Title:
Curve Fitting Using Generalized Fractional Bézier Curve

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Introduction:
Curve fitting is crucial, especially when dealing with a function that can fit data with a high degree of smoothness. Curve fitting has been applied in a variety of areas, including computer-aided geometric design, manufacturing, medical imaging, and animation [2]. Curves such as log aesthetic curve [4] and the classical Bézier curve are widely utilised tools in the curve fitting process due to their advantageous qualities. On the other hand, because the shape of the curve cannot be changed without changing its control points, the Bézier curve has limited flexibility and adjustability. Furthermore, shifting or altering control points to get the correct shape is time-consuming.

Hence, in the present work,the generalized fractional Bézier curve will be used. Since the generalized fractional Bézier curve comprises shape and fractional parameters, and the picture outline can be constructed using the same degree of curve, the curve fitting technique will become easier. Various levels of continuity will be used, notably fractional continuity. Because of the fractional continuity, the position of common points or related joints may be changed along the first curve, giving more options for common point adjustment. Shape parameters contribute to the adjustment of shape locally and globally, hence enhance further the flexibility of curves.

Generalized Fractional Bézier Curve:
The generalized fractional Bézier curve with shape parameters presented in [3] will be used in this paper. For $t \in [0,1]$ and $v \geq 0$, the following function is defined as generalized fractional Bézier curve basis function of degree n with n shape parameters:

$$\begin{gathered}\bar{f}_{i,n}(t) = f_{i,n}(t)\Big(1 + \frac{\alpha_i}{n-i+1}(1 - D_t^{-v}(t)) + -\frac{\alpha_{i+1}}{i+1}(D_t^{-v}(t))\Big),\\ -(n-i+1) < \alpha_i < i,\ \alpha_0 = \alpha_{n+1} = 0,\ i = 0,1,...,n,\end{gathered} \tag{2.1}$$

where $f_{i,n}(t) = \binom{n}{i}(1 - D_t^{-v}(t))^{n-i}(D_t^{-v}(t))^i$ and $D_t^{-v}(t) = \frac{1}{\Gamma(v+2)}t^{v+1}$.
Hence from the basis functions defined in 2.1, the generalized fractional Bézier curve of n degree with n

shape parameters is defined as follows:

$$x(t; v, \alpha_1, \alpha_2, ..., \alpha_n) = \sum_{i=0}^{n} \bar{f}_{i,n}(t; v, \alpha_1, \alpha_2, ..., \alpha_n) P_i, \quad t \in [0, 1], \tag{2.2}$$

where $v \geq 0$, P_i for $i = 0, 1, ..., n$ is the set of control point in $\mathbb{R}^m$.

Fractional Continuity for Generalized Fractional Bézier Curve:

The fractional parameter is a new type of parameter in the generalized fractional Bézier curve. Fractional continuity is a new type of continuity made possible by this fractional parameter [3]. The restriction of parametric and geometric continuity, which allows the curves to be joined at any arbitrary position along the first curve, is addressed by fractional continuity. Special conditions can also be applied to reduce fractional continuity to geometric and parametric continuity. Consider two curves $x_1(t; v_1)$ on $t \in [a, b]$ and $x_2(t; v_2)$ on $t \in [a^*, b^*]$ with fractional parameters v_1 and v_2, respectively, where both curves have a degree of at least $r + 1$. The two curves are F^r continuous if the following condition is satisfied:

$$\begin{cases} x_1(b; v_1) &= x_2(a^*; 0) \\ x_1'(b; v_1) &= \phi_1 x_2'(a^*; 0) \\ x_1''(b; v_1) &= \phi_1^2 x_2''(a^*; 0) + \phi_2 x_2'(a^*; 0) \\ & \vdots \\ x_1^{(r)}(b; v_1) &= \phi_1^r x_2^{(r)}(a^*; 0) + \phi_2^{r-1} x_2^{(r-1)}(a^*; 0) + ... + \phi_r x_2'(a^*; 0). \end{cases} \tag{2.3}$$

The range of the scalar factor, ϕ_i for $i = 0, 1, ..., r$ depends on the degree of continuity r. Generally, for F^r continuity, the range for the scale factors are $\phi_1 > 0$ and $\phi_i \in \mathbb{R}$ for $i = 2, 3, ..., r$.

