Title:
Fairing Algorithm of T-spline Surface Based on Improved Sigmoid Function

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Introduction:

The surface fairing is of substantial significance in CAD, which can improve an aesthetic sense of designing and even reduce the machining difficulty in manufacturing. Compared with a NURBS surface, the T-spline surface [1] reduces the number of control points significantly, especially for the refinement algorithm. What's more, T-spline can simplify the modeling representation and increase flexibility in surface modeling. Based on these advantages, we focus on exploring T-splines. Surface smoothing is an essential link between modeling and analysis, and researchers have demonstrated that surfaces with fairness physical/computation domain will lead to more accurate and reliable analysis results [2]. To achieve the integration of model design and analysis, it's necessary to take further explore on surface fairing. Many scholars have worked on faring the physical domain, the main methods can be classified into three types. Including spline refinement [3], harmonic mapping [4], and nonlinear optimization [2]. But these methods either fails to guarantee the bijection of the mapping or are computationally expensive to optimize. There exist some fairing approaches to discrete surface representation, such as piecewise function [5], Laplacian coordinate [6], and bilateral filtering [7].

To take advantage of the nonlinear optimization method and reduce the computational time consuming, we propose a fairing algorithm for T-spline surfaces based on the Sigmoid function [8]. Sigmoid function, which has widely applied to neural networks, is continuous everywhere and convenient for derivation. This may provide smoothing weight transition in T-surface fairness without oscillation. Inspired by the above, we generalize the improved Sigmoid function as a weighting coefficient to the T-spline surfaces fairing problem. The parameter in the improved Sigmoid function is obtained by fitting another piecewise constant function proposed in [9]. Our method achieves a better result compared with the referenced algorithm, but with less programming by omitting judgment of space angle. Gaussian curvature of the T-spline surface is used to measure the fairness of the surface.

The 1-ring Neighboring Space Angle:

We use the space angle to measure the curvature of surfaces. As visualized in Fig. 1, the 1-ring neighboring control point of p_0 is consisted by a group of blue points that share uniform yellow edges with p_0, i.e. $\{p_1,\ p_2,\ p_3, p_4\}$. The yellow edges represented as $\{\overline{p_0p_1},\ \overline{p_0p_2},\ \overline{p_0p_3},\ \overline{p_0p_4}\}$ is called the 1-ring neighboring edge. And the 1-ring neighboring space angle θ is formed by the sum of angles that are formed by two adjacent yellow edges, as shown in (2.1). When the region contains significant features, the space angle is small, and θ trends to 2π as the region becomes flatten.

$$\theta = \sum_{i=1}^{4} \theta_i \tag{2.1}$$

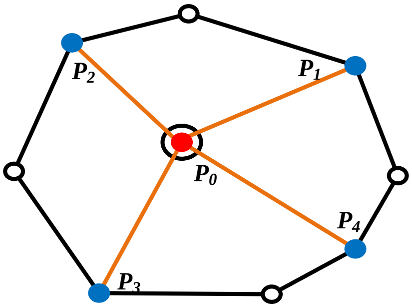

Fig. 1: The 1-ring neighboring control point, edge, and corresponding space angle of the control point p_0 in physical space.

The Improved Sigmoid Function:

The previous smoothing method utilizes the piecewise function to assign the weight in smoothing equation according to the space angle by giving higher weight to similar control points and lower weight to quite different control points. But the weight of the piecewise function mutates when space angles approach $\frac{\pi}{2}, \frac{3}{4}\pi, \pi$, and $\frac{3}{2}\pi$, as shown by the blue line in Fig. 2. To overcome the irregular oscillation at these angles and acquire stable smoothing results, we developed the improved Sigmoid function as the weight. The parameter of the improved Sigmoid function is determined by fitting the piecewise function, we get:

$$w = \frac{1}{1 + 9e^{-\theta}} (0 < \theta \leq 2\pi) \tag{2.2}$$

This expression accomplishes continuous fitting and decreases the programming process by omitting the judgment of space angle. The smooth translation of the weighting function may help to avoid the oscillation when the fairness continues.

