Title:
Convergence and Remeshing Criteria for Fitting Method Based on Iterative Reparameterization via Plane-Stress Model

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Keywords:
Reverse Engineering, Parametric Surface Fitting, Shape Parameterization, Shape Optimization, B-spline

DOI: 10.14733/cadconfP.2020.220-225

Introduction:
In the process of reverse engineering [10] a physical part is used to construct a CAD model. Two main steps of the reverse engineering process are: first part is the digitalization of physical object which commonly performed by the object 3D surface scanning and the second step is constructing a compact surface CAD model. In one of the reverse engineering paths, the object can be divided in a user-selected patches which can be represented by a NURBS surface. For example, in ship hull representation a user might select each side of the hull to be a separate surface patch. In this case, a NURBS surface fitting algorithm is required for each patch separately. As surface fitting is common topic in computer-aided design there already exists a large amount of literature [11] on these topics. The techniques for surface fitting can be divided to parameterization-based and the ones that do not use parameterization. In the parameterization-based methods, the crucial part of the fitting method is the point-cloud parameterization in which parameters u,v are allocated to all points of the 3D point-cloud. These methods can be divided to parameterization of organized [2] and unorganized points [3]. This paper deals with reverse engineering methods in which we assume that 3D model exists in a form of triangulated point cloud (surface mesh), and the objective is to construct a B-spline surface using a user-defined part of the surface.

Triangulated mesh parameterization can be achieved by many different methods. When the parameterization is presented as mapping the 3D surface to planar domain, it can be illustrated that different methods result in different distortions in angles and areas. These distortions are important in reverse engineering, as they lead to a distorted B-spline control-point grid after fitting. Ideally, the fitting method based on parameterization should lead to a smooth B-spline control-point grid but keeping fitting error as small as possible. This is usually a problem in which compromises must be made where one objective is sacrificed for the other. One of the basic parameterization methods is the harmonic mapping [9]. Harmonic map can be computed efficiently (using finite element method (FEM) for example), and the results are good in most simple cases, as the method minimizes deformation of the map (Dirichlet energy). However, for complex examples this method can allocate to few control points to areas which are of interest in engineering cases. Many other different methods exist. A very different method is the one in [4,1], which performs parameterization based on elastic springs. This method normally leads to more distortions but is easy to implement and can be even more computationally efficient. To improve the fitting regarding the capturing of geometric features, method was developed in [5] called the feature sensitive parameterization. This method performs point cloud projections to parametric u-v space such that the areas which contain geometric features are allocated more space in the parameter domain. A drawback of this method is that the smoothness of the resulting B-spline control-point is low as many points are sharply concentrated at the geometric features ignoring possible distortions. The method used for parameterization-based B-spline fitting in this paper can be regarded as a compromise between the harmonic method (very smooth control-point grid, but possible large fitting error) and the feature sensitive parameterization (low error but distorted control-point grid). The method is based on the fitting method used in [6,7]. In this method, initial parametrization is constructed using the harmonic mapping method. The parameterization is subsequently revised using feature-related and smoothness-related metrics. To perform the re-parameterization, plane-stress problem is iteratively solved where the local modulus of elasticity is reduced (equivalent to heating the elastic plate) according to selected set of values.

Surface representation and fitting:
Here, point-cloud is defined as a set of 3D points $\boldsymbol{PC}_i$, where i=1-p. Fig 1. Is an illustration of a point-cloud. The set of points is connected with edges as shown in the figure (triangulation). Here, the geometry represents complete boat-hull. This is a user-defined segment (hull outer surface) that is extracted from full 3D scanned boat-hull. The segment has a boundary B and the boundary has 4 corner points which are also assumed to be user-defined, and the objective of this paper is to investigate methods for B-spline fitting which would afterwards require no additional (of very little) user input.

