Title:

NURBS-based Isogeometric Analysis for Small Deformation of Viscoplastic and Creep Problems

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Introduction:

The observed behaviours of real materials in life or engineering applications always present a certain degree of time dependency [6, 7]. For example, metals, especially under higher temperatures, usually exhibit simultaneously the phenomena of viscoplasticity and creep. The former is a common time dependent plastic deformation and the latter performs the strain-time relationship when deformed at constant stress level. Finite element analysis (FEA) is a main numerical method to deal with these time dependent problems in engineering for many years [8]. Geometrical models are usually discretized into mesh model for FEA, which not only introduces approximation error in the discretization process but also loses certain geometrical information. The gap between CAD and FEA is expected to be bridged by using isogeometric analysis (IGA), proposed by Hughes et al. [2, 3], where the same spline functions are used for both the geometry description in CAD and the variables approximation in FEA. The precise geometry representation and higher-order element continuity of IGA could bring great benefits in the viscoplastic and creep simulation. Therefore, it is a significant and interesting topic to investigate the application of isogeometric analysis on viscoplastic and creep problems.

In this paper, two dimensional small deformation viscoplastic and creep problems are investigated by using NURBS based isogeometric analysis. Viscoplastic materials combining with von-Mises yield function and Perzyna's flow rule are employed [5]. The stresses expression, stress-strain relationship matrix, isogeometric discrete formulations and other important formulas of viscoplastic and creep problems are derived and listed in detail. Numerical examples are investigated to verify the validity of the proposed method through comparing with the results obtained from commercial software ABAQUS and that from existing literatures.

Viscoplasticity and Creep:

In this paper, the small strain problem of the viscoplastic materials is simulated. It is assumed that the total strain, ε, can be separated into elastic, ε^e, and viscoplastic, ε^{vp}, components, so the stress rate can be expressed with the stress-strain relation as

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{D}\dot{\boldsymbol{\varepsilon}}^{e} = \boldsymbol{D}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{vp}\right) \tag{2.1}$$

Considering the property of the viscoplastic materials, it is now necessary to determine a flow rule defining the viscoplastic strains. A simple and explicit form is one in which the viscoplastic strain rate depends only on the current stresses, so that

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \frac{1}{\eta}\langle f(\boldsymbol{\sigma})\rangle \frac{\partial f(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} \tag{2.2}$$

where η is the viscosity parameter. $f(\boldsymbol{\sigma}) = \|\boldsymbol{s}\| - \sqrt{\frac{2}{3}}\sigma_y$ is the yield function in von-Mises type and the notation $\langle\ \rangle$ implies the ramp function $\langle x \rangle = \frac{x+|x|}{2}$.

With the implicit time integration scheme, the viscoplastic strain increment $\Delta\boldsymbol{\varepsilon}_n^{vp}$ generated in time interval $\Delta t_n = t_n - t_{n-1}$ can be calculated as

$$\Delta\boldsymbol{\varepsilon}_n^{vp} = \Delta t_n \left[(1-\theta)\dot{\boldsymbol{\varepsilon}}_{n-1}^{vp} + \theta\dot{\boldsymbol{\varepsilon}}_n^{vp}\right] \tag{2.3}$$

in which $\dot{\boldsymbol{\varepsilon}}_{n-1}^{vp}$ is computed from Eq. (2.2) with $\boldsymbol{\sigma} = \boldsymbol{\sigma}_{n-1}$. While $\dot{\boldsymbol{\varepsilon}}_n^{vp}$ is unknown at time t_n and can be obtained according to Taylor's formula as

$$\dot{\boldsymbol{\varepsilon}}_n^{v} = \dot{\boldsymbol{\varepsilon}}_{n-1}^{vp} + \left(\frac{\partial \dot{\boldsymbol{\varepsilon}}^{vp}}{\partial \boldsymbol{\sigma}}\right)_{n-1} \Delta\boldsymbol{\sigma}_n = \dot{\boldsymbol{\varepsilon}}_{n-1}^{v} + \boldsymbol{H}_{n-1}\Delta\boldsymbol{\sigma}_n \tag{2.4}$$

