Title:

Triangular and Quadrilateral Bézier Discretizations of Trimmed CAD Surfaces and Its Application to the Isogeometric Analysis

Authors:
Xiaoxiao Du, duxiaoxiao@buaa.edu.cn, Beihang University
Gang Zhao, zhaog@buaa.edu.cn, Beihang University
Wei Wang, jrrt@buaa.edu.cn, Beihang University
Mayi Guo, windowsgmy@126.com, Beihang University
Ran Zhang, feliciamail@buaa.edu.cn, Beihang University
Jiaming Yang, williamyjm@163.com, Beihang University

Keywords:
Trimmed NURBS Surfaces, Mixed Bézier Elements, Watertight Model, Isogeometric Analysis

DOI: 10.14733/cadconfP.2020.147-151

Introduction:

Isogeometric analysis of trimmed geometries is a valuable topic and full of challenges, due to the fact that trimming features are widely used but extremely sophisticated. The first work on isogeometric analysis of 2D trimmed NURBS surface is presented by Kim et al.[7], who employed NURBS-enhanced integration scheme [12] for integration of trimmed NURBS elements. Schmidt et al. [11] established the transformation matrix between the control points of untrimmed surface and the reconstructed Bézier patch through the selected sampling points. The so-called nested Jacobian approaches (NEJA) are proposed by Breitenberger et al. [2] from the same team to parameterize the trimmed domain which is decomposed into several subsets for isogeometric analysis of shell structures. Beer et al. [1] presented a double mapping method for analysis of trimmed CAD surfaces. Ruess et al. employed finite cell method (FCM) to deal with the trimming problem [10]. Nagy and Benson [9] approximated the trimmed element with refined control polygons of its boundary curves and established a quadrature rule on the approximated polygon. Marussig et al. gave an excellent review of isogeometric analysis of trimmed CAD models and divided the current approaches into global and local [8]. Xia and Qian [13] converted trimmed CAD surfaces into watertight geometry represented by rational triangular Bézier splines (rTBS) for the subsequent volumetric discretization process.

Considering the whole process of product development, the geometrical defects will influence the downstream applications (e.g., CAM, 3D printing) which need the use of geometries. Therefore, converting a trimmed CAD geometry into a watertight geometry without drawbacks of non-conforming, gap and overlap seems to be a superior approach for isogeometric analysis and other applications. In the present work, isogeometric analysis of trimming problems is solved by converting the trimmed NURBS surface geometry into watertight geometry, which is comprised of tensor-product Bézier patches and triangular Bézier patches. Most area of the trimmed surface is exactly preserved and only the narrow area along the trimming curves is approximated. Trimming curves can be explicitly expressed so that boundary conditions can be easily enforced. The order of the converted watertight model can keep consistent with

Proceedings of CAD'20, Barcelona, Spain, July 6-8, 2020, 147-151

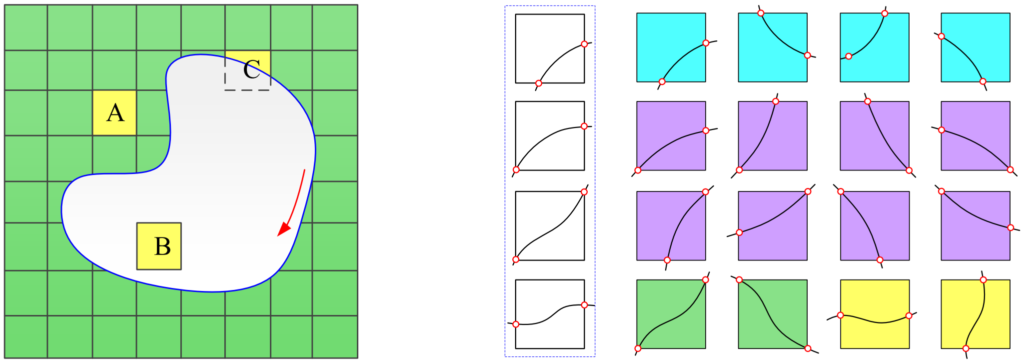

Fig. 1: Left: Categories of NURBS elements(A-valid element, B-invalid element and C-trimmed element); Right: 16 possible templates.

the order of the original trimmed model. In addition, it is easy to deal with multiple trimmed NURBS surface models without additional processing on nonconforming problems. NURBS surface models are widely used in engineering fields, e.g., sheet metal stamping processes. The proposed method is promising to help us solve engineering practical problems by using isogeometric method.