Other important terms regarding the point cloud in this paper are the point-cloud parameterization and its inverse. The parameterization $\mathbf{f}(\mathbf{PC}_i)$ is a mapping operator $\mathbb{R}^3 \to \mathbb{R}^2$ while the inverse operation of the parameterization is called the functionally determined point-cloud $\mathbf{F}(u,v)$ ($\mathbb{R}^2 \to \mathbb{R}^3$). The objective of the parameterization is to assigns parameter (u_i,v_i) to each point i of the point-cloud. Thus, the inverse operator should result in the original geometry. Since the original surface is assumed to be composed of triangles, the functionally determined point-cloud $\mathbf{F}(u,v)$ uses the trilinear interpolation.

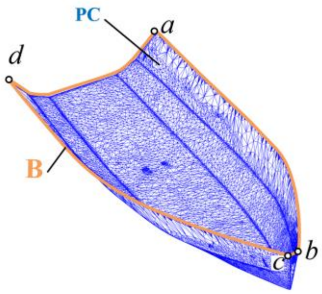

Fig. 1: Point-cloud triangulation for boat hull (B –boundary, with a,b,c and d as corner points; PC – point cloud points).

The parametric surface described by B-splines [8] can be defined as:

$$\mathbf{S}(u,v) = \sum_{i0=0}^{n0}\sum_{i1=0}^{n1} N_{i0,d0}(u)\cdot N_{i1,d1}(v)\cdot \mathbf{Q}_{i0i1}, u,v \in [0,1] \tag{1}$$

where $N_{i0,d0}(u)$ and $N_{i1,d1}(v)$ are the B-spline basis functions with degrees $d0, d1 \in \mathbb{N}$ and (n0+1)×(n1+1) are the respective numbers of the control points (CPs). The dimensionality of the control points $\boldsymbol{Q}_{ij}$, can be arbitrary. Here, we will use only clamped B-spline, but this is not necessary in a general case. Furthermore, the fitting method explained in the next section can be used with any parametric surface such as B-spline, NURBS, T-spline, RBF and others if the surface can be represented as $\mathbf{S}(u,v) = sum(\boldsymbol{B}_i(u,v)\cdot \boldsymbol{Q}_i)$, where i=1-n, (n is the number of basis functions).

Based on point cloud, one can easily construct the functionally determined point-cloud $\mathbf{F}(u,v)$ and then, surface ($\boldsymbol{S}(u,v)$) fitting is performed by minimizing the function (finding control points $\boldsymbol{Q}_i$):

$$A = \sum_{j=1}^{m} \left\| \mathbf{S}\left(u_j, v_j\right) - \mathbf{F}\left(u_j, v_j\right) \right\|^2 \tag{2}$$

Where the parametric coordinates (u_j, v_j) can be located on an equidistant grid in the *u-v* space. In this paper we used 100x100 points grid. This problem can be solved efficiently by converting it in a linear system of equations.

Re-parameterization based on plane-stress model:
This section presents novel additions to the previously devised re-parameterization algorithm variations in [6,7]. The overview of the proposed fitting method algorithm is given in the pseudocode:

Modified adaptive fitting method

Input: triangulated point cloud **PC**, B-spline control point grid dimensions and degree
Output: B-spline control points $\mathbf{Q}_i$
1: **Begin**
2: Harmonic mapping: $(u_i,v_i)=\mathbf{f}(\mathbf{PC}_i)$
3: Construct functionally determined point-cloud $\mathbf{F}(u,v)$ using trilinear interpolation
4: Fitting: find $\mathbf{Q}_i$ a by minimizing function A, Eqn. (2)
5: k=0
6: **do**
7: $k=k+1$
8: Calculate *relaxation field* $R(u,v)$
9: Re-parameterization based on scalar field R: $(u_i,v_i)=\mathbf{f}_\mathbf{R}(R, (u_i,v_i))$ using plane-stress model
10: Construct $\mathbf{F}(u,v)$ by trilinear interpolation
11: Fitting: find $\mathbf{Q}_i$ by minimizing function A, Eqn. (2)
12: Calculate *ConvergenceCriterion*
13: **while** (*ConvergenceCriterion* > *tolerance*)
14: **End**

The first steps of the method (lines 1-6) are the same as in the previous research. The point cloud is parameterized using the harmonic mapping $(u_i,v_i)=\mathbf{f}(\mathbf{PC}_i)$. This allows us to construct a functionally determined point-cloud $\mathbf{F}(u,v)$ using trilinear interpolation which is used for the initial fitting by Eqn. (2).