where $\Delta\boldsymbol{\sigma}_n$ is the stress increment and can be calculated analogously with Eq. (2.1). The expression of stress increment with respect to the unknown displacement increment can be rewritten, by substituting Eqs. (2.3), (2.4) and $\Delta\boldsymbol{\varepsilon}_n = \boldsymbol{B}\Delta\boldsymbol{u}_n$, as

$$\Delta\boldsymbol{\sigma}_n = \boldsymbol{D}_n\left(\boldsymbol{B}\Delta\boldsymbol{u}_n - \Delta t_n\dot{\boldsymbol{\varepsilon}}_{n-1}^{vp}\right) \tag{2.5}$$

where $\boldsymbol{D}_n = \left(\boldsymbol{D}^{-1} + \theta\Delta t_n\boldsymbol{H}_{n-1}\right)^{-1}$ is the matrix of stress-strain relation at time t_n.

Creep problem can be depicted through observing the change in strain over time by applying the load for a long period of time under constant room temperature. In this paper, the primary and secondary creep problems in a two-dimensional analysis are taken into consideration. Creep strain rate can be calculated with the following formulas

$$\dot{\boldsymbol{\varepsilon}}^{c} = \frac{1}{\eta}\|\boldsymbol{s}\|\frac{\partial\|\boldsymbol{s}\|}{\partial\boldsymbol{\sigma}}\ ,\ \dot{\boldsymbol{\varepsilon}}^{c} = \frac{1}{\eta}\|\boldsymbol{s}\|\frac{\partial\|\boldsymbol{s}\|}{\partial\boldsymbol{\sigma}}\cdot t^m \tag{2.6}$$

The secondary creep problem, also known as steady-state creep problem, is directly simplified by assigning the initial yield stress to zero and Eq. (2.2) can be substituted as the former in Eq. (2.6). In the primary creep problem, the creep strain rate is regarded as the function of time which can be written as the latter in Eq. (2.6), where m is a given constant number range $(-1, 0]$ and if $m = 0$ this problem becomes the secondary creep problem. The rest calculation of the creep problem is similar with viscoplastic problem.

Isogeometric Formulations:

Considering the governing equations of the viscoplastic deformation, the incremental form during the time interval Δt_n can be written as

$$\int_{\Omega}\boldsymbol{B}_n^T\Delta\boldsymbol{\sigma}_n d\Omega = \Delta\boldsymbol{f}_n \tag{2.7}$$

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and substituting for $\Delta\boldsymbol{\sigma}_n$ from Eq. (2.5) the above equation becomes

$$\int_\Omega \boldsymbol{B}^T \boldsymbol{D}_n \boldsymbol{B} \Delta \boldsymbol{u}_n d\Omega = \int_\Omega \boldsymbol{B}^T \boldsymbol{D}_n \Delta t_n \dot{\boldsymbol{\varepsilon}}^{vp}_{n-1} d\Omega + \Delta \boldsymbol{f}_n \tag{2.8}$$

Due to the nonlinearity of the viscoplastic problem and the fact that viscoplastic strain rate is approximately calculated at time t_n by Taylor's formula, the residual is inevitable and can be given by $\boldsymbol{R}_n = \int_\Omega \boldsymbol{B}_n^T \boldsymbol{\sigma}_n d\Omega - \boldsymbol{f}_n$. The residual can be added to the incremental equilibrium equation at the next time interval Δt_{n+1} and the complexity caused by the Newton-Raphson iteration can be avoided effectively. Then the discretized isogeometric equations derived for small strain viscoplastic problem are

$$\boldsymbol{K}_{n+1} \Delta \boldsymbol{u}_{n+1} = \boldsymbol{F}_{n+1} \tag{2.9}$$