NURBS Elements Classification:

According to the positional relationship between NURBS elements and trimming curves, NURBS elements on trimmed surface can be classified into valid elements, invalid elements and trimmed elements. Valid and invalid element are completely located within the valid domain or invalid domain of trimmed surface without intersecting with any trimming curves. Trimmed element indicates the element intersecting with trimming curves. As shown in Fig. 1, A, B and C denote valid element, invalid element and trimmed element, respectively. Here element includes parametric element and physical element, where parametric element denotes the grid generated by the knot lines and physical element is the resulted surface element mapping from parametric element. As for trimmed elements, four common cases are considered and extended to 16 templates. Each template corresponds to two different trimmed elements because trimming curve has two opposite directions and only the left-side domain of trimming curve is valid.

Trimmed Elements Detection:

The category of the elements can be decided by the type of corresponding parametric element. Intersections between parametric trimming curves and parametric elements are computed to classify the elements. The detection of trimmed elements contains two tasks: first, the actual detection based on the intersections between knot lines and parametric trimming curves and second, the detection of the related space curve segments together with the projection of its knots to the parameter space. Valid elements and invalid elements can be distinguished by counting the number of intersections between a ray and trimming boundaries including outer trimming boundary curves and inner trimming boundary curves. If the number of intersections is odd and the element is not a trimmed element, then the element can be considered as a valid element. Otherwise it is an invalid element.

Interface Conforming:

Multiple trimmed NURBS surfaces are frequent in engineering model and the interfaces between adjacent

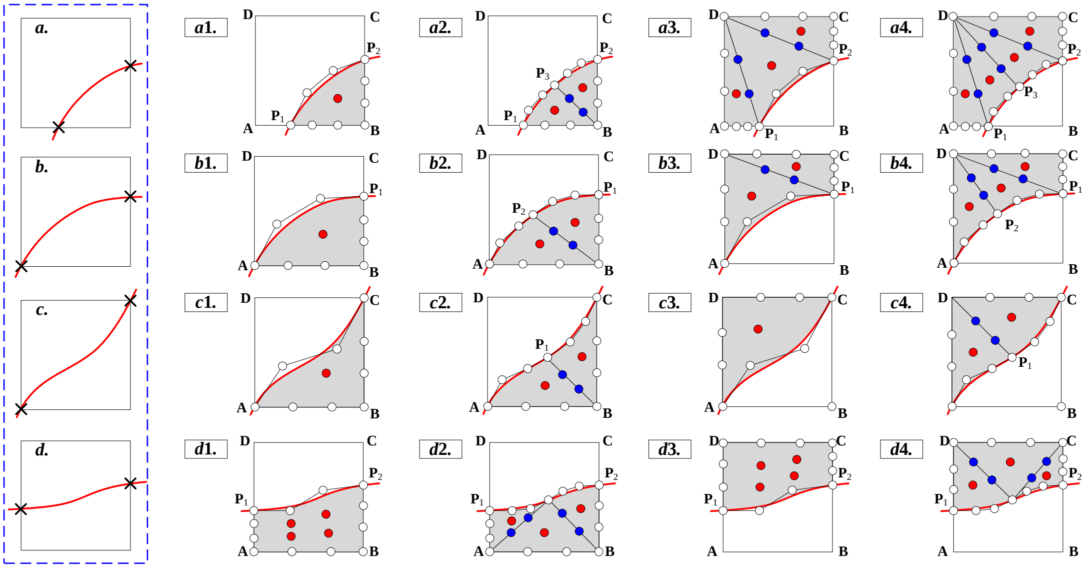

Fig. 2: Conversion of a planar trimmed element into mixed Bézier patch. White dots denote the boundary control points. Blue dots and red dots denote the approximated boundary control points and inner control points of converted Bézier patch, respectively.

surfaces are always inconsistent. Therefore, conforming problem has become a challenging and interesting problem in context of isogeometric analysis, and is widely investigated by using different methods to glue multiple patches, e.g., Nitsche method [3, 5, 14]. In this paper, the relationships between adjacent trimmed or untrimmed surfaces are analyzed firstly and then adjacent boundaries are conformed to build a watertight model.

