Design with 3-2-1 Layout:

The purpose of the fixture is to accurately clamp and locate the workpiece to restrict or control the six degrees-of-freedom of the workpiece in the space; the displacement in X-, Y-, and Z-directions; and the

$\alpha$ ,  $\beta$ , and  $\gamma$  rotation angles along X, Y, and Z; thus, the workpiece is unlikely to become loose during the machining process. The fixture design with the 3-2-1 layout means that the three faces of a workpiece are mutually perpendicular. The first plane XY is confined by three locators, thus constraining three degrees-of-freedom  $\alpha$ ,  $\beta$  and Z. Another plane XZ is confined by two locators, thus constraining two degrees-of-freedom Y and  $\gamma$ . The third plane YZ is confined by one locator, thus constraining the last degree-of-freedom X. Therefore, the "3-2-1" of a fixture design with a 3-2-1 layout is the number of locators on each positioning plane.

#### Fixture Error Calculation and Compensation:

This study used the position information of six locators on a fixture with the 3-2-1 layout to calculate the reference position offset and rotation angle of each axis for TWP command. The calculation procedure is described as follows.

Step 1: Define the part program coordinate frame to be coordinate frame "0," and the position information of locators in coordinate frame "0" is obtained by the machine tool probing system.

Step 2: In terms of a fixture with the 3-2-1 layout, the vectors  $\overline{AB}$  and  $\overline{AC}$  can be obtained from the coordinates  $A(A_x^0, A_y^0, A_z^0)$ ,  $B(B_x^0, B_y^0, B_z^0)$ , and  $C(C_x^0, C_y^0, C_z^0)$  of three locators on the first plane in coordinate frame "0," expressed as Eqn. (3.1), where  $A_z$ ,  $B_z$ , and  $C_z$  are the ideal locator length, as shown in Fig. 1. The normal vector  $\vec{n}$  of this plane will be orthogonal to  $\overline{AB}$  and  $\overline{AC}$ , expressed as Eqn. (3.2).

$$\overline{AB} = (A_x^0 - B_x^0, A_y^0 - B_y^0, A_z^0 - B_z^0 - (A_z - B_z)) \text{ and } \overline{AC} = (A_x^0 - C_x^0, A_y^0 - C_y^0, A_z^0 - C_z^0 - (A_z - C_z)) \quad (3.1)$$

$$\vec{n} = \overline{AB} \times \overline{AC} = (n_x^0, n_y^0, n_z^0) \quad (3.2)$$

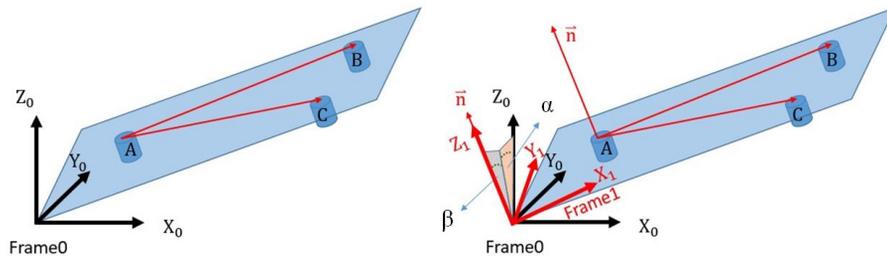


Fig. 1: Relationship between coordinate frame "0" and coordinate frame "1."

Step 3: The angle between normal vector  $\vec{n}$  and  $Z_0$  can be obtained by the cosine theorem, and the component angles  $\alpha$  and  $\beta$  of the angle in  $X_0$  and  $Y_0$  axial directions are calculated and expressed as Eqn. (3.3). The coordinate frame "1" is established by normal vector  $\vec{n}$ , and a relationship between coordinate frame "0" and coordinate frame "1" is established.

$$\alpha = -\tan^{-1}\left(\frac{n_y^0}{n_z^0}\right) \text{ and } \beta = n_x^0 \cdot \left| \frac{1}{n_x^0} \right| \cos^{-1}\left(\frac{\sqrt{(n_y^0)^2 + (n_z^0)^2}}{\sqrt{(n_x^0)^2 + (n_y^0)^2 + (n_z^0)^2}}\right) \quad (3.3)$$

Step 4: The  $Z_1$  axial direction of coordinate frame "1" is parallel to normal vector  $\vec{n}$ , and the HTM  $H_1^0$  is established by rotation angles  $\alpha$  and  $\beta$  as the rotation relationship between coordinate frame "0" and coordinate frame "1," expressed as Eqn. (3.4).

$$H_1^0 = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ \sin\alpha \sin\beta & \cos\alpha & -\cos\beta \sin\alpha & 0 \\ -\cos\alpha \sin\beta & \sin\alpha & \cos\alpha \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.4)$$

Step 5: The coordinates of two locators on the second plane in coordinate frame "1" are  $D(D_x^1, D_y^1, D_z^1)$  and  $E(E_x^1, E_y^1, E_z^1)$ ; another rotation angle  $\gamma$  can be worked out by the cosine theorem and coordinate frame "2" is established on coordinate frame "1." However, considering the locator length, it is different from the former calculation of two angles. As shown in Fig. 2, the distance between locators D and E in the Y-direction of coordinate frame "2" is  $\overline{DS}$ ,  $\overline{RE}$  is the distance between locators D and E in the X-direction of coordinate frame "1," and  $\overline{DR}$  is the distance between locators D and E in the Y-direction of coordinate frame "1." The rotation angle  $\gamma$  is the difference between  $\angle RDE$  and  $\angle SDE$ , expressed as Eqn. (3.5) and Eqn. (3.6).