The difference between fractional continuity and parametric/geometric continuity in the shape point of view is the common point or connected point can be changed by variation of the fractional parameter via fractional continuity. Further and deeper discussion regarding fractional continuity can be seen in [3]. Fractional continuity will be used in the curve fitting process to make much smoother piecewise curves and provide useful tool to draw outline of image.

Curve Fitting Process of Image Using Generalized Fractional Bézier Curve:

In this paper, an image of leaf and a Pikachu as in Figure 1 and 4a respectively, will be used for the curve fitting process. The piecewise generalized fractional Bézier curves will be used to construct the outline of the image. The data points are extracted using the function of "Get Coordinates" in Wolfram Mathematica Software. Different values of shape and fractional parameters will be used to fit the curve on the image. The flowchart for curve fitting process for generalized fractional Bézier curve is shown in Figure 2.

Results for the Curve Fitting Process of Image Using Generalized Fractional Bézier Curve:

For the curve fitting process using generalized fractional Bézier curve, only two common points are chosen to be connected by fractional continuity. Other common points are connected by simple C^0/G^0 continuity. To construct smoother curves, a higher degree of continuity must be used.

Figure 3 illustrates the curve fitting process of Figure 1 using six piecewise cubic fractional Bézier curve. Note that only two joints are connected with F^0 continuity while the rest are C^0 continuity. The process of curve fitting is shown side by side with the respective curvature comb. The curvature comb is only shown for the curves that be connected with F^0 continuity. The double sided arrows indicate that change in position of the common point that have been connected using fractional continuity due to the

