Title:
Interactive G^1 and G^2 Hermite Interpolation Using Coupled Log-aesthetic curves

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Introduction:

Aesthetic curves are primarily used in computer-aided design to meet the high aesthetic requirements of the industry. Levien et al. stated that the log-aesthetic curve is the most promising curve for aesthetic design [3] and a large number of research papers are published since their introduction.

Yoshida et al. [6] analyzed the properties of the log-aesthetic curve and derived a general formula from the relationship between the arc length and the radius of curvature of the curve. The authors are also presented an interactive algorithm to draw a log-aesthetic curve segment, which is an essential approach in the log-aesthetic design and several algorithms have been developed based on their method.

In the algorithms dealing with log-aesthetic curve, normally there are restrictions on the curve, consequently drawing may not be possible for the given boundary condition depending on the shape parameter [1, 7]. A possible solution is the discretization of the log-aesthetic curve, proposed by Yagi et al. [5].

In this work, we propose a new interactive G^1 Hermite interpolation method based on the algorithm of Yoshida et al. [6]. The new method generates coupled log-aesthetic curve segments that may include inflection point or curvature-extremal point. Moreover, the user has control over the curvature of the first point that makes the approach capable of G^2 joining log-aesthetic curve segments as well.

Generating a log-aesthetic curve segment:

The general formula has been derived by Yoshida et al. [6]. They have defined a point $P(\theta)$ of the log-aesthetic curve with tangential angle θ, expressed in the complex plane as:

$$P(\theta) = \begin{cases} \int_0^\theta \mathrm{e}^{(1+i)\Lambda\psi}\, d\psi & \text{if } \alpha = 1 \\ \int_0^\theta \big((\alpha-1)\Lambda\psi + 1\big)^{\frac{1}{\alpha-1}} \mathrm{e}^{i\psi}\, d\psi & \text{otherwise,} \end{cases} \tag{2.1}$$

where $\alpha \in \mathbb{R}$ is the shape parameter and $\Lambda \in \mathbb{R}^+$ is a transformation parameter. When $\alpha \neq 1$, all of the aesthetic curves are congruent under similarity transformation, depending on the value of $\Lambda(\neq 0)$.

Appropriately selecting α, the following curves can be generated: clothoid ($\alpha = -1$), Nielsen's spiral ($\alpha = 0$), logarithmic spiral ($\alpha = 1$), circle involute ($\alpha = 2$), or the circle itself ($\alpha = \pm\infty$).

From the relationship between the tangential angle θ and the arc length s, a point $C(s)$ on the aesthetic curve can be defined on the complex plane as ([6]):

$$C(s) = \begin{cases} \int_0^s \exp(i\frac{1-\mathrm{e}^{-\Lambda u}}{\Lambda})\,du & \text{if } \alpha = 0 \\ \int_0^s \exp(i\frac{\log(\Lambda u+1)}{\Lambda})\,du & \text{if } \alpha = 1 \\ \int_0^s \exp(i\frac{(\Lambda\alpha u+1)^{(1-\frac{1}{\alpha})}-1}{\Lambda(\alpha-1)})\,du & \text{otherwise.} \end{cases} \tag{2.2}$$

Eq. (2.2) and Eq. (2.1) represent the same curve.

The tangential angle θ and arc length s may have upper or lower bounds depending on the value of α (because of the negative bases of the fractional exponents):

	Tangential angle (θ)			**Arc length (s)**		
	$\alpha < 1$	$\alpha = 1$	$\alpha > 1$	$\alpha < 0$	$\alpha = 0$	$\alpha > 0$
Upper bound:	$\frac{1}{\Lambda(1-\alpha)}$	-	-	$-\frac{1}{\Lambda\alpha}$	-	-
Lower bound:	-	-	$\frac{1}{\Lambda(1-\alpha)}$	-	-	$-\frac{1}{\Lambda\alpha}$

Table 1: Upper and lower bound of θ and s.

Besides the general equations of the log-aesthetic curve, Yoshida et al. also presented an interactive algorithm [6] to generate a log-aesthetic curve segment by specifying three so-called control points (similarly in case of a quadratic Bézier curve) and α. The idea is to search for a curve segment that fits a similar triangle defined by the control points, using a bisection method on Λ. The curve is drawn from the first control point A to the last C, while point B specifies the change of the tangential angle θ_Δ between the endpoints (see Figure 1).

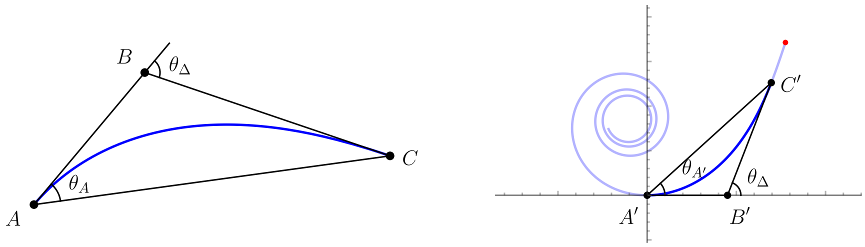
The main drawback of their algorithm is that the position of the control points and the value of α highly restrict the region where the curve can be drawn. In [6], the area where the curve segment can be found is called the drawable region. If α is less than 0 or greater than 1, this drawable region is getting to be drastically smaller (see e.g. Figure 9 in [6]). To extend the capability of their algorithm, we introduce the new, coupled log-aesthetic curves.