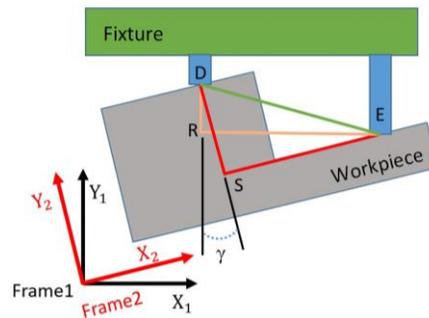


Fig. 2: Plane view of relationship between coordinate frame "1" and coordinate frame "2."

$$\angle RDE = \tan^{-1}\left(\frac{\overline{RE}}{\overline{DR}}\right) \text{ and } \angle SDE = \cos^{-1}\left(\frac{\overline{DS}}{\sqrt{(\overline{DR})^2 + (\overline{RE})^2}}\right) \quad (3.5)$$

$$\gamma = -\tan^{-1}\left(\frac{\overline{RE}}{\overline{DR}}\right) + \cos^{-1}\left(\frac{\overline{DS}}{\sqrt{(\overline{DR})^2 + (\overline{RE})^2}}\right) \quad (3.6)$$

Step 6: The HTM  $H_2^1$  of coordinate frame "2" and coordinate frame "1" is established by rotation angle  $\gamma$ , expressed as Eqn. (3.7).

$$H_2^1 = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.7)$$

Step 7:  $H_1^0$  is multiplied by  $H_2^1$  to obtain the HTM  $H_2^0$  of coordinate frame "2" and coordinate frame "0," expressed as Eqn. (3.8), where  $\alpha$ ,  $\beta$ , and  $\gamma$  are the angular deviation.

$$H_2^0 = \begin{bmatrix} \cos\beta \cos\gamma & -\cos\beta \sin\gamma & \sin\beta & 0 \\ \cos\gamma \sin\alpha \sin\beta + \cos\alpha \sin\gamma & \cos\alpha \cos\gamma - \sin\alpha \sin\beta \sin\gamma & -\cos\beta \sin\alpha & 0 \\ -\cos\alpha \cos\gamma \sin\beta + \sin\alpha \sin\gamma & \cos\gamma \sin\alpha + \cos\alpha \sin\beta \sin\gamma & \cos\alpha \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.8)$$

Step 8: The position of each locator in coordinate frame "0" is calculated by  $H_2^0$ , expressed as Eqn. (3.9), to obtain the locator position in coordinate frame "2."

$$\begin{bmatrix} A_x^2 & B_x^2 & C_x^2 & D_x^2 & E_x^2 & F_x^2 \\ A_y^2 & B_y^2 & C_y^2 & D_y^2 & E_y^2 & F_y^2 \\ A_z^2 & B_z^2 & C_z^2 & D_z^2 & E_z^2 & F_z^2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = (H_2^0)^{-1} \cdot \begin{bmatrix} A_x^0 & B_x^0 & C_x^0 & D_x^0 & E_x^0 & F_x^0 \\ A_y^0 & B_y^0 & C_y^0 & D_y^0 & E_y^0 & F_y^0 \\ A_z^0 & B_z^0 & C_z^0 & D_z^0 & E_z^0 & F_z^0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (3.9)$$

Step 9: The locator position in coordinate frame "2" is compared with the ideal locator position in a fixture with the 3-2-1 layout to obtain the position deviation vector  $d$ , expressed as Eqn. (3.10) to Eqn. (3.11); and the HTM  $H_3^0$  of coordinate frame "3" and coordinate frame "0" is established, expressed as Eqn. (3.12).

$$d_x = F_x^2 - F_x; \quad d_y = D_y^2 - D_y = E_y^2 - E_y; \quad d_z = A_z^2 - A_z = B_z^2 - B_z = C_z^2 - C_z \quad (3.10)$$

$$d = [d_x \quad d_y \quad d_z \quad 1]^T \quad (3.11)$$

$$H_3^0 = \begin{bmatrix} \cos\beta \cos\gamma & -\cos\beta \sin\gamma & \sin\beta & d_x \\ \cos\gamma \sin\alpha \sin\beta + \cos\alpha \sin\gamma & \cos\alpha \cos\gamma - \sin\alpha \sin\beta \sin\gamma & -\cos\beta \sin\alpha & d_y \\ -\cos\alpha \cos\gamma \sin\beta + \sin\alpha \sin\gamma & \cos\gamma \sin\alpha + \cos\alpha \sin\beta \sin\gamma & \cos\alpha \cos\beta & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.12)$$

Step 10: When any point on a workpiece with an ideal position is calculated by HTM, the point on the workpiece with the actual position can be obtained; thus, HTM  $H_3^0$  is the fixture error analysis result. The fixture error compensation imports the analysis result (angular deviations  $\alpha$ ,  $\beta$ , and  $\gamma$ , and offsets  $d_x$ ,  $d_y$ , and  $d_z$ ) into the reference position offset of the TWP command and the rotation angle of each axis; thus, the effect of fixture error on the geometric accuracy of workpiece dimension can be reduced.