Fig. 1: Image of a leaf

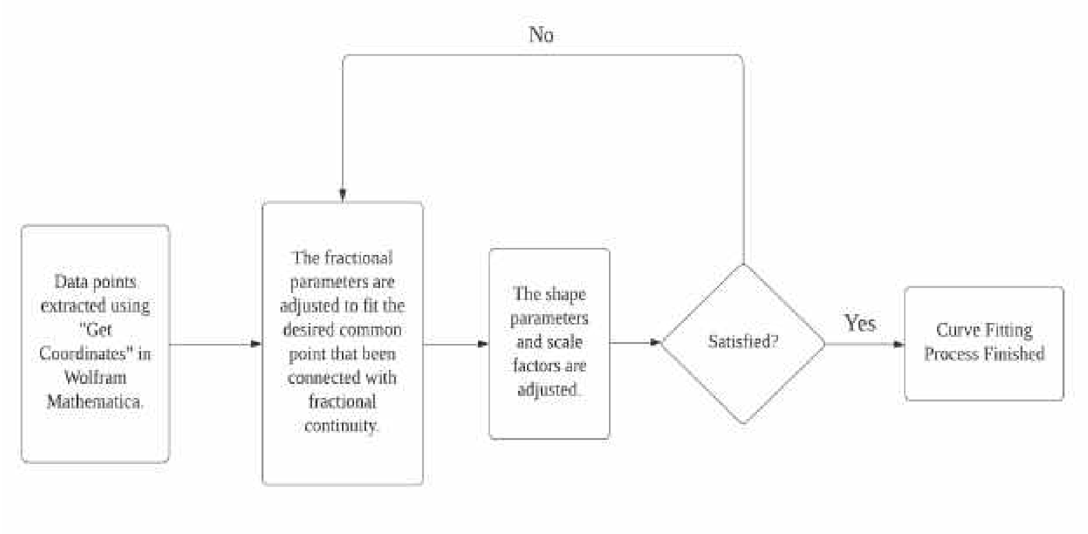
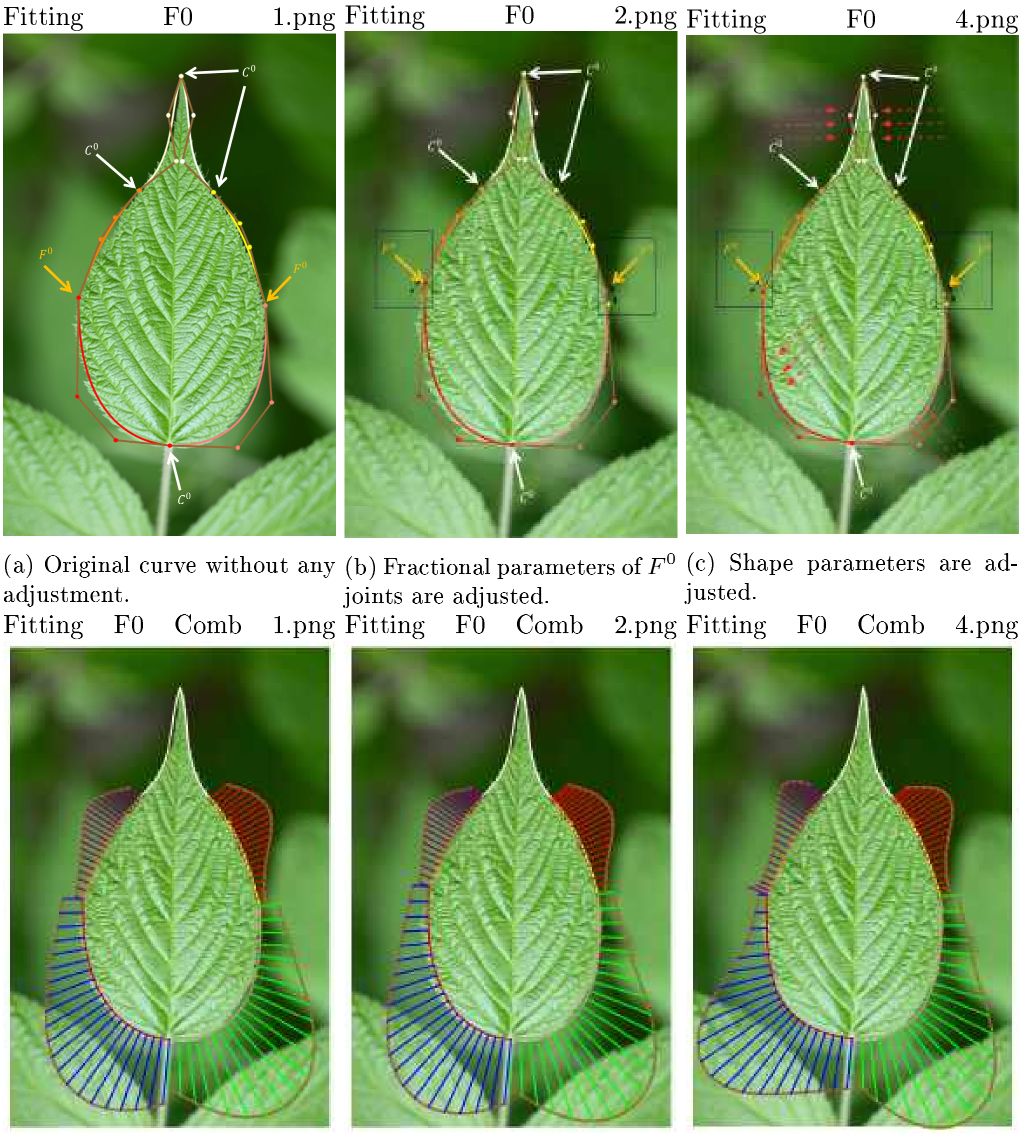


Fig. 2: Flowchart of curve fitting using generalized fractional Bézier curve

change of spectrum values of fractional parameters. While, the dashed brown arrow represent the curve moved due to the manipulation of shape parameters. The fractional and shape parameters are manually adjusted, and the parameters value can be chosen arbitrarily. However, the fractional continuity will be formed automatically when the fractional continuity condition is satisfied. Figure 4 shows the curve fitting process of Figure 4a using fifteen piecewise cubic fractional Bézier curve. Only six joints are linked with F^0 continuity, while the others are connected with C^0 continuity.

