Title:
A G^2 Interpolation Scheme for Polar Surface

Authors:
Jianzhong Wang, jwangf@uky.edu, University of Kentucky
Fuhua Cheng, cheng@cs.uky.edu, University of Kentucky

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Introduction:

Subdivision surfaces have been widely used in CAD, gaming and computer graphics. Catmull-Clark subdivision (CCS) [1], based on tensor product bi-cubic B-Splines, is one of the most important subdivision schemes. The surfaces generated by the scheme are C^2 continuous everywhere except at extraordinary points, where they are C^1 continuous. A shortcoming inherent in CCS surfaces is the ripple problem, that is, ripples tend to appear around an extraordinary point with high valence. In the past, research focused on improving curvature distribution at extraordinary points. However, with quad mesh structure of CCS surfaces, ripples could not be avoided in high valence cases. To handle this artifact, Polar surface are studied by a number of researchers. A Polar surface has a quad/triangular mixed mesh structure. A bi-cubic Polar subdivision scheme is presented in [2] that sets up the control mesh refinement rules for Polar configuration so that the limit surface is C^1 continuous and curvature bounded. A Polar surface handles high valence cases well, but there are some issues to solve for connecting them to Catmull-Clark meshes. For instance, because of the mismatch on the mesh between radial subdivision and Catmull-Clark subdivision, in [3], given a polar vertex of valence n, at the k^{th} level, its generalized bi-cubic subdivision scheme generates 2^k subfaces and expands the valence to $n2^k$. Recently a new subdivision scheme was developed in [4], this new scheme subdivides triangular faces in Polar Catmull-Clark (PCC) mesh without generating exponential number of subfaces and without doubling valences in each subdivision step and its limit surface is G^2 on Polar parts.

The polar surface can handle high valence very well, however, all current polar subdivision schemes are approximating, i.e. the generated limit surface will not interpolate the given control mesh. Given the complexity of quad/triangular mesh structure, no known interpolation scheme was developed yet. However, since many applications require an interpolation scheme, Polar surface is not well adopted in CAD/CAM. In this paper, we present a G^2 interpolating scheme on Polar surface, such that it can be used in high precision CAD/CAM application. Our new scheme is based on Bezier crust [5][6], where an interpolating surface was generated by parametrically adding Catmull-Clark subdivision surface and a special selected bi-degree 5 Bezier surface. Although scheme of Bezier crust handles quad faces only, we show by conversion, we can handle triangular faces in Polar mesh as well. The curvature continuity of generated limit surface of our new scheme is consistent with the corresponding Polar surface, in case of the PCCS [4], it is G^2 on Polar parts.

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Main Idea:
A typical Polar mesh structure is shown in Fig. 1(a), where all control points are regular (valence 4) except at Polar extraordinary point (surrounding faces are triangular). A more flexible scheme is to embed Polar mesh into Catmull-Clark mesh structure into a mesh structure called Polar Catmull-Clark (PCC) mesh[4], such that extraordinary point can also occur at non-Polar faces (Fig. 1 (c)). This new scheme generates a G^2 surface at Polar parts and keeps the same surface property at Quad parts as CCS. By far, current Polar schemes are all approximating, however most application in CAD/CAM require an interpolating scheme, here we propose an interpolation scheme on Polar mesh, especially we focus on subdivision scheme used in [4].

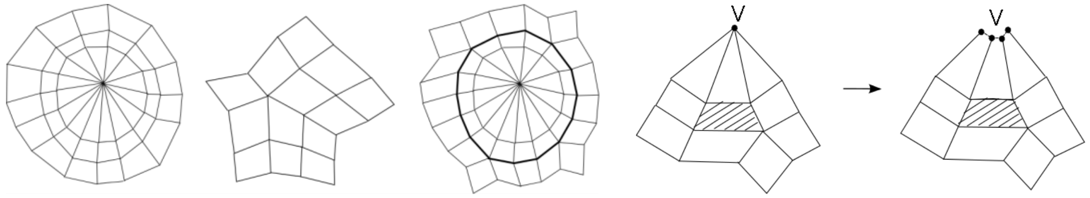

Fig. 1: (a) Polar mesh, (b) Catmull-Clark mesh, (c) Polar Catmull-Clark mesh, (d)-(e) mesh conversion of triangular faces into quad faces at Polar extraordinary points

A Polar triangular face during subdivision can be converted into quad faces by vertex splitting (Fig. 1 (d), (e)), such that we may apply the interpolation schemes currently used for CCS subdivision.