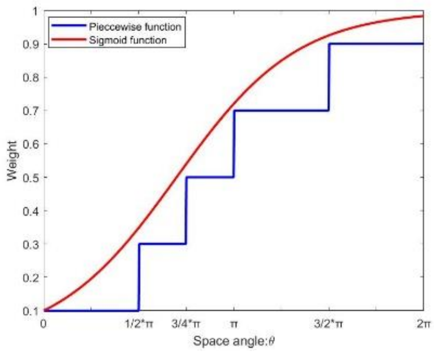

Fig. 2: The fitting result of the Sigmoid function.

Fairing Method and Error:

We adapt the control point movement method to smooth the T-spline surfaces. As demonstrated in expression (2.3), the control points in k iterations are calculated from the weighted average of the control points and their 1-ring neighboring control point in $k-1$ iterations.

$$p_i^{\,k} = wp_i^{k-1} + (1-w)\frac{1}{n}\sum_{j=1}^{n} p_j^{k-1} \tag{2.3}$$

where p_i represents the current control point, $\{p_j\}_{j=1}^{n}$ is the 1-ring neighboring control point of p_i, w is determined by the improved Sigmoid function, and k , which is used as a superscript, indicates the number of iterations.

Significant features should be maintained through smoothing, as in reference, we limit the movement of the control point if the distance between k and $k-1$ iterations is larger than the average length of its 1-ring neighboring edge. Hence, the movement constraint is expressed as following:

$$||p_i^k - p_i^{\,k-1}||_2 \le \frac{1}{n}\sum_{j=1}^{n}||p_j^{\,k-1} - p_i^{\,k-1}||_2 \tag{2.4}$$

where the $||p_i^k - p_i^{\,k-1}||_2$ is called an offset.

The fairing error is measured by the average Euclidean distance (AED) of the control points between k iterations and the origin model. The representation is in (2.5), wherein N is the number of control points, p_i^k indicates the control point i in k iterations, and p_i^0 expresses control point i in the origin model.

$$AED = \frac{\sum_{i=1}^{N}||\, p_i^k - p_i^0 \,||_2}{N} \tag{2.5}$$

Flowchart of the Algorithm:

The proposed T-Spline surface fairing method is accomplished by traversing all the control points on the surface, and the iteration is adaptive according to the presetting ADE. The whole flowchart of the proposed algorithm is shown in Fig. 3. The left part controls the loop of the program through ADE and the traverse of all control points. The right part circled by red dot lines is the core structure of the fairness method as illustrated above.

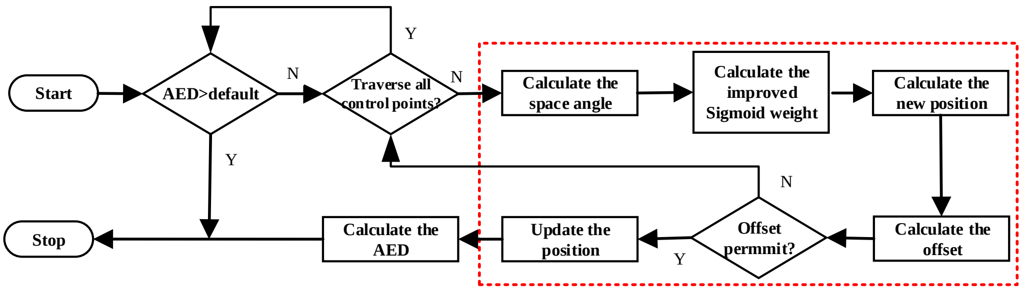

Fig. 3: The flowchart of our fairing algorithm.

Experimental and Discussion:

In this section, a comparison is executed with the fairing method based on a piecewise function [9] to verify the validity of our method. The SunFace model contains substantial details and the component information appears in table 1. The same default AED of 0.1 is given. And the literature's method and our method are shown in Fig. 4. (c-d) and (e-f) respectively. Although our method iterates more times, it achieves a better smooth result. As shown in figure 4, the fairing effects are obvious, especially in the eyes, nose, and mouth. Although depressions with abrupt high Gaussian curvature are repaired above the two eyeholes, our method reaches significant fairness on the nostril and at the corners of the mouth. Under the improved Sigmoid weight, our method eliminates the vibration problem caused by piecewise function smoothing and performs robustness even iteration increases.

