Title:
Efficient Extraction of Information from Correlation Matrix for Product Design

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Introduction:
Product functions are decided in the design stage. A variety of design concepts can be proposed for the product functions to meet customer requirements (CRs) and design constraints. Quality Function Deployment (QFD) is an approach for mapping CRs to technical requirements (TRs) and guiding designers to search product solutions. House of quality (HOQ) is an essential mapping tool in QFD which helps designers to find TRs of design concepts and their correlations. Using HOQ, relations of CRs and TRs can be identified to find their effects. In the existing HOQ template, effects of TRs on each other are represented in a correlation matrix which is a triangle-shaped 'roof' of HOQ. Correlations of TRs are represented with signs or numbers to express the degree of relations between TRs. However, information based on this representation is not clear enough to provide designers details in considering all of TRs interactions [2]. Another problem in the roof of HOQ is that the intersection between two TRs shows only their effect degree, we cannot conclude whether they have the same effect on each other. For example, a TR may have a strong effect on another TR, but it doesn't necessarily mean that the reverse is the same.

Information in the roof of HOQ is also seldom used by designers as it is difficult to quantify interactions of TRs [2]. Some researchers combined effects of the matrix with TR's weight to rank them. For example, a weighted average method was used to prioritize and rank TRs [4]. But if TRs have an equal strength of correlations, their weights will remain the same. In another research, a method was introduced to search correlations between TRs for importance weights [1], but this method decides correlation coefficients at the criterion level, not the sub-criterion level. In another study, an improved QFD framework was presented for the service quality management [3]. An integrated QFD-AHP-ANP approach was used for analysis of relations and correlations of TRs. However, this method is time-consuming as it requires many pairwise comparisons. It is necessary for designers to know whether a TR has more effect than others. Although there are a few studies addressed effects of the correlation matrix of HOQ on the QFD process, based on our knowledge, none of them provides a systematic method to decide TRs interactions and their influence on each other for design solutions using information of the roof part in HOQ. Moreover, even literature on the prioritization of TRs is vast, a significant gap exists in the efficient extraction of information from the correlation matrix. To overcome the above-mentioned limitations, this paper introduces a method for analyzing the roof data of HOQ to extract additional information of correlations between TRs, which will help designers in TRs selections. The method integrates decision-making trial and evaluation laboratory (DEMATEL), analytic network process (ANP), and QFD to model interrelations of TRs. DEMATEL is used for understanding of correlation and interdependence of TRs through analysis of TRs in cause and effect relations. The final weight is obtained from an ANP super-matrix. Design of a hand rehabilitation device in the case study verifies the proposed method.

Main Idea:

Error! Reference source not found. shows a HOQ based on known CRs and TRs of a specific product. Data are collected from users and experts to indicate degrees of the direct effect of TRs on each other in the correlation matrix with a scale range from 0 to 4 (from “no effect (0)” to “very high effect (4)”). In a common HOQ as shown in Figure 1, it is assumed that TRs have the equal effect in intersections of the correlation matrix, which results in a triangular shape of the roof. However, this simplified assumption will affect the TRs selection in design. Therefore, the suggested form of the correlation matrix is a non-symmetric square matrix. The proposed method can abstract design information from both triangle-shaped and square-shaped correlation matrix. An initial matrix A can be formed either by converting the triangular shape correlation matrix into a symmetric square matrix or using a non-symmetric square matrix.

