Title:
A Redundant Constraint Identification Method Based on Constraint Space

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Introduction:
Geometric constraint solving is essentially a kind of constraint satisfaction problem, which expresses geometric objects such as lines, circles, arcs, cylinders and constraints such as parallel, tangent, distance dimensions between them. Geometric constraint solving is one of the core issues of parametric modeling. Along with the rapid development of engineering applications, this problem has been extensively studied in the past three decades and a lot of achievements have been made. However, there are still some open problems to be studied, and one of the most important research fields is redundant constraint processing. Redundant constraints are common in practical parametric modeling, which directly affects the correctness of the decomposition of geometric constraint system, efficiency and stability of geometric constraint solving methods. The existence of redundant constraints seriously restricts the practicability and engineering of geometric constraint solver. Currently, some researchers have made a lot of meaningful studies on the identification of redundant constraints. They put forward some enlightening methods to solve some practical problems. However, due to the complexity of redundancy issues, there is not yet formed a generally applicable and dominant research idea.

This article focuses on the discussion of redundant constraint processing. We first illustrate the harm of redundant constraints in actual solutions through some examples, and then describe both the previous efforts and their limitations. Finally, we propose an effective method and apply it to the application developed by the author. Based on a concept called constraint space, this method transforms constraints into basis of the constraint space, and determines the existence of redundant constraints by analyzing the intersection of the constraint space. This method provides a new idea for the realization of degree of freedom（DOF） analysis. It is effective and general when applied in combination with graph theory.

Problem Formulation:
Generally speaking, if there is a constraint that can be expressed by other constraints in a constraint system, its addition or deletion will not affect the existing solution results, the constraint can be viewed as a redundant constraint, which is also called consistently redundant constraint. For example, in the application of 3D assembly, if two planes have both coplanar and parallel constraints, the parallel constraint will be regarded as a redundant constraint.

Here is the formal definition:

Definition 1[4]. Suppose that the algebraic equations corresponding to a geometric constraint system is $f_o = f_1 = \cdots = f_s = 0$, where $f_o, f_1, \cdots, f_s$ is the polynomials in $\Re[x_1, x_2, \cdots, x_n]$, and the affine variety $\mathrm{W}(f_1, \cdots, f_s) \subset k^n$ is the set of all solutions of the system of equations $f_1 = \cdots = f_s = 0$. If $\mathrm{W}(f_1, \cdots, f_s) = \mathrm{W}(f_0, f_1, \cdots, f_s) \neq \varnothing$, then the constraint equation f_0 is called a redundant constraint equation.

For the sake of flexibility, almost all of the geometric constraint solvers allow the user to add arbitrary constraints into the system. As a result, redundant constraints exist in many practical applications.

Related Work:

The existing of redundant constraints not only causes practical engineering application problems, but also affects the correctness of the research results of geometric constraint solving. Methods based on decomposition-combination strategy are the mainstream way for solving geometric constraints, the method proposed in Reference [1] is a representative. The correctness of this approach depends on the correctness of the bipartite graph decomposition results. Once redundant constraints exist, there are errors in the decomposition results of bipartite graph. More details can be found in Reference [1].

Gröbner basis method is based on symbolic algebra theory. It is a complete and stable method to judge the properties of constraint equations directly from the algebraic theory. It is suitable for determining whether the affine variety of the two algebraic equations are equal or not according to definition 1. Gao et al. [2] proposed a symbol method to apply this theory to redundant equation systems. The Reference [3] points out that the time complexity of computing Gröbner basis is exponential. When the scale of constraint equations is large, the response speed will be too slow to be suitable for solving practical geometric constraints with high interactive performance requirements.

QR decomposition is an extensive and effective method for solving all eigenvalues of small and medium-sized matrices. Meanwhile, it is also used in geometric constraint solving and redundant constraint analysis. The basic principle of identifying redundant constraints is based on rank loss of Jacobian matrix. If the rank of the matrix has a loss, it indicates that there is a redundant constraint equation. At this time, the row transformation of matrix R is carried out to eliminate non-zero elements other than the main diagonal elements as much as possible. Then, according to whether there are non-zero elements other than the principal diagonal elements in matrix R, the current redundant equations are determined.

Michulucci et al. [6] proposed the Witness method which is called WCM to detect redundant constraints in geometric constraint systems. Firstly, WCM generates a projection part which contains all the constraints that are still satisfied under the projection transformation, then constructs a Witness point by solving the constraint equations in the projection part. Finally, it calculates the Jacobian matrix at the Witness point which is used to detect the redundant constraints under the principle similar to QR decomposition.

Both QR decomposition and WCM are not theoretically complete. They give correct results only if the redundant constraints contained in the equation system are linear correlation. Besides, a series of numerical calculations are also needed to identify redundancy. Therefore, when the system scale is large, the calculation efficiency will also be affected.

