Title:
Application of Unstructured T-spline Continuity Elevation Algorithm in Surface Merging

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Introduction:
In CAGD, B-spline has been used to describe the free-form shapes in miscellaneous occasions and is widely used in product modeling. With the introduction of weight, B-spline can be upgraded as NURBS, so primary analytic surfaces such as spherical and cylindrical surfaces can be represented. The unified expression of free surfaces as well as primary analytic surfaces by NURBS has long become the backbone for CAD software, but there are still the following drawbacks in existing systems: (1) a single patch cannot express the complex model; (2) it is difficult to obtain watertight geometries in surface merging; (3) for the limitation of the rectangular grid, a large number of superfluous control points will be introduced during local refinement. To overcome these drawbacks, T-spline was proposed by Sederberg et al.[5], of which the T-junction breaks through the limitation of the control points' rectangular topology, so (2) and (3) of the above drawbacks are annihilated. By introducing extraordinary points into T-spline, with which the T-spline is called as unstructured T-spline, (1) of the above drawbacks is also solved, But unstructured T-spline results in some problems: (1) only C^0 continuity can be obtained around the extraordinary point; (2) the traditional local refinement algorithm[4] can't be used in the vicinity of the extraordinary points. These problems affect the accuracy of modeling and analysis, even failing to meet some design situations with high continuity requirements.

The T-spline technology is promising if it fully accommodates the requirement of Isogeometric Analysis(IGA). While the C^0 continuity limitation around the extraordinary point is not wanted because one of IGA's advantages is that it offers a framework of high order analysis approach. In order to more closely combine T-spline modeling with IGA, in this paper we demonstrate certain use case of merging multiple NURBS surfaces into one unstructured T-spline surface and then elevate the continuity around extraordinary point, after which the IGA can be implemented. This approach can effectively exploit the potential of unstructured T-spline technology in modeling as well as in analysis, and promote the possible advent of a new design pattern on the basis of integrating geometric modeling and performance evaluation more closely.

T-spline Foundation:
The T-spline uses a point-spline approach to define the surface. Each control point is associated with a basis function, and the T-spline surface is obtained by multiplying the control points' coordinates with corresponding basis functions and then accumulating them. Unlike NURBS basis functions which are defined based on knot vectors, T-spline basis functions are defined based on knot interval vectors, which are obtained based on T-mesh topology. Knot interval reflects the distance between two

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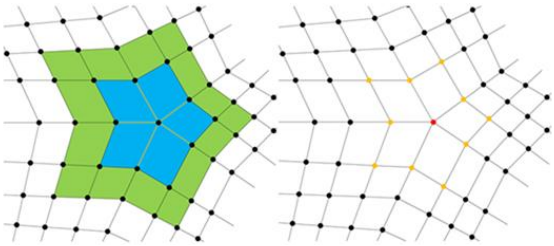
adjacent knots, so knot interval vectors and knot vectors can be converted to each other. When given knot interval vectors $[d_0,d_1,d_2,d_3]\times[e_0,e_1,e_2,e_3]$, it can be converted to knot vectors. The bi-cubic T-spline basis function $B_i(u,v)$ is equivalent to the B-spline basis function, which can be constructed from the associated knot vectors, the specific method to obtain knot interval vectors is detailed in [5].

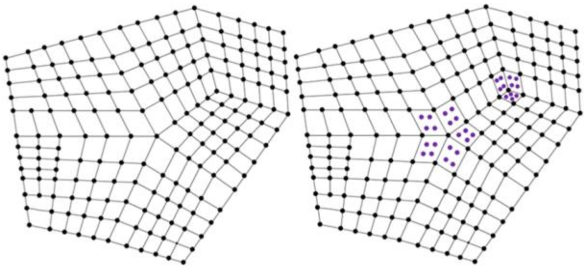
The valence, denoted by μ, is the number of edges that emanate from a certain point. In unstructured T-spline, extraordinary points are those internal control points with $\mu\neq 4$ that are not T-junctions or boundary control points with $\mu>3$, spoke edges are edges emanating from the extraordinary point. Extraordinary points affect the surrounding faces and control points. For the convenience of subsequent discussion, we classify related faces and control points into different categories: (1) irregular faces are faces that are in contact with extraordinary points; transition faces are faces that are in contact with irregular faces and do not belong to irregular faces; the remaining faces are called regular faces, as shown in Fig. 1(a); (2) irregular control points are extraordinary points; transition control points are control points on irregular faces that do not belong to irregular points; the remaining control points are called regular control points, as shown in Fig. 1(b).

