Title:
Interpolation of Point Sequences with Extremum of Curvature by Log-aesthetic Curves with G^2 continuity

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Introduction:

Yan et al.[1] create a quadratic Bezier curve sequence that interpolates control points and takes local maxima and minima of curvature only at the control points called κ-curves. Their interpolation method guarantees G^2 continuity at control points except at inflection points and guarantees only G^1 continuity there. Recently there are a lot of researches on curves for aesthetic design, and among them the log-aesthetic curve is indicated by some research to be the most promising curve for its ability of controling the curvature [2].

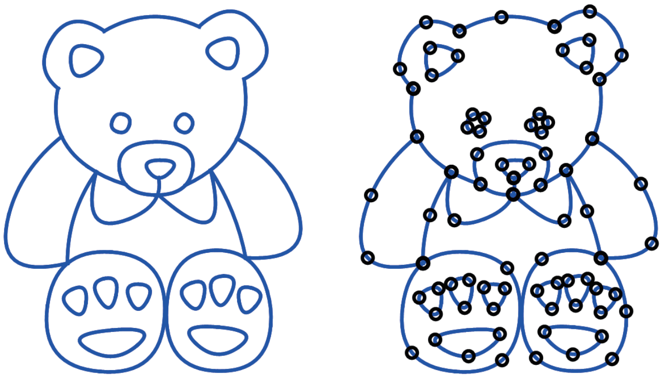

Fig. 1: An example of a κ-curve[1].

Interpolation of Points With LA Curve:

Aesthetic curves were proposed by Harada et al.[3] as such curves whose logarithmic distribution diagram

Proceedings of CAD'20, Barcelona, Spain, July 6-8, 2020, 313-317

of curvature (LDDC) is approximated by a straight line. Miura et al.[4, 5] derived analytical solutions of the curves whose logarithmic curvature graph (LCG): an analytical version of the LDDC [3] are strictly given by a straight line and proposed general equations of aesthetic curves and their solutions as log-aesthetic curves. Furthermore, Yoshida and Saito[6] analyzed the properties of the curves expressed by the general equations and developed a new method to interactively generate a curve by specifying two end points and the tangent vectors there with three control points as well as the slope of the straight line of the LCG. In this paper, we propose a new method that enables deformation by shape parameters, guarantees that G^2 continuity at the control points by replacing the quadratic Bezier curve with a log-aesthetic curve.

General Equations of Aesthetic Curves:

In this section, Logarithmic Curvature Graph (LCG) which is given by a straight line (linear LCG), is used to describe LAC equation. The equation for LCG which has a slope, α, is the fundamental equation of LAC [4]:

$$\log(\rho\frac{ds}{d\rho}) = \alpha\log\rho + C \tag{2.1}$$

where s is the arc length of a curve,
ρ is the radius of curvature,and
C is the constant.

The following is obtained by differentiating and substituting $\Lambda = e^{-C}$ (Λ is the shape parameter of LAC) into the equation where Λ is in the range [0,∞] ,

$$\frac{ds}{d\rho} = \frac{\rho^{\alpha-1}}{\Lambda} \tag{2.2}$$

Integrating equation (2) and rewriting ρ in terms of arc length, s :

$$\rho = \begin{cases} e^{\Lambda s} & \alpha = 0 \\ (1+\Lambda\alpha s)^{\frac{1}{\alpha}} & \text{otherwise} \end{cases} \tag{2.3}$$

ρ can be expressed in terms of turning angle, $\theta(s)$ by using equation (2) into $\frac{d\theta(s)}{ds} = \frac{1}{\rho}$ and then integrating it
we obtain:

$$\rho = \begin{cases} e^{\Lambda\theta(s)} & \alpha = 0 \\ (1+(\alpha-1)\Lambda\theta(s))^{\frac{1}{\alpha-1}} & \text{otherwise} \end{cases} \tag{2.4}$$

ρ varies from 0 to ∞ , s and $\theta(s)$ have various upper bounds and lower bounds which depend on the α value. The upper and lower bounds of s and $\theta(s)$ with respect to α can be obtained from [6].

$$P(\theta) = \begin{cases} \{\int_0^\theta e^{\psi\Lambda}\cos(\psi)d\psi, \int_0^\theta e^{\psi\Lambda}\sin(\psi)d\psi\} & \alpha = 1 \\ \{\int_0^\theta (1+(\alpha-1)\Lambda\psi)^{\frac{1}{\alpha-1}}\cos(\psi)d\psi, & \\ \quad \int_0^\theta (1+(\alpha-1)\Lambda\psi)^{\frac{1}{\alpha-1}}\sin(\psi)d\psi\} & \text{otherwise} \end{cases}$$

