Title:

Visualization of the Shape Information of Curves using Two-tone Pseudo Coloring

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Keywords:
logarithmic curvature graph, logarithmic torsion graph, shape information, log-aesthetic curves, two-tone pseudo coloring

DOI: 10.14733/cadconfP.2022.27-31

Introduction:

In this work, we present a new method for visualizing the shape information of curves in terms of logarithmic curvature graphs (LCGs) and logarithmic torsion graphs (LTGs)[6] using two-tone pseudo coloring [4]. The slope of LCG or LTG gives us the information related to the curvature or torsion function of log-aesthetic (space) curves. In the LCG (or LTG), however, it is not easy to understand which point of the curve corresponds to which point of the LCG (or LTG). We propose a method for visualizing the LCG (or LTG) slope in which users can read the approximate value of the LCG (or LTG) slope within a specific range.

Review of LCGs and LTGs:

Let κ and s be the curvature and the arc length, respectively. The linearity of logarithmic curvature graph (LCG) is

$$\log\left(\kappa\frac{ds}{d\kappa}\right) = -\alpha\log\kappa + c, \tag{2.1}$$

where α is the slope and c is a constant. The linearity of logarithimic torsion graph (LTG) is similarity defined by replacing κ, α and c by torsion τ, LTG slope β, and a constant d. In case of LCG, when $\alpha = -1, 0, 1$ or 2, the curve becomes the Clothoid, Nielsen's spiral, a logarithmic spiral or the circle involute. The planar curve whose LCG is linear is called *log-aesthetic curve* [5]. The space curve whose LCG and LTG are both linear is called *log-aesthetic space curve* [6]. See [7] for more details about LCGs and LTG

An LCG is graph whose horizontal and vertical axes are $\log\kappa$ and $\log\left(\kappa|\frac{ds}{d\kappa}|\right)$, respectively. Fig. 1 shows a curve segment with monotonically varying curvature and its LCG. As is shown in Fig. 1, it is not clear which point of a curve corresponds to which point in the LCG.

Shape Information regarding the Slopes of LCGs and LTGs:

The slope of an LCG in terms of radius of curvature is derived by Gobithaasan et al. [3]. In this section we drive an equation in terms of curvature. We also derive the equation for the slope of an LTG.

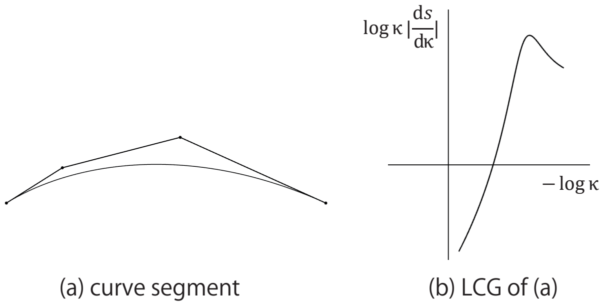

Fig. 1: A curve segment and it LCG

The slope of LCG α is

$$\begin{aligned}\alpha &= -\frac{\mathrm{d}\log\left(\kappa\frac{\mathrm{d}s}{\mathrm{d}\kappa}\right)}{\mathrm{d}\log\kappa} = -\frac{\mathrm{d}\log\left(\kappa\frac{\mathrm{d}s}{\mathrm{d}\kappa}\right)}{\mathrm{d}\kappa}\frac{\mathrm{d}\kappa}{\mathrm{d}\log\kappa} = -\frac{\frac{ds}{d\kappa}+\kappa\frac{d^2s}{d\kappa^2}}{\kappa\frac{ds}{d\kappa}}\kappa \\ &= -1-\kappa\frac{d^2s}{d\kappa^2}\frac{d\kappa}{ds} \qquad (2.2)\end{aligned}$$

The derivation of the LTG slope is similar. By replacing α and κ in Eq. (2.2) with β and τ, respectively, we can derive the following equation.

$$\beta = -1-\tau\frac{d^2s}{d\tau^2}\frac{d\tau}{ds} \qquad (2.3)$$

By computing α (and β for space curves), we can obtain the shape information of curves related to log-aesthetic (space) curves. The shape information is independent of the position, rotation, and scaling of the curves. The value of α or β tells us that the curvature or torsion function is best approximated by the curvature or torsion function of log-aesthetic (space) curves.

Visualizing the Shape Information using Two-tone Pseudo Coloring:

In this section, we visualize the shape information using the two-tone pseudo coloring [4] proposed by Saito et al. In the two-tone pseudo coloring, a user can read out approximate values from the colors of the paint.