IGA has additional requirements compared to geometric modeling. To meet these requirements, each irregular face introduces 4 face points and forms the analysis space[6], each face point is associated with a face point basis function, the calculation of face points is detailed in [6]. The original T-mesh and its control points are called the design space, and the design space belongs to the analysis space. Fig. 2 gives examples of control points.



(a) (b)

Fig. 1: (a) The classification of faces, (b) The classification of control points. The irregular faces, transition faces, and regular faces are marked with blue, green, and white, respectively; irregular points, transition points, and regular points are marked with red, orange, and green, respectively; spoke edges are represented by yellow lines in (a).



(a) (b)

Fig. 2: (a) Control points in the design space, (b) Control points in the analysis space. Compared with the design space, in the analysis space, each irregular surface contains 4 face points. The classification of faces and control points is the same in both spaces, in addition, face points do not participate in the above classification of the control points.

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Bézier Extraction:
We assume that the reader is familiar with Bézier extraction. According to it, T-spline control points P can be transformed into Bézier control points Q^0: $Q^0 = EP$, where E is called extraction matrix. The transpose matrix E^T, called extraction operator M, reflects the transformation relationship between Bernstein basis functions $B^0(u,v)$ and T-spline basis functions $B^V(u,v)$, i.e. $B^V(u,v) = MB^0(u,v)$, which indicates that each row vector of the M can define a T-spline basis function. The row vectors are called extraction coefficients. In fact, the extraction coefficients reflect the corresponding Bézier control points, and modifying the extraction coefficients equivalent to modify the Bézier control points.

Smoothing Spoke Edge:
Based on D-Patch framework[1][3][6] and basis function truncation[1], using a half-edge data structure to express T-spline surfaces, we take a split-then-smoothen approach to achieve spoke edges smoothness for design space and analysis space, respectively. Unlike the direct modification of Bézier control points[6], for subsequent IGA, we modify the Bézier control points indirectly by modifying extraction coefficients, resulting in continuity enhancement. The procedure can be described as follows:

(1) In the design space, the approach is used for irregular faces, and obtain C^1 continuity crossing spoke edges. The approach is applied as: (a) For irregular face, we obtain all the basis functions that have support on this face and call them support basis function; (b) Obtain the extraction coefficients $(C_0^i, C_2^i, \ldots, C_{15}^i)(i = 1,2,\ldots,\mu)$ defining a support basis function on each irregular face, and arrange the extraction coefficients in the manner of Fig. 3(a); (c) Perform a 2×2 split of this extraction coefficients, then 4 new extraction coefficients $C_j^{i,pq}(i = 1,2,\ldots,\mu; j = 0,1,\ldots,15; p,q = 0,1)$ will be obtained, which are numbered as shown in Fig. 3(b), this can be regarded as using the De Castaljau algorithm at $u = u_0 / 2$, $v = v_0 / 2$ of the parameter domain to split it into 4 sub-element domains; (d) Repeat steps (b) and (c) for each support basis function, then the face is split into 4 sub-faces; all irregular faces are split, as shown in Fig. 3(d). (e) Modify the extraction coefficients $C_5^{i,00}, C_6^{i,00}, C_9^{i,00}$ using Eqn. (4.1), and Π^+ is smooth matrix[3].