Proceedings of CAD'20, Barcelona, Spain, July 6-8, 2020, 313-317

Log-aesthetic Curve Generation Algorithm:

As shown in Figure 1, we generate a sequence of aesthetic curves with G^2 continuity by the following algorithm:

1. Input a sequence of control points P_i.
2. Generate κ-curves interpolating the control points P_i.
3. Calculate the tangential angle of κ-curves connection as initial value of tangent angle θ_{ei} of LA curve.
4. Calculate the line length l_i between control points P_i, the inner angle of the control polygon θ_i, so another angle between the tangential line and the connection line $\theta_{fi} = \pi - \theta_i - \theta_{ei}$
5. Generate LA curves with a specified α and a triangle located at the coordinate origin and supposed to be similar to the triangle defined by the control points.
6. Find Λ that satisfies the similar conditions of two triangles with bisection method.
7. Translate, roltate and scale the LA curve to the trangle defined by the control points.
8. Calculate the curvature $\kappa_{i,1}, \kappa_{i+1,0}$ of the LA curves at the control points.
9. Compare the curvature difference $\Delta\kappa_i$ at the control points. The G^2 connection is achieved if, for example, $\Delta\kappa_i < 10^{-6}$. If it is not satisfied, find the θ_{ei} that satisfied the condition of $\kappa_{i,1} = \kappa_{i+1,0}$.
10. Take the new θ_{ei} to Step 8. to recalculate the curvature of each LA curve segment at control points until $\Delta\kappa_i$ meets the accuracy requirement.

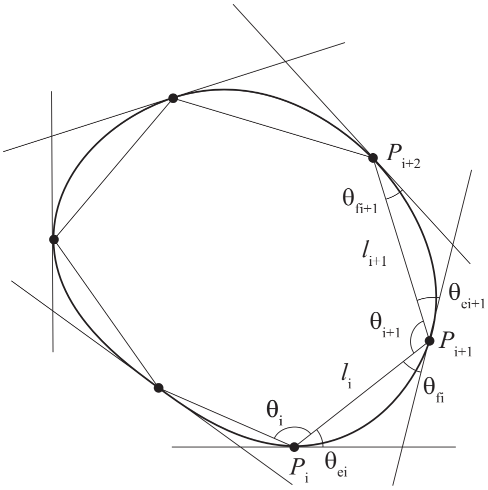

Fig. 2: The notation for control points and tangential angles of LA curves.

Examples of LA curve generation:

As shown in Figure 2, we input six green control points, and form a control polygon. The α is specified to be -0.5. At first the algorithm generates κ-curves interpolating the control points, then calculate the tangent line at the control points, using the tangential angle as the initial turning angle of each LA curve segment. After iterating about 50 to 100 times, the difference of the curvature $\Delta\kappa$ satisfied the precison of 10^{-6}. The purple lines represent the control polygon and the pink ones are tangential lines. The κ-curve is drawed by green lines while the LA curve is in red. We can find there is some difference of their shapes between those curves.

Conclusions and Future Work:

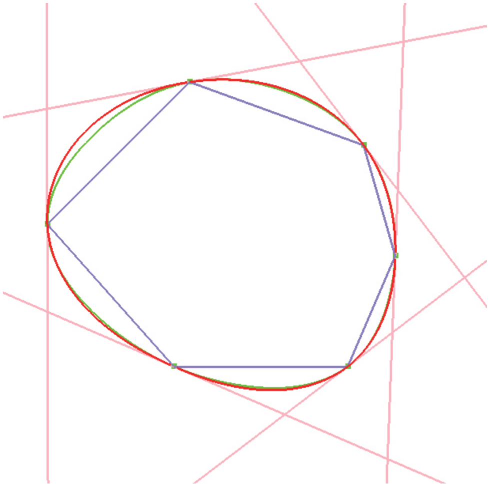

Fig. 3: The comparetion between κ-curve and LA curve. The red line represents LA curve and the green line represents κ-curve.

This paper proposes a new method that enables deformation by shape parameters, guarantees that G^2 continuity at the control points by replacing the quadratic Bezier curve with a log-aesthetic curve. We have compared the shape between κ-curves and log-aesthetic curves. But we only use C-shape Log-aesthetic curves to interpolate the control points, our next work in progress includes adapting S-shape log-aesthetic curves to interpolate the inflection points.

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Proceedings of CAD'20, Barcelona, Spain, July 6-8, 2020, 313-317