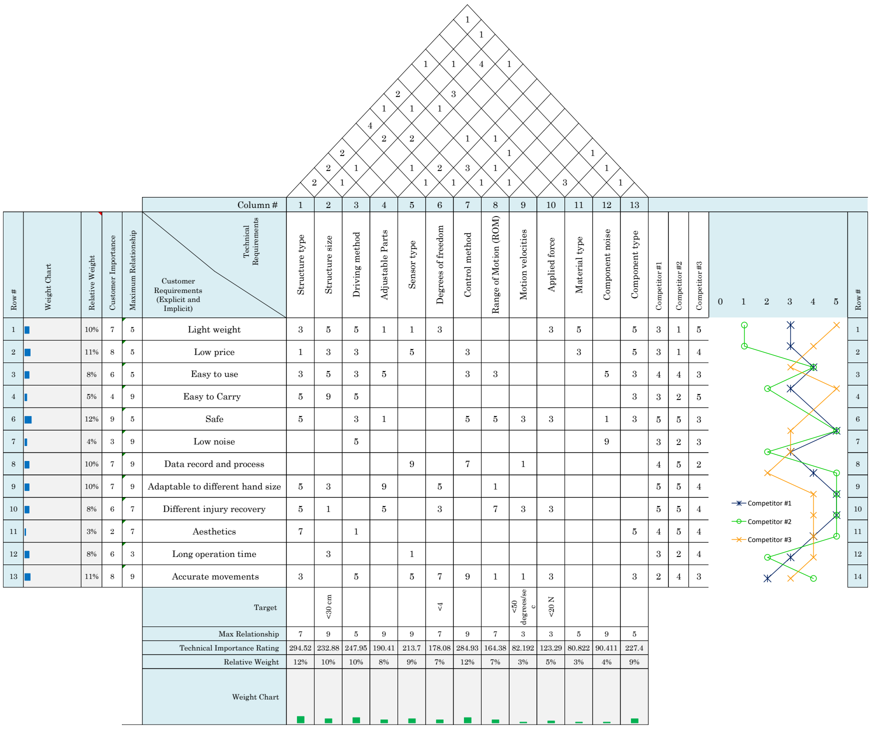

Fig. 1: HOQ based on known CRs and TRs of a hand rehabilitation device.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nj} & \cdots & a_{nn} \end{bmatrix} \tag{1}$$

A normalized initial influence-relation matrix N is formed by normalizing average matrix A.

$$N = \frac{A}{max\left(\sum_{j=1}^{n} a_{ij}, \sum_{i=0}^{n} a_{ij}\right)} \tag{2}$$

$$\lim_{m\to\infty} N^m = [0]_{n\times n}, 0 \le x_{ij} \le 1 \tag{3}$$

The total influence matrix T is a n × n matrix as follows.

$$\begin{aligned} T &= N + N^2 + \cdots + N^m \\ &= N(I + N + N^2 + \cdots + N^{m-1}) \\ &= N(I + N + N^2 + \cdots + N^{m-1})(I-N)(I-N)^{-1} \\ &= N(I-N)^{-1}, \quad When \lim_{m\to\infty} N^m = [0]_{m\times m} \end{aligned} \tag{4}$$

Where I is an $n \times n$ unit matrix. TRs can be grouped based on similar criteria. For example, we can form a cost sub-matrix including manufacturing cost and operation cost. The structure of total-influenced matrix for sub-criterion T_c is shown below:

$$T_c = \begin{array}{cc} & \\ D_1 & \begin{array}{c} c_{11} \\ \vdots \\ c_{1m_1} \end{array} \\ \vdots & \begin{array}{c} c_{i1} \\ \vdots \\ c_{im_i} \end{array} \\ D_i & \\ \vdots & \begin{array}{c} c_{n1} \\ \vdots \\ c_{nm_n} \end{array} \\ D_n & \end{array} \overset{\begin{array}{ccccc} D_1 & \cdots & D_j & \cdots & D_n \\ c_{11}\cdots c_{1m_1} & \cdots & c_{i1}\cdots c_{im_i} & \cdots & c_{n1}\cdots c_{nm_n} \end{array}}{\begin{bmatrix} T_c^{11} & \cdots & T_c^{1j} & \cdots & T_c^{1n} \\ \vdots & & \vdots & & \vdots \\ T_c^{i1} & \cdots & T_c^{ij} & \cdots & T_c^{in} \\ \vdots & & \vdots & & \vdots \\ T_c^{n1} & \cdots & T_c^{nj} & \cdots & T_c^{nn} \end{bmatrix}} \tag{5}$$

Where D_n symbolizes the nth criterion; C_{nm} represents the mth sub-criterion in the nth criterion. Unweighted super matrix Q is a transpose of normalized total influence-relation matrix T_c^{norm} .

$$Q = (T_c^{norm})' \tag{6}$$

The weighted super matrix is calculated by multiplying normalized total influence-relation matrix of dimensions T_D^{norm} and unweighted super matrix as follows.

$$W = T_D^{norm} \times Q \tag{7}$$

The weighted super matrix is then limited to a large power α until it converges in a stable super-matrix. Influential weights will be used in HOQ to obtain the final weight of TRs.