Fig. 2 shows the comparison of continuous coloring and two-tone pseudo coloring. In both the coloring methods, the color is gray at $\alpha = 0$ and red at $\alpha = 1$. In continuous coloring, the color changes continuously by linearly interpolating the color from gray to red. In two-tone pseudo coloring, as the value α changes from 0 to 1, the height of the color changes. For example, if the height of the red color is approximately $\frac{1}{3}$ of the total height and the other color is gray, we can read out the approximate value as $\alpha = \frac{1}{3}$. See Fig. 2(b). In contrast, in continuous coloring shown in (a), it is not easy to read out the approximate value from interpolated colors because the human eyes' color perception is affected by nearby colors, making it difficult to correctly distinguish subtle differences. Note that, in two-tone pseudo coloring, if the upper one-third of the total height of the coloring is red and the lower two-thirds is gray, we can still read out the approximate value as $\frac{1}{3}$. Therefore, we can read out the approximate value when color inversion between top and bottom occurs. More details on two-tone pseudo coloring are described in [4].

As shown in Fig. 3, we set the colors of two-tone coloring blue, gray, red, or orange when $\alpha = -1$, 0, 1 or 2, respectively. These values of α are particularly meaningful because the curve becomes the Clothoid,

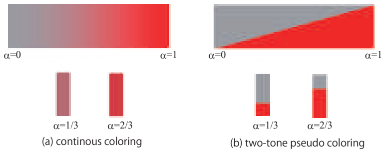

Fig. 2: Comparison of continuous coloring and two-tone pseudo coloring

Fig. 3: Two-tone pseudo coloring for α and β

Nielsen's spiral, a logarithmic spiral, or the circle involute corresponding to the value of α. The color is dark cyan when α is less than -2 or green when α is greater than 3. The same coloring is used for β.

Figure 4 shows a visualization of the shape information of polynomial Bézier curve with monotonically varying curvature. From a to b, the curve is changing from the logarithmic spiral ($\alpha = 1$) to the circle involute ($\alpha = 2$). From b to c, the curve is changing from the circle involute ($\alpha = 2$) to the logarithmic spiral ($\alpha = 1$). The arc length from c to d is the longest. From c to d, the curve is changing more slowly from the logarithmic spiral ($\alpha = 1$) and getting closer to the Nielsen's spiral ($\alpha = 0$).

Figure 5 shows polynomial cubic Bézier curves with a curvature maximum (a) and a curvature minimum (b). The color becomes dark cyan near the curvature maximum and green near the curvature minimum.

Figure 6 is an example of a cubic Bézier curve that is close to a logarithmic spiral. In the figure, the control points are manually placed so that the curve gets closer to a logarithmic spiral ($\alpha = 1$). Initially, the control points are placed so that they roughly satisfy the condition of typical Bézier curves[1], which are known to get closer to logarithmic spirals as the degree gets higher. In typical Bézier curves, $\frac{|\mathbf{P}_{i+2}-\mathbf{P}_{i+1}|}{|\mathbf{P}_{i+1}-\mathbf{P}_{i}|}$ and the angle between $\mathbf{P}_{i+1} - \mathbf{P}_i$ and $\mathbf{P}_{i+2} - \mathbf{P}_{i+1}$ are constants. Then the control points are fine-tuned so that the coloring approaches the single red color. Manually finding the placement of control points is relatively easy for $\alpha = 1$. For other α, finding the placement of control points is not easy since the geometric information like typical Bézier curves are not known.

Figure 7 shows examples of rational cubic Bézier curves optimized for $\alpha = -1, -0.5, 0, 0.5, 1, 1.5$ and 2 with the same G^1 Hermite interpolation condition. Note that, for example, when $\alpha = 2$, the coloring is not exactly single yellow because rational cubic Bézier curves cannot exactly represent log-aesthetic curves.

Figure 8(a) is an example of space curves, where α and β are shown on the curvature comb and torsion comb, respectively. Similarly, as in the case of curvature, if the coloring of a portion of the torsion comb is almost red, we can obtain the information that the torsion function is close to the case of $\beta = 1$, which