### Experimental Results and Discussion:

A CNC five-axis machining center is combined with a FANUC Series 31i-MODEL B5 controller to implement the fixture error compensation by TWP command [6]. The experimental process of fixture error compensation proceeds as follows: locate and fix the fixture; use the Renishaw OMP400 machine tool probing system to measure the relative positions of the fixture locators and the fixture reference point; substitute the position information of each locator in the fixture error computing process; calculate the fixture error; and import the result into the TWP command. This study designs a workpiece with step and hole features, as shown in Fig. 3. The dimension of the two features is measured to determine whether there is a significant difference after compensation, and the rate of improvement is compared. The center position of the hole and hole height on the XY plane and the roundness are measured in order to confirm whether the tilted machining plane leads to an elliptical hole. The possible causes of machining plane tilt include the errors of XY angles  $\alpha$  and  $\beta$ . In addition to the step height, the Y-direction parallelism is measured to determine the error of Z-axis angle  $\gamma$ .

The feature dimension measurement results before and after compensation are shown in Tab. 1. This study used the ZEISS CONTURA G2 coordinate measuring machine to measure the workpieces before and after compensation. As shown in Tab. 1, the hole machining result shows that the position and dimension obviously improved. The Z-axis rate of improvement is 91.53%, the minimum rate of improvement is 80.47%, and the average rate of improvement is 85%. However, it is difficult to see the improvement in roundness detection as the ellipticity of the hole is low before compensation; thus, the rate of improvement is only 39.68%. In terms of the step machining result, the average rate of improvement is 56%; therefore, the fixture error can be effectively compensated by this compensation method.

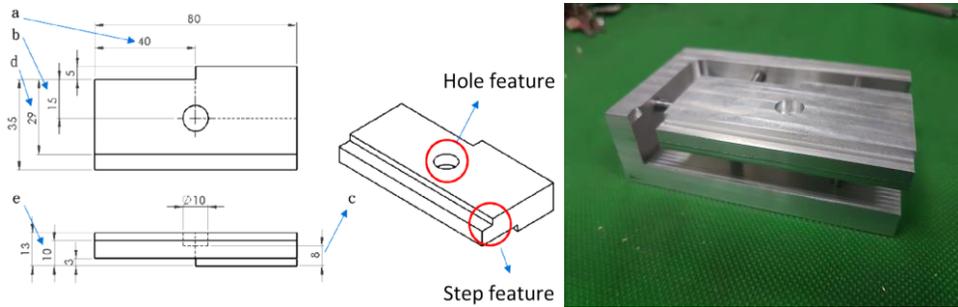


Fig. 3: Workpiece and fixture design for machining experiments.

Feature	Test item	Before compensation [mm]	Error [mm]	After compensation [mm]	Error [mm]	Rate of improvement
Hole	X-coordinate position (a)	40.1505	0.1505	39.9706	-0.0294	80.47%
	Y-coordinate position (b)	15.0802	0.0802	14.9860	-0.0140	82.54%
	Depth(c)	8.1170	0.1170	7.9901	-0.0099	91.53%
	Roundness	0.0126		0.0076		39.68%
Step	Y-coordinate position (d)	29.0888	0.0888	28.9638	-0.0362	59.23%
	Step height (e)	10.2052	0.2052	10.0977	0.0977	52.38%
	Parallelism	0.0775		0.0323		58.32%

Tab. 1: Machined workpiece inspection result.

### Conclusions:

This study used a machine tool probing system to measure the position and orientation of fixture locators. Then, it calculated the errors resulting from the fixture locators by the cosine theorem and HTM according to the measurement result. The TWP command roll-pitch-yaw of the five-axis machine tool controller was set using the calculated values for error compensation, where the fixture rotation and offset operations were automatically implemented by the TWP command of the five-axis machine tool controller, and the workpiece can compensate the fixture error to increase the geometric dimension accuracy of the workpiece. Finally, the fixture error compensation was tested, the workpiece was designed with step and hole features, and the results before and after fixture error compensation were compared to confirm the feasibility and effect of the methodology of this study. The experimental results showed that the geometric dimension accuracy of the workpiece improved after fixture error compensation. The hole machining and step machining results showed average improvement rates of 85% and 56%, respectively. Therefore, the fixture error compensation method designed in this study can effectively reduce the effect of fixture error on the geometric dimension accuracy of the workpiece.

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