Proposed Method:

In 3D assembly design, the relative position relationship between components is determined by matching relationship. The relationship is actually a kind of geometric constraint. The restriction of geometric constraint on components is called degree of constraint (DOC), including translation degree of constraint (TDOC) and rotation degree of constraint (RDOC).

Definition 2. The restriction of a constraint on the movement direction of components in space is called constraint space (CS), which includes translation constraint space (TCS) and rotation constraint space (RCS). It takes a set of restricted direction vector of the component as a basis (as shown in Tab.1, $v_i, i = 1,2,3$). The translation constraint space and the rotation constraint space take the set of restricted translation direction vectors and rotation direction vectors as their bases, respectively.

According to definition 2, the calculation of constraint space basis is equivalent to determining the restricted translation and rotation direction vectors of the constrained objects. Taking the coplanar constraint as an example, this paper introduces the calculation method of the basis of translation and rotation constraints:

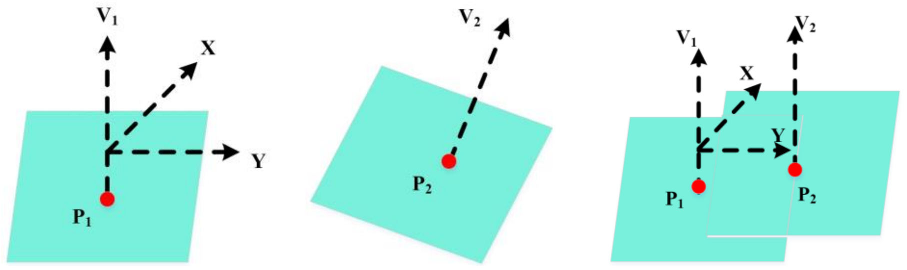

Fig. 1: The coplanar constraint of two planes: (a) Plane1, (b) Plane2, (c) Coplanar result.

Constraint type	*TDOC*	*TCS*	*RDOC*	*RCS*
Plane parallel	0	$\{0\}$	2	$L^R(v_1^R, v_2^R)$
Coaxial	2	$L^T(v_1^T, v_2^T)$	2	$L^R(v_1^R, v_2^R)$
Coplanar	1	$L^T(v_1^T)$	2	$L^R(v_1^R, v_2^R)$
Co-Point	3	$L^T(v_1^T, v_2^T, v_3^T)$	0	$\{0\}$
Plane distance	1	$L^T(v_1^T)$	2	$L^R(v_1^R, v_2^R)$
Line distance	1	$L^T(v_1^T)$	2	$L^R(v_1^R, v_2^R)$
Line-to- plane distance	1	$L^T(v_1^T)$	1	$L^R(v_1^R)$

Tab. 1: Constraints, degree of constraint and constraint space.

It is assumed that plane2(P_2) in Fig.1(b) moves relatively to plane1(P_1) in Fig.1(a). In order to satisfy the coplanar constraint, it is necessary to restrict the translational movement of P_2 along the direction v_1, which constitutes the basis of the translation constraint space, then the translation constraint space $L^T(v_1)$ is obtained. Similarly, it is necessary to restrict the rotational movement of P_2 along the X and Y directions. X and Y constitute the basis of the rotation constraint space $L^R(X,Y)$, and then the rotation constraint space is obtained. Tab.1 lists some common 3D geometric constraints and degrees and constraint spaces of them.

It can be seen from the above that if there are multiple constraints between component A and component B, once there exists same base vector between translation constraint spaces of different constraints, it shows that the translation direction is repeatedly restricted, that is, there is a redundant translation constraint. The judgment of the rotation constraint is similar.

Based on the principle discussed above, the method of identifying redundant constraints applied to the constraint graph of geometric constraint system is given as follows:

Step1: Construct constraint graph $G(V,E)$ [5];

Step2: Decompose constraint graphs into constraint subgraph sequences [5];

Step3: Calculate the translation constraint space $L_j^{\,T}(j=1,2,\cdots,n)$ and rotation constraint space $L_j^{\,R}(j=1,2,\cdots,n)$ of each edge in each constraint subgraph, n is the number of geometric constraints;

Step4: Traversing the nodes of the constrained subgraph to find node paris, the number of edges between which is greater than 1. According to

$$L_s \cap L_t = \{v_i^s \in L_s \mid \left|v_i^s \times v_j^t\right| = 0, s < t\},\ i,j=1,2,3,\ s,t=1,2,\cdots n, \tag{1}$$

the redundant constraints in the system are identified.