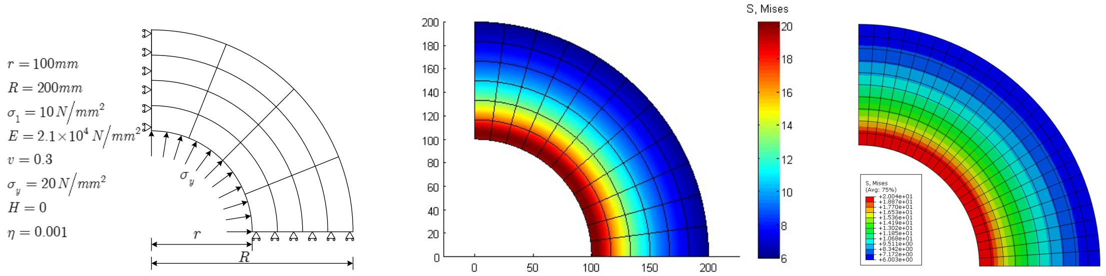
where

$$\boldsymbol{K}_{n+1} = \int_\Omega \boldsymbol{B}^T \boldsymbol{D}_{n+1} \boldsymbol{B} d\Omega \ , \ \boldsymbol{F}_{n+1} = \int_\Omega \boldsymbol{B}^T \boldsymbol{D}_{n+1} \Delta t_{n+1} \dot{\boldsymbol{\varepsilon}}^{vp}_n d\Omega + \Delta \boldsymbol{f}_{n+1} + \boldsymbol{R}_n \tag{2.10}$$

Analogously, the isogeometric formulations of the creep problem can be expressed as previous.

Numerical Examples:

As given in Fig. 1(a), a thick cylinder subjected to a constant internal pressure was previously studied in [4] by using finite element method and is also investigated here by employing isogeometric method. Plane strain condition and von Mises yield criterion are considered. Figure 1(b) shows the contour of the Mises stress based on the IGA method. For the comparison, the problem is also simulated in ABAQUS [1] and the stress result is provided in Fig. 1(c).



(a) Problem description (b) IGA result (c) ABAQUS result

Fig. 1: Problem description and stress result comparison. (a) The description and boundary condition of a thick quarter cylinder, (b) the result from IGA program, (c) the result from ABAQUS.

In the second example, we studied the creep deformation problems of a square plate under various combinations of boundary conditions, element types and creep stages. Three cases are described as shown in Figs. 2-4 (a). Material properties including Young's modulus $E = 2.0 \times 10^5 N/mm^2$, Poisson ratio $\nu = 0.3$ and $m = -0.5$ are used. The total creep time for these tests is set as 1000 hours. The results of magnitude displacement are calculated by our IGA program and ABAQUS, respectively and shown in Figs. 2-4 (b) and (c).

From the comparison of the displacement contours given in Figs. 2, 3 and 4, it can be found that the IGA results agree very well with that of ABAQUS. As shown in Fig. 5, the relationships between time and the magnitude of displacement for the above creep examples are compared between using IGA

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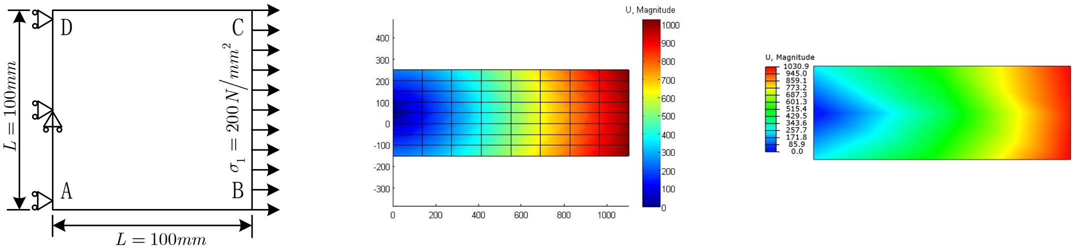
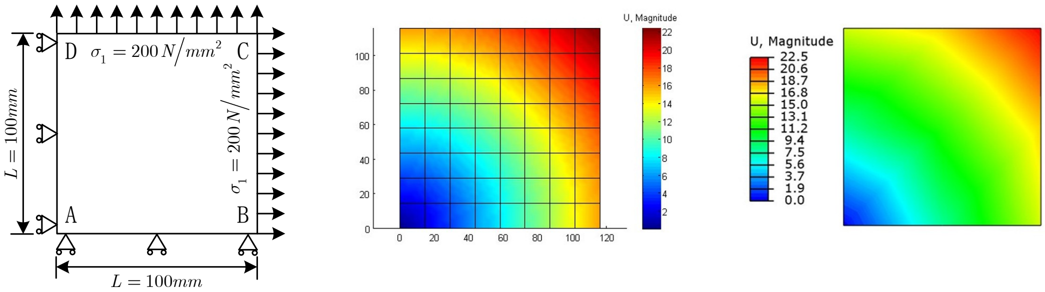