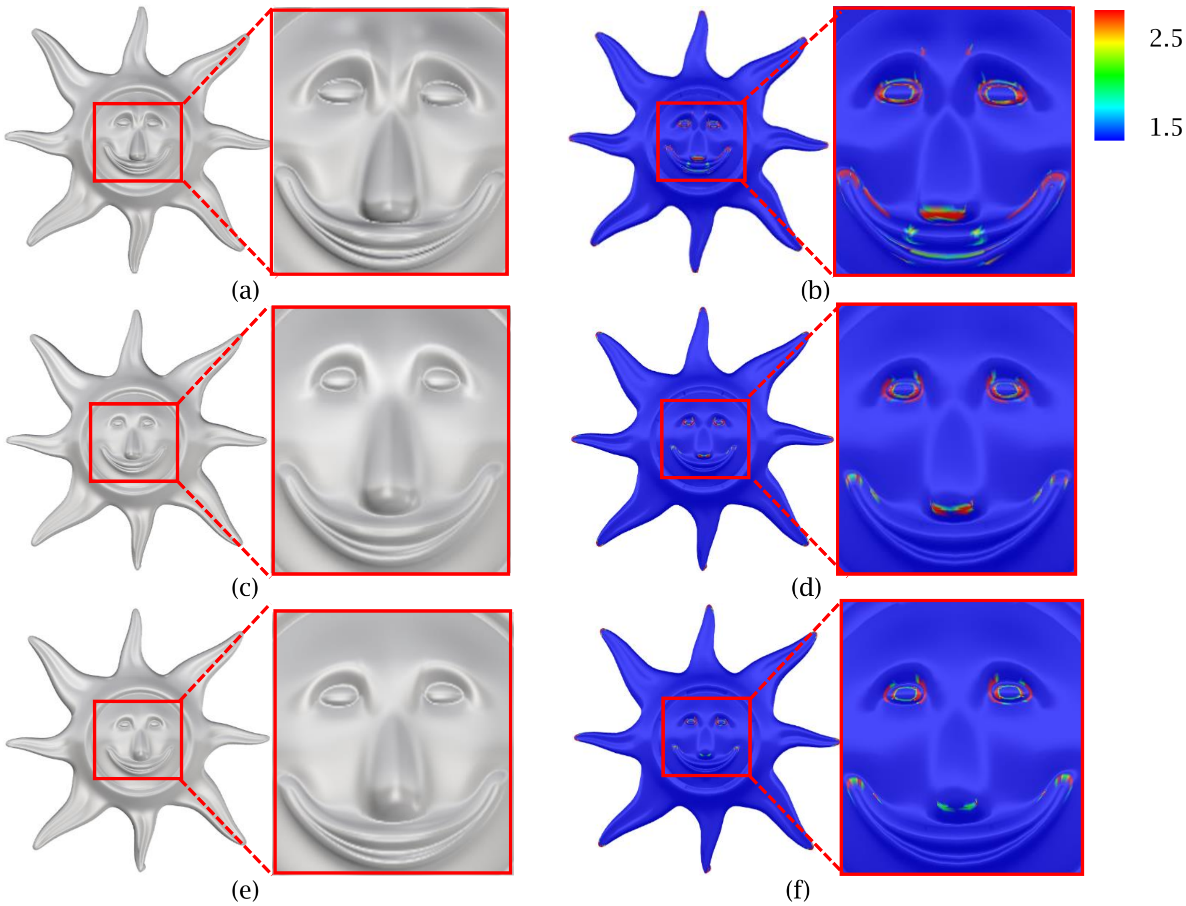

Fig. 4: A compared experiment is executed between our method and algorithm [9]. The first row is the rendering and Gaussian curvature of the Sunface model, the second row (c-d) shows the denoised results of the literature [9] by presetting AED 0.1, and the third row (e-f) is our method on the same default errors.

SunFace model	*Model information*				*iteration*	*AED*	*TIME(s)*
	face	*edge*	*vertex*	*Control point*			
Algorithm [9]	1024	2052	1060	1091	4	0.10	11.25
Our algorithm					20		49.25

Tab. 1: SunFace model information and performance data.

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Conclusions:

We developed a T-spline fairness method based on the weighted movement of control points, the weight is determined by an improved Sigmoid function. The proposed Sigmoid weight reduces the programming procedure of space angle judging, and this continuity enhances the stability of fairness work without oscillation.

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