Title:
Research on Post-processing Method of Topology Optimization Model

Authors:
Han Gao, 250620579@qq.com, Sichuan University
Lei Xu, xulei@scu.edu.cn, Sichuan University
Yuanhao Hu, 810363492@qq.com, Sichuan University
Zhanling Guo, 3030649850@qq.com, Sichuan University

Keywords:
Topology Optimization, Boundary Diffusion, Variable Density Method, Jagged Boundary, Sensitivity Filtering Method, Boundary Smooth Post-processing Method

DOI: 10.14733/cadconfP.2022.367-371

Introduction:

Structural topology optimization is an effective structural optimization method, which has become a hot research topic in the field of finite element analysis. The variable density method is usually used to solve the structural topology optimization problem due to the advantages of less design variables and high efficiency. However, this method also has disadvantages such as network dependence and boundary diffusion which would increase the geometric complexity and optimization time cost. In addition, the optimization results often bring a gray transitional boundary, the ideal smooth boundary cannot be obtained by traditional method of curve approximation or curve fitting, and it is impossible to determine whether the boundary extraction and model reconstruction can be successful. At the same time, the optimization model needs to be attached to the grid during topology optimization, so it is unavoidable that the edge of the optimization result has a jagged boundary. This not only increase the manufacturing difficulty, but also increases the complexity of the structure and makes model reconstruction difficult.

Aiming at the problems of boundary diffusion and jagged boundary existing in the topology optimized model based on the variable density method, a topology optimization post-processing method using the partition sensitivity filtering and the ordinary least squares is proposed. The partition weighted sensitivity filtering method is to divide the sensitivity filtering area into two parts, and use different weighting factors to weight the inner and outer areas respectively to remove the gray value to obtain a topology optimization structure with clear boundaries. The ordinary least squares curve fitting is to make the sum of the squares of the errors between the extracted boundary points and the fitting points reach the minimum value as much as possible, and the curve formed by the fitting points is an ideal fitting curve, which can make the jagged boundary become smooth. Some typical examples verify the effectiveness and feasibility of the method in suppressing boundary diffusion and solving jagged boundary problems under the conditions of single load, multiple loads and elements with different densities. By analyzing the model before and after post-processing, it is verified that this post-processing method can effectively obtain the topology optimization structure with clear and smooth boundaries, and the stress distribution is more uniform, which can reduce the difficulty of model reconstruction and manufacturing.

Post-processing Related Mathematical Models:

The variable density method has become one of the most popular methods for solving continuum topology optimization problems. In topology optimization based on the variable density method, numerical instability such as grid dependence and checkerboard often occur. Sigmund [1] proposed a sensitivity filtering method that introduces a minimum filtering radius for distance-weighted averaging, which effectively solves problems such as checkerboard and grid dependencies. However, there are still difficulties such as low optimization efficiency and boundary diffusion. Therefore, it is difficult and inaccurate to perform edge extraction directly. To solve the problem of boundary diffusion, a partitioned weighted sensitivity filtering method is proposed. The schematic diagram of partition sensitivity filtering area is shown in Fig.1. Its essence is to divide the original sensitivity filtering area into two sub-areas, and use different weight factors in different sub-areas. The boundary diffusion phenomenon of a large number of gray units can be effectively suppressed, and a clear topology optimization structure boundary can be obtained. The mathematical expression of its weighting factor $\tilde{H}_g$ is:

$$\tilde{H}_g = \begin{cases} 1 & , dist(k,i) \le r_n \\ \eta * r_{\min} * dist(k,i)^{(r_{\min}-\mu)} * e^{\left[-k*dist(k,i)^{r_{\min}}\right]} & , r_n \le dist(k,i) \le r_{\min} \end{cases} \tag{2.1}$$

The η and μ are the correction coefficients of the exponential function, the maximum value of the weighting factor in area I can be changed by adjusting η, and the trend of weighting factor change in area II can be changed by adjusting μ. The linear weighting represented by the dotted line in Fig. 2 is the weighting principle image of the Sigmund sensitivity filtering method, and the solid line is the principle image of the partition sensitivity filtering method weighted by the exponential function of different μ values. The partition weighting factor can ensure that the cells in region I play a leading role in the influence of the sensitivity of the objective function, and in region II, the exponential function can achieve a significant reduction in the weight when it is close to the boundary, further weakening the sensitivity of cells in region II to the central cell. In order to measure whether the optimization model satisfies the optimization requirements after the partition sensitivity filtering process, measures such as dispersion rate, gray rate and flexibility value are introduced. After using this method to optimize several typical topology optimization examples in the MTLAB-2019a software environment, each measurement index changes into a better direction. Comprehensively considering the influence of weight factors on the optimization results, the sensitivity filtering method on the basis of Eqn. (2.1) is expressed as:

$$\frac{\bar{\partial} C}{\partial l_e} = \frac{1}{\sum_{i=1}^{n} \tilde{H}_g} * \sum_{i=1}^{n} \tilde{H}_g l_i \frac{\partial C}{\partial l_i} \tag{2.2}$$

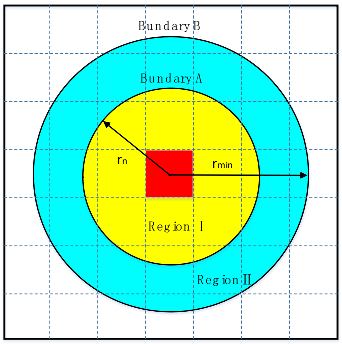

Fig. 1: Area division diagram.