The crucial part of the algorithm is the re-parameterization (line 9), which is now different than in previous papers and it is based on solving plane-stress problem. Once the initial parameterization is performed using harmonic mapping, each 3D point is projected to 2D space $(u_i,v_i)=\mathbf{f}(\mathbf{PC}_i)$. In the reparameterization, plane-stress problem is solved for displacement, and this displacement is used to modify the (u_i,v_i) points. The solution of the displacement operator depends the selected *relaxation field* $R(u,v)$. The problem is solved using roller-type boundary conditions. The relaxation field is used to modify the local plate modulus of elasticity by E_{new}=E/(1+ $R(u,v)$), where E=1 is the selected as the initial modulus of elasticity. To define the problem, the Poisson's ratio ν has to be selected. In this paper, several tests were performed, and only small differences were notices among which selecting the negative value ν=-0.5 has shown slightly better smoothness. The problem was solved by FEM using 99x99 grid of four-node rectangular elements (which corresponds to 100x100 nodes, the same as fitting point-grid). Solving the plane-stress problem provides the displacement fields $\Delta u(u,v)$ and $\Delta v(u,v)$, and the reparameterization $\mathbf{f}_\mathbf{R}$ is obtained as $(u_i,v_i)= (u_i+\Delta u(u_i,v_i), v_i+\Delta v(u_i,v_i))$.

There are two addition changes compared to previous variations in [6,7]. First, convergence criterion in previous papers was while to perform the iterations while the mean square error is decreasing, but here we have shown that different criteria might be slightly better suited for engineering applications. Similarly, different variants of the relaxation field $R(u,v)$ are tested here.

Proceedings of CAD'20, Barcelona, Spain, July 6-8, 2020, 220-225

Results for different converence criteria tests:
Selected objects for test cases are shown in the Fig. 3. Relaxation surface was used as in paper [7].

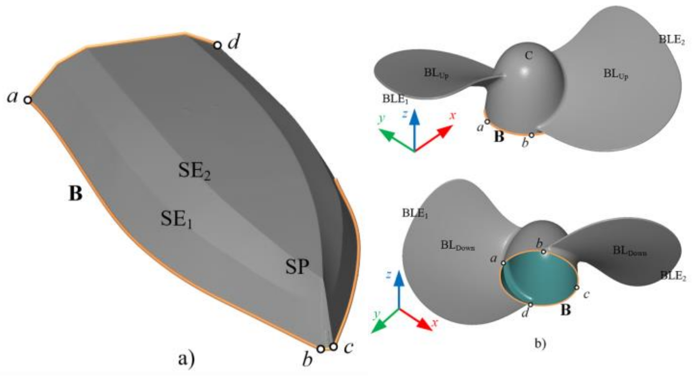

Fig. 2: Selected test objects: a) Ship hull and b) Ship propeller.