Bézier Elements Conversion from Trimmed Elements:

In the process of converting trimmed NURBS surface to mixed Bézier patches, NURBS surface is firstly extracted as tensor product Bézier piecewise patches by using knot insertion algorithm. Bézier patches corresponding to valid elements are preserved and the patches related to invalid elements are removed. Now the problem is simplified as how to convert trimmed elements to mixed Bézier patches, which can be divided into two parts: boundary curves construction and inner control points generation. As illustrated in Fig. 2, the trimmed elements are split into several sub-patches. The boundary curves of the sub-patches are firstly constructed and inner control points are subsequently generated by using a so-called discrete Coons method [6], which is also employed to construct higher-order elements from linear unstructured mesh for analysis [4]. Note that only bi-cubic NURBS surfaces are considered in this paper and the presented method could be extended to higher degrees' case.

Numerical Example:

In this example, we consider a multi-trimmed T-junction pipe shell problem based on the Reissner-Mindlin shell theory. The T-junction pipe is built with two orthogonally intersected pipe and the interface of the pipes is filleted. Due to the symmetry property, a half of the T-junction pipe is modeled with three NURBS patches as depicted in Fig. 3. The edges of the horizontal pipe are clamped and the top edge of the vertical pipe is subjected to an edge load P. A fillet surface R_3 is generated from the interface

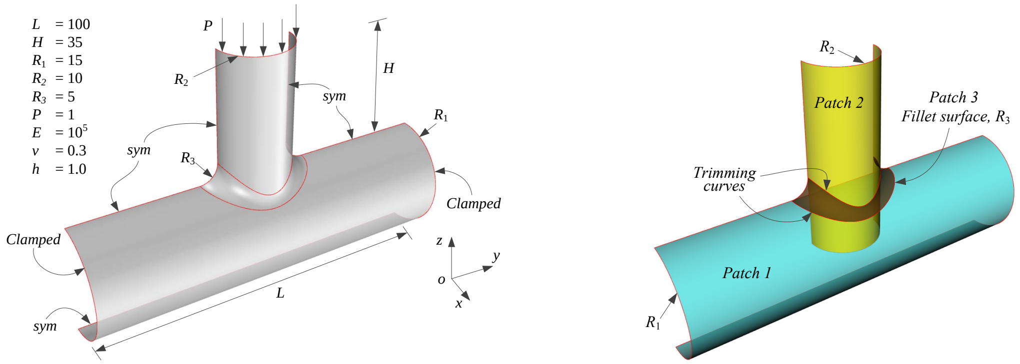

Fig. 3: Left: Problem definition; Right: modeling of the T-junction pipe.

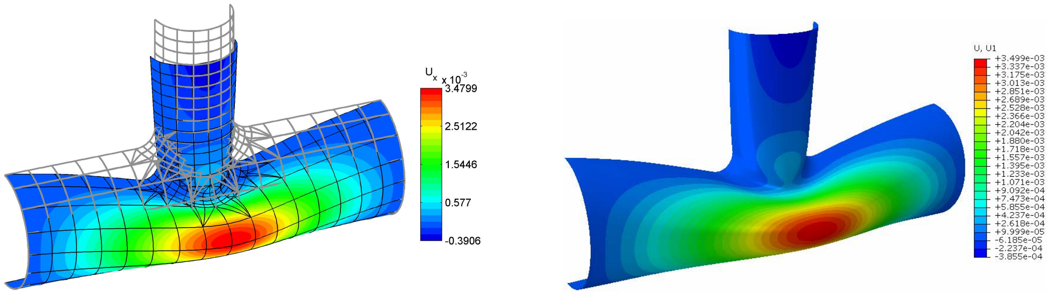

Fig. 4: Left: Displacement U_x using our method; Right: Displacement U_x using ABAQUS.

of the intersected semi-cylindrical patch 1 with radius R_1 and semi-cylindrical patch 2 with radius R_2. Multi-trimmed T-junction pipe model is discretized into 242 bi-cubic quadrilateral Bézier patches and 116 cubic triangular Bézier patches. The number of control points is 2821 in total. Reissner-Mindlin shell theory is employed to study the deformation of the pipe model. Figure 4 presents the comparison of displacement in x directions between present results and that of ABAQUS where the T-junction pipe is discretized into 22246 quadratic shell elements and 67309 nodes to calculate the displacement results. The present results are found in good coincidence with that of ABAQUS from the comparison.