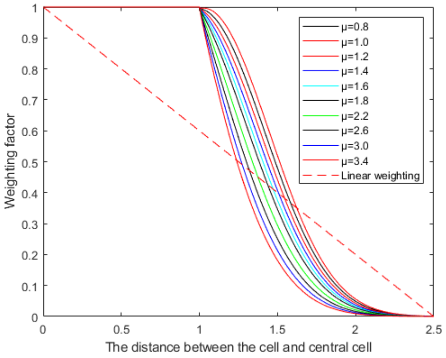

Fig. 2: Exponential function principle image.

At present, the most commonly used topological optimization boundary smoothing methods mainly include interpolation and fitting. The interpolation method is a method of determining the simulation function through known points, and it is only suitable for the situation where the data is accurate and the amount of data is small, otherwise it may cause a large local error. In the boundary smoothing of the topology optimization results, the amount of data at the boundary point is very huge, and the error is unavoidable, so in order not to introduce the original error, it is preferred to use curve fitting to determine the function. As the most common method of curve fitting, the ordinary least squares is widely used because it is more accurate and practical. It is widely applied as the standard regression method for analytical calibrations, and it is usually accepted that this regression method can be used for quantification starting at the limit of quantification [2]. After the gray value is removed by the partition sensitivity filtering method, the jagged boundary points of the topology optimization result are extracted. A smooth curve is formed by fitting the quadratic spline curve piecewise of the least-squares method to the discrete corner point Set P. If setting $y = f(x)$ to minimize the sum of squared errors, then:

$$\begin{cases} S = \sum_{i=1}^{n} \delta_i^2 = \min \sum_{i=1}^{n} [f(x_i) - y_i]^2 \\ f(x) = \alpha_1 r_1(x) + \alpha_2 r_2(x) + \ldots + \alpha_m r_m(x), m < n \end{cases} \tag{2.3}$$

The obtained curve is checked for goodness by using the error sum of squares discriminant method to detect the fit between the curve and the model boundary, its correlation index C can be expressed as:

$$C = \frac{\sum_{i=1}^{n} (f_i - \widehat{f}_i)^2}{\sum_{i=1}^{n} (f_i - \bar{f}_i)^2} \tag{2.4}$$

Here, the f_i is the measured value of the corner point. The $\widehat{f}$ is the theoretical value of the fitted curve. The $\bar{f}$ is the mean value. The smaller the C value of the fit, the better the curve fit. By merging the independent smooth curves, a closed region with smooth boundary can be obtained, and the final topology optimized structure with clear and smooth boundary can be obtained. This method effectively removes the jagged boundary in the topology optimization and greatly reduces the manufacturing difficulty of the optimized structure.

Topology Optimization Post-processing Flow:

The post-processing operation of topology optimization can effectively remove gray cells and jagged boundaries, improve the boundary smoothness of the topology structure and the performance related to manufacturing, and then improve the overall quality of structural topology optimization. A topology optimization post-processing method based on the variable density method is proposed in this paper. Firstly, finite element analysis is performed on the target model, and the partitioned weighted sensitivity filtering method is used to remove the gray cells present at the boundary to obtain a structure with clear boundaries. Then, the obtained results are subjected to a binarization operation to obtain optimized results with density values fully distributed as 1 and 0. Next, edge detection is performed to extract the boundaries. Finally, corner points are extracted from the boundary lines to obtain a sample point set, and the point set is used as a benchmark for curve fitting to obtain a topological optimization model structure with clear, smooth and easy manufactured boundaries. The post-processing flow chart of the method is shown in Fig. 3.

The topology optimization results obtained by the variable density method, because the weight factor decreases linearly from the central unit to the unit where the filter boundary is located, so although it can ensure that the unit close to the center obtains a higher weight value, the low-density unit close to the filter boundary and cells that are farther apart from the center will have a larger effect on the sensitivity of the center cell. The boundary of topology optimization will be over-smoothed, which is prone to the problem of boundary diffusion and a large number of gray units.