$$\begin{bmatrix} a_5 \\ a_6 \\ a_9 \end{bmatrix} = \Pi^+ \begin{bmatrix} A_5 \\ A_6 \\ A_9 \end{bmatrix} \tag{4.1}$$

$$\Pi^+ = \begin{bmatrix} \Pi_7^+ & \Pi_8^+ & \Pi_9^+ \\ \Pi_4^+ & \Pi_5^+ & \Pi_6^+ \\ \Pi_1^+ & \Pi_2^+ & \Pi_3^+ \end{bmatrix}, \ \Pi_i^+ = \begin{pmatrix} \Pi_{i,11}^+ & \cdots & \Pi_{i,1\mu}^+ \\ \vdots & \ddots & \vdots \\ \Pi_{i,\mu 1}^+ & \cdots & \Pi_{i,\mu\mu}^+ \end{pmatrix} \tag{4.2}$$

$$\Pi_{i,jk}^+ = p_i^{(j-k)\%\mu} \tag{4.3}$$

where $p_1^j = p_4^j = p_7^j = 0$, $p_2^j = \frac{1}{2\mu}(1 + \cos(2\psi - j\varphi))$, $p_3^j = p_5^j = \frac{1}{2\mu}(1 + \cos(j\varphi))$, $p_6^j = \frac{1}{2\mu}(1 + \cos(2\psi + j\varphi))$, $p_8^j = p_9^j = \frac{1}{2\mu}$ and $p_1^j = p_4^{-j}, p_2^j = p_6^{-j}, p_3^j = p_5^{-j}, p_7^j = p_7^{-j}, p_8^j = p_9^{-j}$, a_k and $A_k (k = 5,6,9)$ are column vectors of extraction coefficients of dimension μ, for example $A_5 = \left[C_5^{1,00}, C_5^{2,00}, \cdots, C_5^{\mu,00}\right]^T$. The extraction coefficients to be modified are marked in Fig. 3(e) using red; (f) The affected face, edge, and vertex extraction coefficients should be updated.

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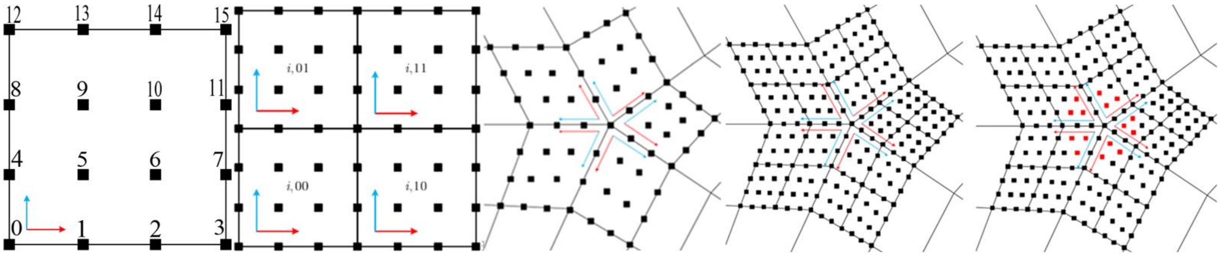
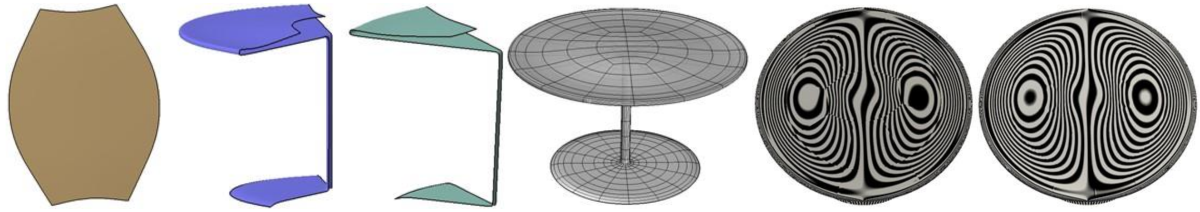


(a) (b) (c) (d) (e)

Fig. 3: (a) The distribution of extraction coefficients, (b) Numbering of the 4 sub-element, (c) and (d) are distribution of extraction coefficients on the irregular faces before and after splitting, (e) Extraction coefficients marked in red of irregular faces that need to be changed to smooth spoke edges.