Conclusions:

In this paper, a novel approach is proposed to convert the trimmed surfaces into mixed Bézier patches for isogeometric analysis. The converted models are watertight and can keep the original model geometrically exact except the parts along the narrow area of trimming curves, where the approximated error depends on the flatness of the trimmed elements and can be reduced by refining the NURBS surface with knot insertion algorithm. The comparison of presented results and that of ABAQUS or existing literatures shows the validity of the proposed method.

Acknowledgement:

The work is supported by the Natural Science Foundation of China (Project Nos. 61572056 and 61972011).

References:

[1] Beer, G.; Marussig, B.; Zechner, J.: A simple approach to the numerical simulation with trimmed CAD surfaces. Computer Methods in Applied Mechanics and Engineering, 285, 776–790, 2015. http://doi.org/10.1016/j.cma.2014.12.010.

[2] Breitenberger, M.; Apostolatos, A.; Philipp, B.; Wüchner, R.; Bletzinger, K.U.: Analysis in computer aided design: Nonlinear isogeometric B-Rep analysis of shell structures. Computer Methods in Applied Mechanics and Engineering, 284, 401–457, 2015. http://doi.org/10.1016/j.cma.2014.09.033.

[3] Du, X.; Zhao, G.; Wang, W.: Nitsche method for isogeometric analysis of Reissner–Mindlin plate with non-conforming multi-patches. Computer Aided Geometric Design, 35, 121–136, 2015. http://doi.org/10.1016/j.cagd.2015.03.005.

[4] Du, X.; Zhao, G.; Wang, W.: Mixed Bernstein-Bézier construction from unstructured mesh for higher-order finite element analysis of plates and shells. Computer-Aided Design & Applications, 17(2), 362–383, 2020. http://doi.org/10.14733/cadaps.2020.362-383.

[5] Du, X.; Zhao, G.; Wang, W.; Fang, H.: Nitsche's method for non-conforming multipatch coupling in hyperelastic isogeometric analysis. Computational Mechanics, 65, 687–710, 2020. http://doi.org/10.1007/s00466-019-01789-x.

[6] Farin, G.; Hansford, D.: Discrete Coons patches. Computer Aided Geometric Design, 16(7), 691–700, 1999. http://doi.org/10.1016/S0167-8396(99)00031-X.

[7] Kim, H.J.; Seo, Y.D.; Youn, S.K.: Isogeometric analysis for trimmed CAD surfaces. Computer Methods in Applied Mechanics and Engineering, 198(37), 2982–2995, 2009. http://doi.org/10.1016/j.cma.2009.05.004.

[8] Marussig, B.; Hughes, T.J.: A review of trimming in isogeometric analysis: challenges, data exchange and simulation aspects. Archives of computational methods in engineering, 25(4), 1059–1127, 2018. http://doi.org/10.1007/s11831-017-9220-9.

[9] Nagy, A.P.; Benson, D.J.: On the numerical integration of trimmed isogeometric elements. Computer Methods in Applied Mechanics and Engineering, 284, 165–185, 2015. http://doi.org/10.1016/j.cma.2014.08.002.

[10] Ruess, M.; Schillinger, D.; Özcan, A.I.; Rank, E.: Weak coupling for isogeometric analysis of non-matching and trimmed multi-patch geometries. Computer Methods in Applied Mechanics and Engineering, 269, 46–71, 2014. http://doi.org/10.1016/j.cma.2013.10.009.

[11] Schmidt, R.; Wüchner, R.; Bletzinger, K.U.: Isogeometric analysis of trimmed NURBS geometries. Computer Methods in Applied Mechanics and Engineering, 241, 93–111, 2012. http://doi.org/10.1016/j.cma.2012.05.021.

[12] Sevilla, R.; Fernández-Méndez, S.; Huerta, A.: NURBS-enhanced finite element method (NEFEM). International Journal for Numerical Methods in Engineering, 76(1), 56–83, 2008. http://doi.org/10.1002/nme.2311.

[13] Xia, S.; Qian, X.: Isogeometric analysis with Bézier tetrahedra. Computer Methods in Applied Mechanics and Engineering, 316, 782–816, 2017. http://doi.org/10.1016/j.cma.2016.09.045.

[14] Zhao, G.; Du, X.; Wang, W.; Liu, B.; Fang, H.: Application of isogeometric method to free vibration of Reissner–Mindlin plates with non-conforming multi-patch. Computer-Aided Design, 82, 127–139, 2017. http://doi.org/10.1016/j.cad.2016.04.006.