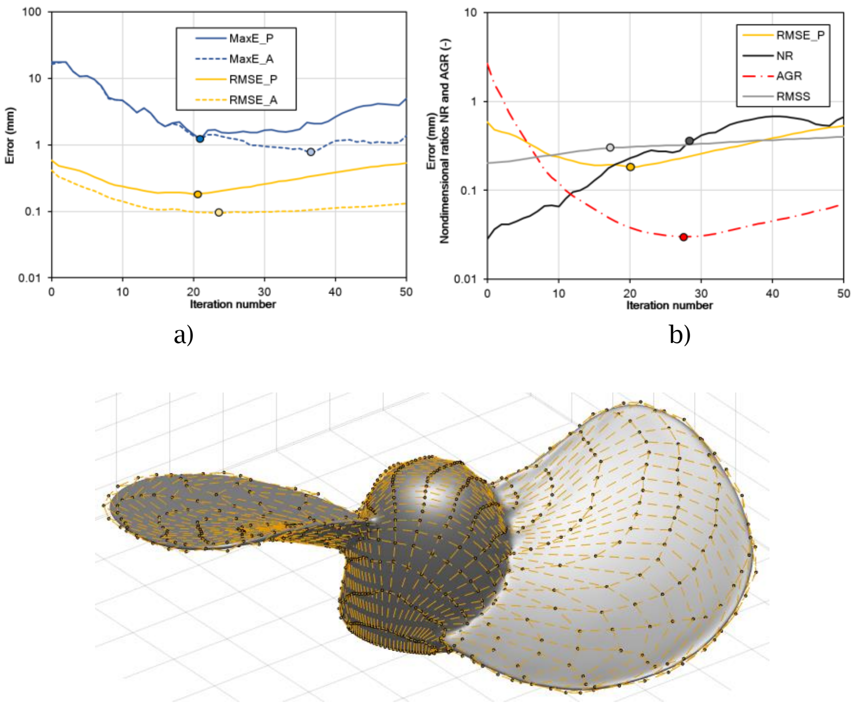

Fig. 3: Iterative fitting of propeller: a) Fitting error based on P – equal-parameter distance and A – actual distance, b) Different considered convergence criteria (NR – noise-based, AGR – area-based and skewness-based) and c) fitting result (iteration = 28).

The selected convergence criteria were monitored during the iterative fitting and the results for the propeller case are shown in Figure 3. When observing the error-related values, the minimal value can be considered as the converged result. Different points are obtained if the maximum error or the RMSE is used. Also, these points are different when using equal-parameter distance (_P) or actual distance (_A). The most representative error might be the RMSE based on the actual distance, which is obtained at 24th iteration. However, the maximum error is still reduced up to 35th iteration while the RSME remains practically the same up to 34th iteration. So, 34th iteration might be considered as the best solution here. However, it is computationally expensive to calculate the actual error during fitting. The remaining considered criteria are shown in Figure 9b. Slightly different results were obtained for the ship hull test case, but in both cases, the best criteria for convergence was the noise-related criterion.

Results for different relaxation field tests:

Here, we performed different variations of the relaxation surface used as in paper [7]. One of the variations was removing the noise by from the relaxation field using the same filter as earlier. The results are partly shown in figure 5. If closely observed, it can be seen that the B-spline control point grid has a more straight-lined segments when the noise is removed using the previous filter.

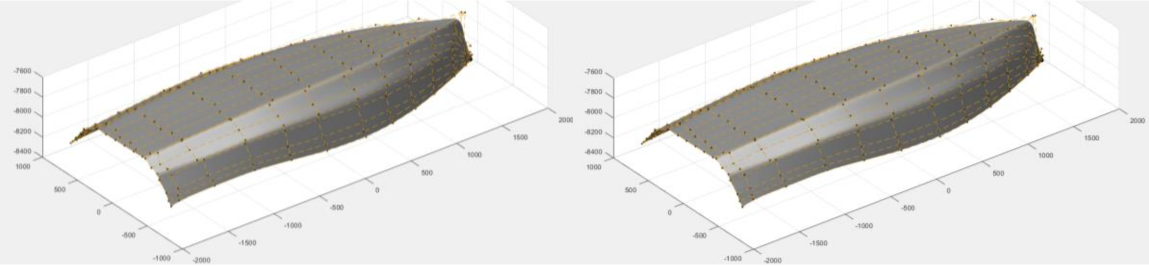

Fig. 5: Ship hull fitting for relaxation field: a) Without removing noise and b) removed noise.

Conclusions:

Good fitting of a smooth B-spline surface is possible to conduct on complex single-patch surfaces using the proposed reparameterization method. As the method is iterative, one of the main problems is to select an appropriate criterion for convergence of the method. It was shown that mean square error is not suited for convergence criterion, but noise-related criterion performed much better. Tests in relaxation field variations showed only slight variations and might require further research.

Acknowledgements:

This work was supported by the Croatian Science Foundation [HRZZ-IP-2018-01-6774].

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