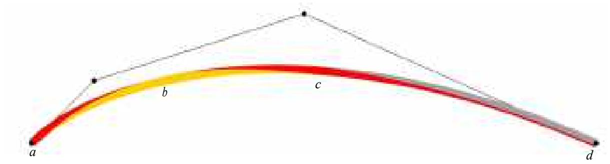

Fig. 4: Visualization of cubic polynomial Bézier curve

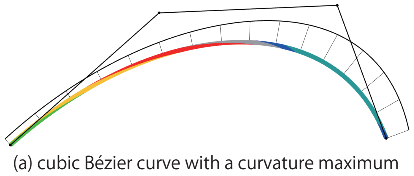

Fig. 5: Polynomial cubic Bézier curves with curvature extrema

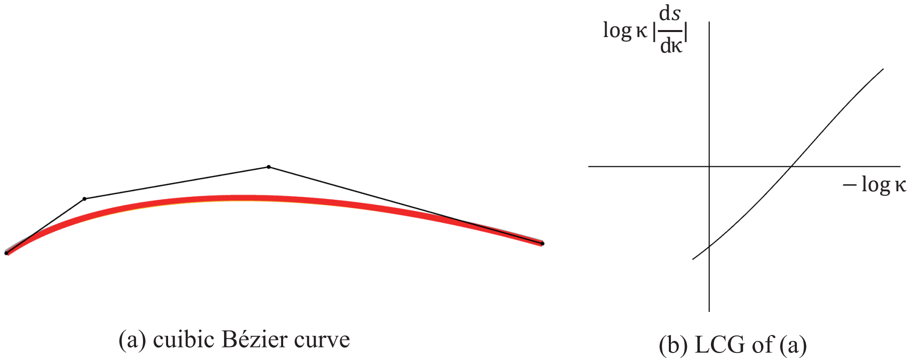

Fig. 6: Cubic Bézier curve close to a logarithmic spiral and its LCG

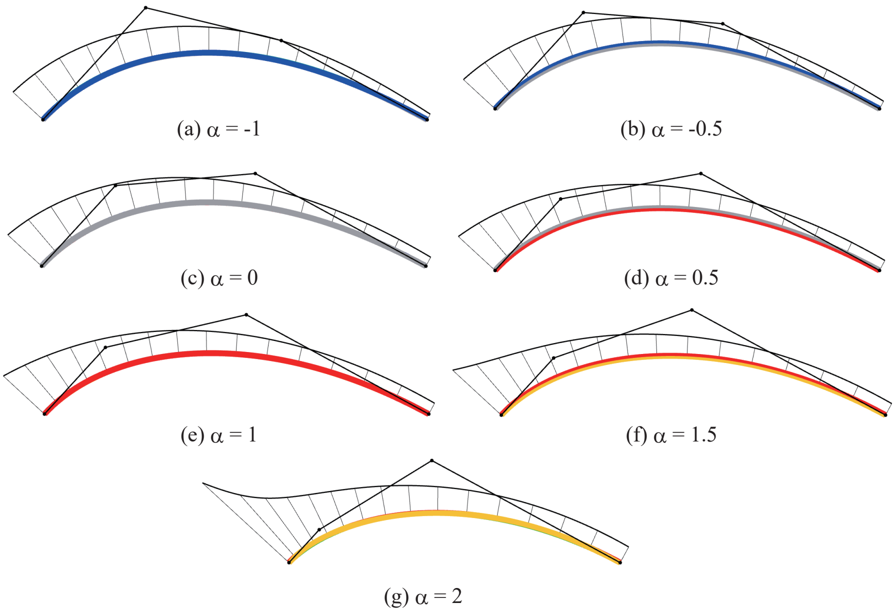

Fig. 7: Rational cubic Bézier curves optimized for specific α

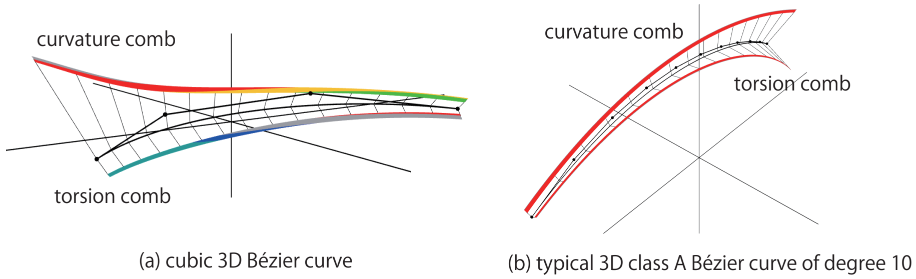

Fig. 8: 3D polynomial cubic Bézier curves

means the torsion is close to be inversely proportional to the arc length. Figure 8(b) is a typical 3D class A Bézier curve [1] of degree 10. Under the same G^1 Hermite condition, it is known that typical 3D class A Bézier curves get closer to the 3D extension of logarithmic spirals ($\alpha = 1$ and $\beta = 1$) as the degree gets higher [7]. Therefore, the coloring of α and β is both close to be single red. The space curve with $\alpha = 1$ and $\beta = 1$ is a 3D logarithmic spiral, and the curve $\alpha = -1$ and $\beta = -1$ is the 3D Clothoid curve. By visualizing the shape information of space curves, we can see the similarity to these curves.

Conclusions:

In this paper, we proposed a method for visualizing the LCG or LTG slope using two-tone pseudo-coloring. For each point of a curve, users can read the approximate value of α or β if it is within a specified range. The method is implemented using C++ and the visualization is fully interactive. In the final paper, more details about the visualization and the analysis of various planar curves in terms of LCG slopes will be described.

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