One recent interpolation scheme on CCS is developed to add together CCS parametric surface and an G^2 offsetting surface called bi-degree 5 Bezier crust [5], such that the interpolation surface will keep the same curvature continuity as its underlying CCS surface. This interpolation scheme was designed to handle meshes consisting of only quad faces. The Polar surface has triangular face, so it cannot be directly applied. However as shown in Fig. 1 (d) & (e), a Polar triangular face will be treated as a quad face during subdivision process and it has the same (u,v) parameterization as quad faces [4], such that the scheme of Bezier crust can also be applied to these triangular faces.

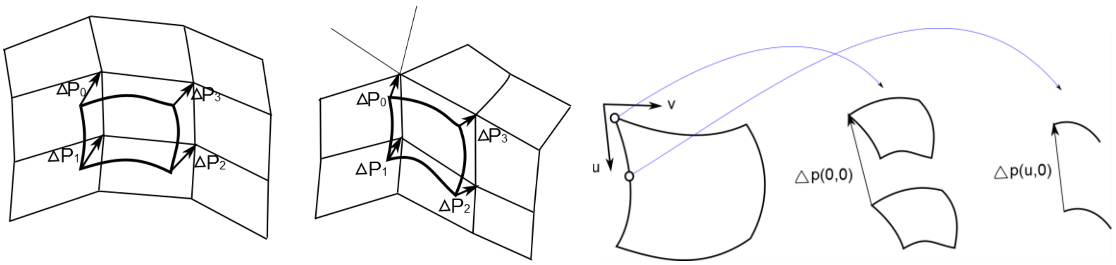

Figure 2: (a) & (b) difference vectors between control points and corresponding CCS limit points, (c), (d) & (e) How can CCS surface and an offsetting surface be added to form an interpolation surface. Bottom of (d) & (e): faces and curves of CCS limit surface; top of (d) & (e) the interpolating surface by adding an offsetting surface of bi-degree 5 Bezier crust working on the given difference vectors.

The parametric form of our new interpolation scheme on Polar parts is as follows:

$$\bar{S}(u,v) = S(u,v) + \Delta\mathbf{p}(u,v)$$

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Where S(u,v) is the parametric form of Polar surfaces, while Δp(u,v) is the parametric form of bi-degree 5 Bezier crust [5] with

$$\Delta\mathbf{p}(u,v) = \sum_{i=0}^{5}\sum_{j=0}^{5} \mathbf{b}_{i,5}(u)\mathbf{b}_{j,5}(v)\Delta\mathbf{P}_{i,j},$$

Where $b_{i,5}(u)$ and $b_{j,5}(v)$ are coefficient for parametric (u,v) surface.