Fractional continuity is applied to shift the position of the common point to suit where the curve should be joined. Fractional continuity can be used as an alternative to the subdivision method and has less computational time especially when involving a high degree of curves. Fractional continuity can be used to control the extrapolated points. Using fractional continuity, the common/connected point can be



(a) Original curve without any adjustment.

(b) Fractional parameters of F^0 joints are adjusted.

(c) Shape parameters are adjusted.

(d) Curvature comb of (a).

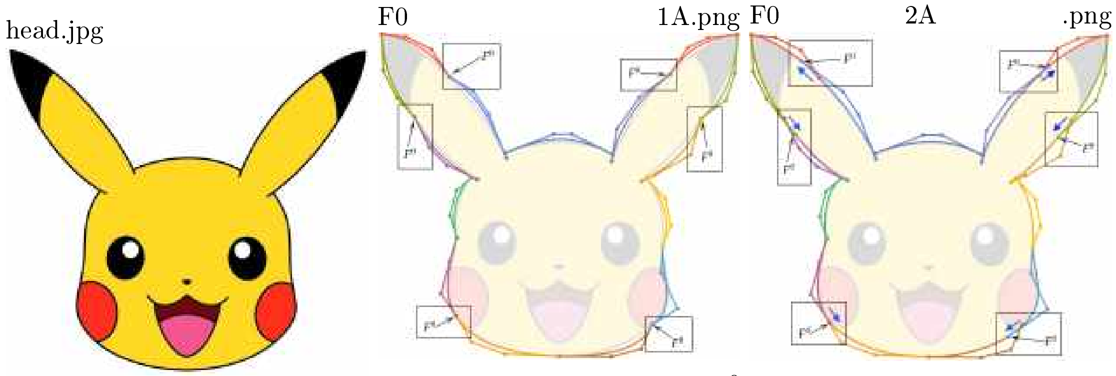
(e) Curvature comb of (b).

(f) Curvature comb of (c).

Fig. 3: Curve fitting of a leaf with F^0 continuity.

controlled along the first curve by adjusting the value of the fractional parameter so it can be adjusted to fit the outline of the shape.

Shape parameters give the users more control over the shape of the curve. The shape parameters enable the shape to be change locally especially if there a specific part of the curve that only needed to be changed. Changing one control points of the Bézier curve can affect the whole shape of the curve. This will cause the curve fitting process using classical Bézier curve taking a significant amount of time compare to use the generalized fractional Bézier curve.



(a) Pikachu from Pokémon.

(b) Fractional parameters of F^0 joints are adjusted.

(c) Shape parameters are adjusted.

Fig. 4: Curve fitting of Pikachu with F^0 continuity.

However, there lies a limitation using the generalized fractional Bézier curve, which it has complex computation compared to the classical Bézier curve. Fractional continuity also has more complex equation than parametric/geometric continuity. Nevertheless, at the cost of complex computation, the generalized fractional Bézier curve provides easier manipulation of shape and adjustability of curve. While fractional continuity also has lower computational cost time compared to the subdivision method (for splicing the curve before connecting with second curve) [3]. Thus, generalized fractional Bézier curve and fractional continuity offering useful tool for designing especially in the curve fitting.

Conclusions:

The generalized fractional Bézier curve is employed in the curve fitting technique in this study. The curve fitting process becomes faster and easier due to its shape and fractional parameters. The fractional continuity is seen to be a beneficial tool, especially when adjusting the common point along the first curve. It can also be used in place of the subdivision method. The shapes of the curves may be changed further using shape parameters and scale factors without changing the control points. As a result, the generalized fractional Bézier curve is a great tool for curve fitting.

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