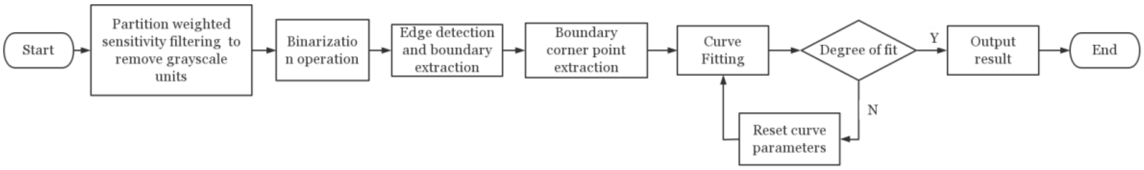

Fig. 3: Topology optimization post-processing flow chart.

Now taking a two-dimensional plane stress structure as an example, the design area is 80mm×80mm, and the mesh division is 80×80. The left end is restrained in full plane, and the upper and lower right corners of the structure are subjected to vertical upward and vertical downward loads respectively, and the model is restrained and loaded as shown in Fig. 4. The result of topology optimization of the Michelle structural model shown in Fig. 5.

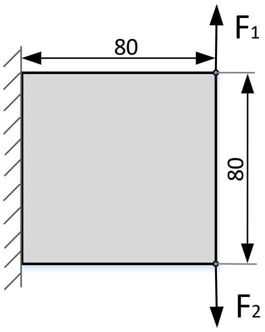

Fig. 4: Michelle structural design area.

Fig. 5: Original image after topology optimization.

The model after topology optimization presents a large number of gray transitional boundaries, and accurate image boundaries cannot be obtained by using the binarization method. It is also very difficult to perform boundary smoothing directly through curve approximation or curve fitting. Therefore, the method of partition sensitivity filtering is proposed to remove the gray value, and then the intermediate density unit is completely removed by binarization, and the optimized result with a jagged boundary and completely clear black and white is obtained. The result is shown in Fig. 6.

Fig. 6: Image with grayscale values removed.

The optimization results are processed by erosion dilation edge processing and jagged boundary extraction. The obtained result is suppressed by non-maximum value in the neighborhood, and the point where the local maximum is located is the corner point. Then to record the obtained corner data and to determine each corner of the boundary. The coordinate origin is further determined, and the size of the discrete cells divided in advance is used as the length of the coordinate unit, and the coordinates of each corner point are determined to form a point set. The corner point extraction result is shown in Fig. 7.

The ordinary least-squares quadratic spline curve segmentation fit to the set of discrete corner points is used to form a smooth curve. The obtained curve is checked for goodness by using the error sum of squares discriminant method to detect the fit between the curve and the model boundary. The spline interpolation operation is adopted for the curve set to obtain a closed curve group, and then the model structure of the boundary smoothing is obtained. The final post-processing model is shown in Fig. 8. This method effectively removes the jagged boundary in the topology optimization and greatly reduces the difficulty of manufacturing optimized structure.

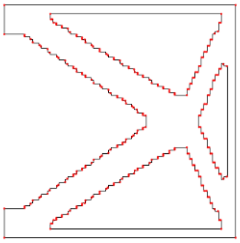

Fig. 7: Image after boundary point extraction.

Fig. 8: Smoothed image after post-processing.

Conclusions:

In the process of topology optimization, the variable density method often come with numerical instability such as checkerboard phenomenon and grid dependence. The optimization results have problems such as gray cells and jagged boundaries, which will affect the manufacturing performance of the optimized structure. Aiming at the above problems, Sigmund introduced the sensitivity filtering method of distance weighted average by introducing the minimum filtering radius, while this paper divides the original sensitivity filtering area into new sub-areas, and uses different sensitivity filtering factors for different areas. It is ensured that while improving the ability of the close-range unit at the center to affect the sensitivity of the central unit, the influence of the far-distance unit at the center on the sensitivity of the central unit is reduced, and the problem of boundary diffusion is further weakened to obtain a structure with a clear boundary. Then, the post-processing method of the least squares fitting curve is adopted to obtain an ideal model with clear and smooth boundaries, which reduces the difficulty of manufacturing optimized models. Some typical arithmetic models are optimized and post-processed from single loads, multiple loads, and different mesh densities and volume ratios, respectively. The experimental results show that the method can effectively avoid the phenomena of boundary diffusion and jagged boundary of the optimized results, and make them easier to produce optimized results, while ensuring the structural performance within the permitted range. When the structural performance is guaranteed to be within the allowable range, the problems of gray cells and jagged boundaries can be effectively solved, and the manufacturing performance of the optimized results can be improved at the same time.

Acknowledgement:

This paper is supported by the Major Science and Technology Project of Sichuan Province (Item Number: 2020ZDZX0013).

References:

[1] Sigmund, O.: Design of Material Structures Using Topology Optimization, Thesis Technical University of Denmark Dept of Solid Mechanics, 1994.

[2] Sanchez, J.M.: Linear calibrations in chromatography: the incorrect use of ordinary least squares for determinations at low levels, and the need to redefine the limit of quantification with this regression model, Journal of Separation Science, 43(13), 2708-2717, 2020.