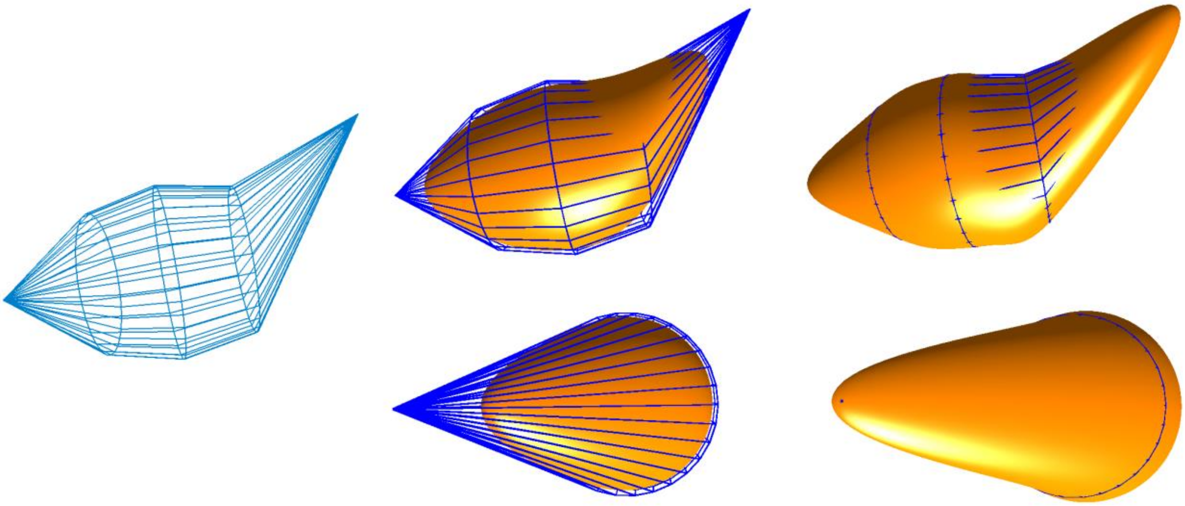

Fig. 3: A Polar example. Left: Polar mesh; middle: two views of PCCS surface; right: two views of our new Interpolation surface.

Fig. 3 and 4 show examples of applying our new interpolation scheme. We can see that the interpolating surface does not have ripples or creases which are commonly found while performing CCS.

Conclusion:
In this paper, we introduce a new interpolating scheme to handle Polar parts. We show that with our new scheme, we could handle triangular faces the same as that of quad faces, such that a G^2 continuity can be guaranteed at Polar parts.

Experiment results show that a high quality interpolation surface (Fig. 3 & Fig. 4) can be obtained by adding bi-degree 5 Bezier crust parametrically to underlying CCS surface

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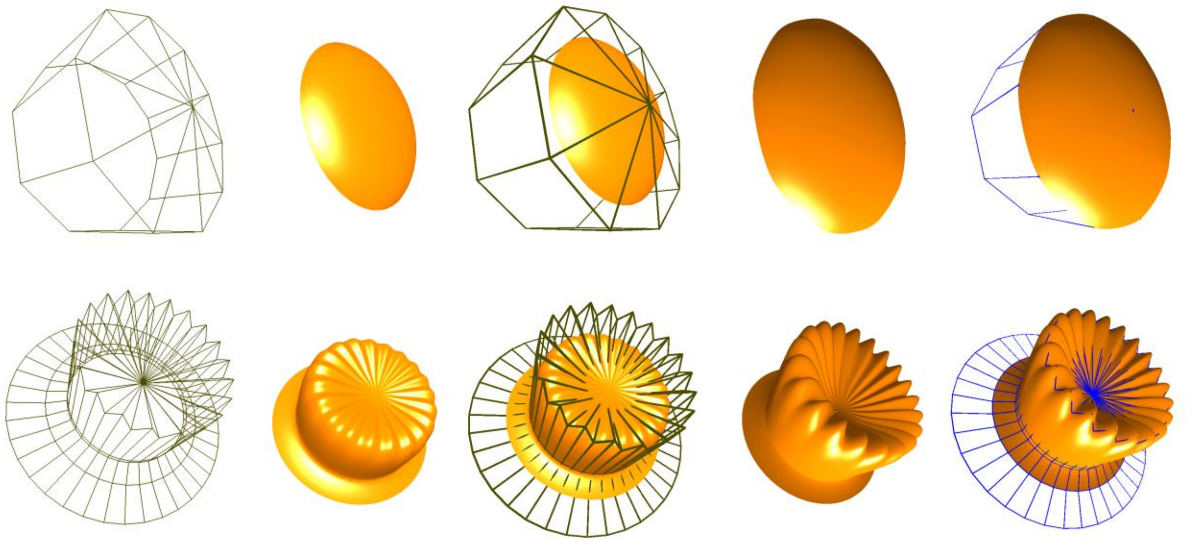

Fig. 4: two examples of interpolation on Polar mesh, (a) mesh, (b) Polar limit surface, (c) mesh with limit surface together, approximating, (d) interpolation surfaces (e) mesh shown with interpolation.

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