$$Influential\ weights = \lim_{\alpha\to\infty} (W)^{\alpha} \tag{8}$$

<u>Case study:</u>

Design of a hand rehabilitation device is a complex decision-making process. There are some TRs with close correlations to affect each other. In HOQ, these TRs are decided to meet CRs. It is not enough to decide TRs' weights only based on their interactions with CRs in the traditional HOQ method. It is therefore necessary to find interrelations of TRs using the correlation matrix in the roof of HOQ. 13 TRs were identified based on benchmarking products. The initial correlation matrix was also obtained. Two scenarios were investigated to compare with the traditional HOQ method. The first one used the triangular shape roof of HOQ to form a symmetric initial correlation matrix. The second used a square asymmetric correlation matrix. After normalizing the initial data, total-influenced matrix T_c was formed using Eqn. (5) for both cases. In matrix Tc, D and R were calculated by adding elements of each row and column in the matrix, respectively. Elements of each row (D) were added for each TR to find its effectiveness on other TRs. Elements of each column (R) were added for each TR to find the total effects (both direct and indirect) received from other factors, or effectiveness of variables. These parameters show effects of TRs on each other. In Figure 2, horizontal vector (D+R) is amount of TR interactions. In other words, it shows amount of TR interactions and a central role of the product. Vertical vector (D-R) indicates the effect power of each TR. Therefore, a positive (D-R) shows the causal parameter, otherwise, the parameter receives influence from other TRs. Figure 2 shows prominence-causal relations obtained by the total influence-relation matrix Tc. For the first case, as the initial matrix is symmetric, all of D-R values are zero, meaning that they have the same effect on each other. The horizontal axis however shows important TRs. For the second case, it can be concluded that the structure type has a big impact on other TRs, and motion velocities and structure size have effect from other TRs.

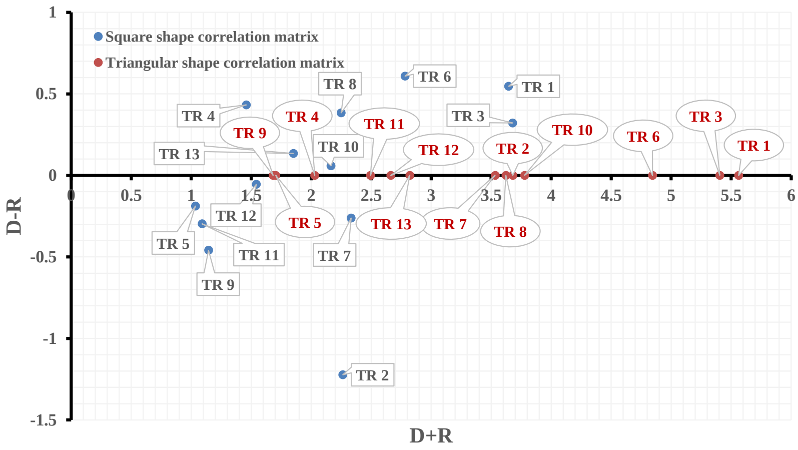

Fig. 2: DEMATEL prominence-causal relationship of TRs.