New Λ bisection method using coupled log-aesthetic curves:

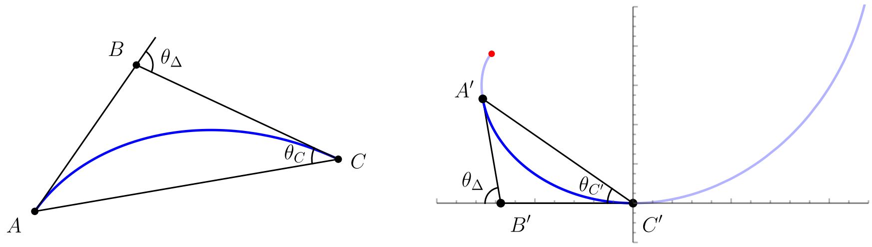
The coupled log-aesthetic curve is defined by mirroring the original curve at the bounds. In case of $\alpha < 1$, θ has an upper bound (see Table 1). Although, it does not restrain the algorithm until $\alpha \geq 0$ because at the point of $\theta = \frac{1}{1-\alpha}$ the arc length s is infinite. Therefore, using coupled log-aesthetic curve is only required in the cases of $\alpha > 1$ and $\alpha < 1$ (see Figure 2).

In our approach, a new Λ bisection method is used, improving the method provided in [6] in order to find a coupled log-aesthetic curve segment that fits the triangle ABC. The desired curve segment is specified by two points (A and C) and two vectors $\vec{v_A}$ and $\vec{v_C}$, where $\vec{v_A}$ is the tangent vector of the curve at point A, and $\vec{v_C}$ defines the direction of the tangent line at C. The point B is the intersection point of the tangent lines. The difference of the tangential angle between the first and last endpoint (θ_Δ) is obtained by calculating the angle between $\vec{v_A}$ and $\vec{v_C}$.

The new Λ bisection with the given geometric data determines a coupled log-aesthetic curve segment with an arbitrary value of $\alpha \in \mathbb{R}$, providing a solution to design using aesthetic curves with only a minor boundary condition.



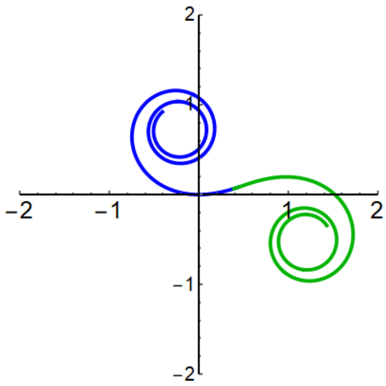
(a) $\alpha \leq 1$



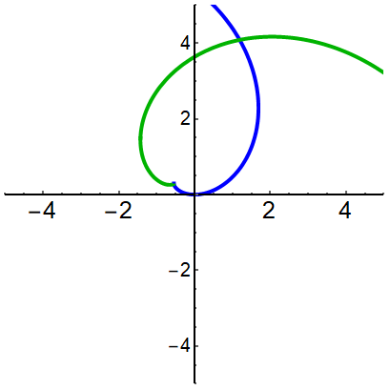
(b) $\alpha > 1$

Fig. 1: Different configurations. The red point is the bound of the curve (based of Fig. 7 in [6]).

<u>Controlling the tangent length and curvature at the first point using an α bisection method:</u>
Regarding Harada et al. [2], α is related to the impression of the curve. However, it is difficult to choose a suitable value for it, therefore, it is a common practice to fix the shape parameter in the design process with log-aesthetic curves. The novel algorithm determines the shape parameter α to match also the length of the given vector $\vec{v_A}$. Since the same geometric data with different α parameter require different Λ values, an exact calculation is not possible, thus the suitable value is found by another bisection method.



(a) $\alpha < 1$



(b) $\alpha > 1$

Fig. 2: Coupled log-aesthetic curves, defined by concatenating the mirrored parts of the original curve.

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The log-aesthetic curve has the property that the length of the tangent equals the radius of curvature, hence the user also controls the curvature of the first point, which makes the algorithm capable of G^2 joining log-aesthetic curve segments.

An example can be seen in Figure 3. Regarding [4], designing fonts using aesthetic curves yields better results than using standard free-form curves because the design of font variation is more accessible and productive with these curves (e.g. interpolation between cubic Bézier-curves may fail to preserve even G^1-continuity).

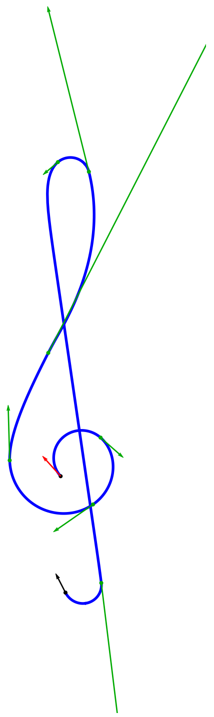

Fig. 3: The violin (G) clef designed using the presented algorithm. The extended log-aesthetic curve segments are connected at the green points with G^2 continuity (both tangents and curvatures are match). The curve segments are controlled by the position of the points, by the direction of the tangent vectors (black, green, and red arrows) and by setting the length of the first tangent vector (red).

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