(2) In the analysis space, the transition point basis function T can be expressed as a linear combination[1] as $T=\sum_{i=1}^{16} c_i m_i$, wherein m_i denotes the face point basis function of the face that contact transition point, c_i is called combination coefficient. Among the 16 face point basis functions, where there must be irregular face's face point basis functions, which causes the reuse. To avoid this situation, the contribution of irregular face's face point basis functions should be discarded. In the analysis space, the algorithm is applied to irregular and transition faces, and the steps to obtain C^1 continuity crossing spoke edges are as follows: (a) For an irregular or transition face, obtain support basis functions and extraction matrix E. In the extraction matrix, set extraction coefficients defining the irregular point and transition point basis functions to zero; (b) Truncate transition point basis functions and irregular point basis functions, then the new extraction coefficients defining this basis function will be obtained; (c) The new extraction matrix can be obtained by applying the algorithm in (1) to the irregular faces.



(a) (b) (c) (d) (e) (f)

Fig. 4: (a) (b) (c) are 3 NURBS surfaces, which can be used to merge into to obtain the table surface, of which surfaces in (b) and (c) are used twice, (d) is a T-spline table surface with 4 extraordinary points, (e) The zebra pattern on the top surface of the table before smoothing, (f) The zebra pattern on the top surface of the table after smoothing. We can see that: the zebra pattern near the extraordinary point before smoothing is discontinuous, indicating that only C^0 continuity; the zebra pattern near the extraordinary point after smoothing is continuous but not smooth, indicating that C^1 continuity.

Numerical Examples:

As shown in Fig. 4, we merge together multiple NURBS faces into a single T-spline surface and use the above algorithm to enhance the continuity around extraordinary points. Apply a distributed load downward on the top surface of the table model and a fixed constraint on the bottom edge, the material parameters are taken as: $E=3\times10^{11}$, $\nu=0.3$, $h=0.03$, $F=1.0$, we perform a simulation

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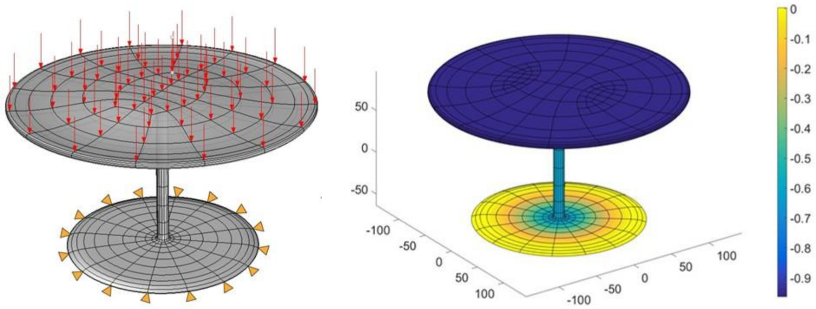
analysis and Fig. 5 shows the colors plots of vertical displacement. This analysis case shows that the model after smoothing has C^1 continuity, and the continuity analysis of the model before and after smoothing are shown through zebra pattern in Fig. 4, which can lay sound foundation for IGA.



(a) (b)

Fig. 5: (a) Boundary conditions applied for the bottom side of the table, and distributed load applied for the top surface of the table, (b) Plot of the numerical result of the table: vertical displacement.

Conclusion:

To enhance the spoke edge continuity appears in unstructured T-spline surfaces, the design space and analysis space continuity enhancement algorithms were given respectively in this paper. As shown in Fig. 4, when input multiple NURBS faces, they can be merged into one single unstructured T-spline surface, then the algorithm is executed so that the spoke edges have C^1 continuity. This procedure could be used to convert existing multiple NURBS surface model into T-spline surface model with at least C^1 inner continuity, which is easy to carry out IGA. With the technologies presented in this paper, more complex models from design department are possibly to be processed, thus offering a promising method to realize IGA scenarios in industry level practices.

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