Row #	Column #	1	2	3	4	5	6	7	8	9	10	11	12	13
		TR 1: Structure type	TR 2: Structure size	TR 3: Driving method	TR 4: Adjustable Parts	TR 5: Sensor type	TR 6: Degrees of freedom	TR 7: Control method	TR 8: Range of Motion (ROM)	TR 9: Motion velocities	TR 10: Applied force	TR 11: Material type	TR 12: Component noise	TR 13: Component type
1	TR 1: Structure type	0.10477	0.1586	0.11379	0.21583	0.08954	0.18991	0.11533	0.16658	0.07326	0.11004	0.07466	0.07294	0.12013
2	TR 2: Structure size	0.10496	0.06707	0.06804	0.13018	0.11997	0.11323	0.05639	0.09844	0.04663	0.05593	0.09122	0.0479	0.09365
3	TR 3: Driving method	0.11058	0.09991	0.10139	0.07166	0.07302	0.07418	0.14136	0.10333	0.15768	0.17545	0.1497	0.2628	0.13095
4	TR 4: Adjustable Parts	0.07875	0.07178	0.0269	0.05121	0.03001	0.06564	0.02899	0.04095	0.01865	0.02786	0.02468	0.02063	0.07256
5	TR 5: Sensor type	0.02744	0.05557	0.02303	0.02521	0.04662	0.0504	0.0926	0.03213	0.10011	0.02134	0.01713	0.01815	0.02285
6	TR 6: Degrees of freedom	0.16544	0.14905	0.0665	0.1567	0.14325	0.1041	0.10528	0.18536	0.06554	0.09083	0.05639	0.0442	0.0691
7	TR 7: Control method	0.07326	0.05413	0.09241	0.05047	0.19192	0.07677	0.06269	0.0875	0.131	0.08504	0.05207	0.06103	0.08957
8	TR 8: Range of Motion (ROM)	0.1085	0.09688	0.06925	0.0731	0.06829	0.13859	0.08971	0.07712	0.11913	0.0521	0.03962	0.0454	0.04909
9	TR 9: Motion velocities	0.02217	0.02132	0.0491	0.01547	0.09884	0.02277	0.0624	0.05535	0.03929	0.02455	0.02236	0.07626	0.02372
10	TR 10: Applied force	0.07476	0.05742	0.12265	0.05187	0.0473	0.07084	0.09095	0.05434	0.05513	0.08149	0.20845	0.08359	0.10559
11	TR 11: Material type	0.03348	0.06182	0.06908	0.03034	0.02507	0.02903	0.03676	0.02728	0.03313	0.13759	0.06647	0.0929	0.08664
12	TR 12: Component noise	0.03493	0.03466	0.12951	0.02708	0.02836	0.0243	0.04601	0.03339	0.1207	0.05892	0.09922	0.08203	0.08701
13	TR 13: Component type	0.06094	0.0718	0.06836	0.10088	0.03782	0.04024	0.07153	0.03824	0.03976	0.07885	0.09802	0.09216	0.04913

Tab. 1: Weighted super matrix of the device of case study one.

Weighted super matrix Eqn. (7) was formed by normalizing and processing total-influenced matrix T_c. Table 1 shows the weighted super matrix for case study one. The influential weights were converged after four iterations (Eqn. (8)) for a stable matrix of TR weights. By using the proposed method, we can extract more information from correlations in the roof matrix and re-rank final weights of TRs. Table 2 shows TR weights based on the proposed method and traditional approach. The result shows that some TRs' weights have changed compared to those generated by the traditional method. Attentions should be paid to these changes in design. For example, in case study one, the weight of deriving method was increased to indicate its impact on other TRs like motion velocities, and the reduced weight of sensor type shows its weak interactions with other TRs. Although using the triangular shape correlation matrix can improve weighting method based on the proposed method, the result obtained by the square shape correlation matrix is closer to reality as they fully considered interactions of TRs correlations in two directions.

	TR 1: Structure type	TR 2: Structure size	TR 3: Driving method	TR 4: Adjustable Parts	TR 5: Sensor type	TR 6: Degrees of freedom	TR 7: Control method	TR 8: Range of Motion (ROM)	TR 9: Motion velocities	TR 10: Applied force	TR 11: Material type	TR 12: Component noise	TR 13: Component type
Traditional HOQ	0.12216	0.09659	0.10284	0.07898	0.08864	0.07386	0.11818	0.06818	0.03409	0.05114	0.03352	0.0375	0.09432
Proposed method (triangle shape correlation matrix)	0.12695	0.09183	0.11467	0.06168	0.05985	0.09216	0.09946	0.07655	0.03685	0.06761	0.04446	0.04858	0.07937
Proposed method (square shape correlation matrix)	0.12965	0.10701	0.12125	0.05831	0.06783	0.07334	0.10325	0.06615	0.04361	0.06272	0.04046	0.04902	0.07741

Tab. 2: TRs weights based on the proposed method and traditional approach.

Conclusions:

This paper introduced a combined method for modeling TRs correlations in HOQ to decide TR's weights in product design. It is identified that TRs interactions cannot be fully considered using the triangular shape correlation matrix in the traditional HOQ. A square roof of the HOQ template is recommended. The proposed method can model the influence and interdependence among TRs, which is neglected in traditional QFD methods. The case study of design for a hand rehabilitation device verified the proposed method. Detailed evaluations of the proposed method will be presented in the full paper using design solutions of the hand rehabilitation device.

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