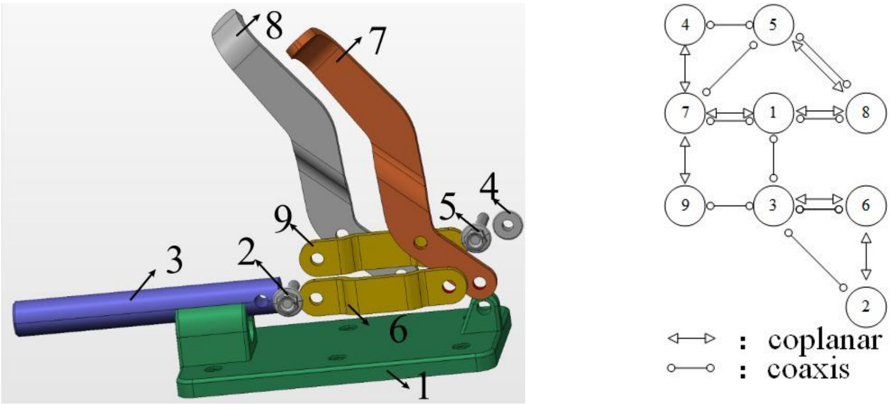

Fig. 2: (a) Example to be solved, (b) Constraint graph.

Fig. 2(a) shows a linkage mechanism composed of 9 components. Fig.2(b) is its constraint graph, containing 7 coplanar constraints and 9 coaxial constraints. In our solving system, each component has 7 variables, each coplanar constraint has 3 constraint equations, and each coaxial constraint has 4 constraint equations. As a result, the problem to be solved is of 63 variables, 57 constraint equations and 8 redundant constraint equations.

According to Tab.1, a translation constraint space and a rotation constraint space are constructed for each edge in Fig. 2(b). Traversing the nodes of the graph to find the cases where the number of edges between two nodes is greater than 1. Four pairs of nodes (1,7), (1,8), (3,6) and (5,8) are achieved. According to formula (1), it is determined that each node pair contains 2 redundant constraint equations, a total of 8.

The comparison of efficiency and accuracy between the proposed method and the existing methods is given in Tab. 2. The referenced methods are commonly used QR method and maximum matching (MM) method. NV, NE and NRE is short for the number of variables, equations and redundant equations, respectively. The calculation time of QR and MM methods is the average time of 1000 executions.

Methods	*NV*	*NE*	*NRE*	*Time/ms*	*Number of identifications*	*Accuracy*
QR				141	6	75%
MM	63	57	8	0.39	0	0%
Ours'				1.2	8	100%

Tab. 2: The comparison of efficiency and accuracy of redundancy identification methods.

It can be seen from Tab.2 that QR method can't identify all the redundant constraint equations, and its efficiency is poor compared with the other two methods. Although the efficiency of MM method is the best among these methods, it fails to identify any of the redundant constraint equations. The proposed method performs well in both efficiency and accuracy.

The constraint graph in Fig.2(b) is transformed into a set of nonlinear equations, and the equations are solved by Newton-Raphson iterative method to obtain the solution results before and after eliminating redundant constraints, as shown in Fig.3 (b) and Fig.3 (c) respectively.

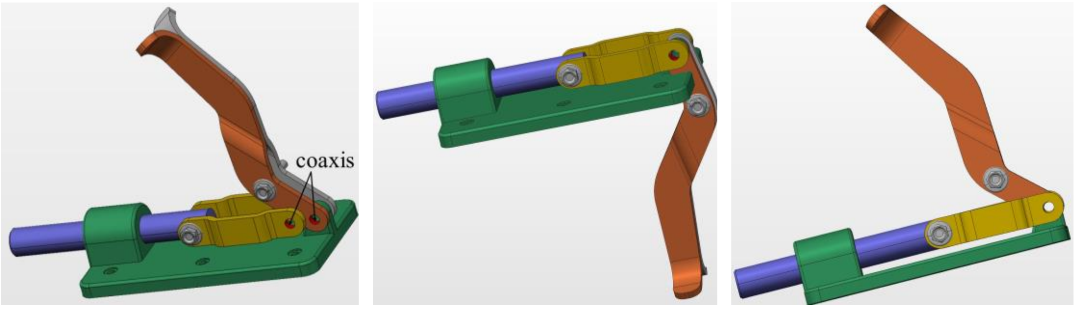

Fig. 3: The results of numerical solution: (a) Unsolved, (b) Solution with redundant constraints, (c) Solution without redundant constraints.

Conclusions:

In this paper, a new method for identifying redundant constraints in geometric constraints systems is proposed. By restricting the motion direction of the constrained object under geometric constraints, a set of vectors are constructed as a basis to generate constraint spaces. According to the intersection operation of different constraint space basis, the redundant constraints are identified. This method is integrated into the solution of geometric constraints to improve its stability and efficiency. In the future, this method can also be combined with screw theory to solve more practical problems.

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