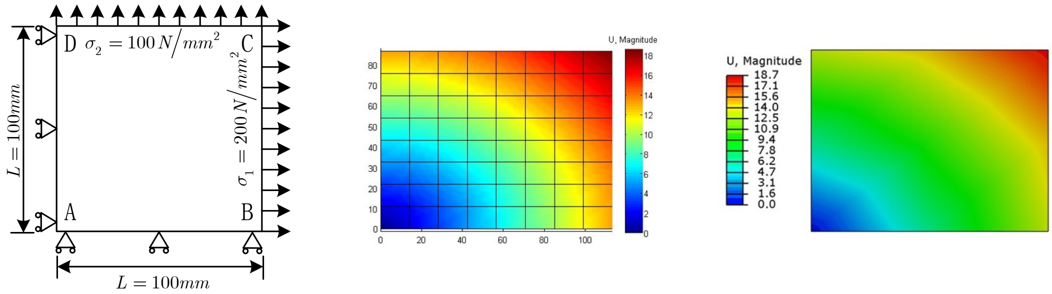
(a) Problem description (b) IGA result (c) ABAQUS result

Fig. 2: Problem description and displacement result comparison. (a) The description and boundary condition in secondary creep, (b) the result from IGA program, (c) the result from ABAQUS.



(a) Problem description (b) IGA result (c) ABAQUS result

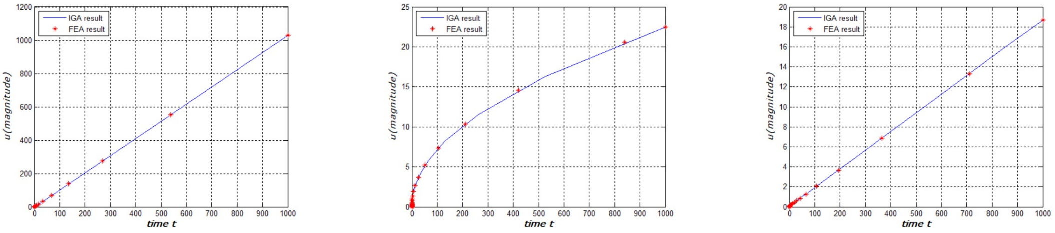
Fig. 3: Problem description and displacement result comparison. (a) The description and boundary condition in primary creep, (b) the result from IGA program, (c) the result from ABAQUS.



(a) Problem description (b) IGA result (c) ABAQUS result

Fig. 4: Problem description and displacement result comparison. (a) The description and boundary condition in secondary creep, (b) the result from IGA program, (c) the result from ABAQUS.

and FEA in ABAQUS, respectively. It can be found that for each example, the displacement-time curves obtained by using different methods agree well with each other. Note that the curve in Fig. 5(b) agrees with the Eq. 2.6(2) which expresses the non-linear relationship between the creep strain rate and time. While the curves in Figs. 5(a) and 5(c) reflect the linearity of the Eq. 2.6(1).



(a) Creep problem in Fig. 2 (b) Creep problem in Fig. 3 (c) Creep problem in Fig. 4

Fig. 5: Displacement-time curve for the three creep cases obtained by IGA and FEA in ABAQUS. Horizontal axis denotes time and the vertical axis denotes the magnitude of displacement on the top right corner of the square plate.

Conclusions:

In this work, isogeometric analysis has been successfully employed for simulation of viscoplastic and creep problems. Classical benchmark examples are investigated by using IGA and FEA to verify the validity of the proposed method. In practical engineering, some mechanical parts are always working under high temperature, e.g., aeroengine gas turbines. The simulation of such cases should consider the viscoplasticity of the materials. Therefore, future works will focus on the isogeometric analysis of these practical problems.

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References:

[1] Abaqus: Abaqus user's manual. version 6.14, 2014.

[2] Cottrell, J.A.; Hughes, T.J.; Bazilevs, Y.: Isogeometric analysis: toward integration of CAD and FEA. John Wiley & Sons, 2009.

[3] Hughes, T.J.; Cottrell, J.A.; Bazilevs, Y.: Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. Computer methods in applied mechanics and engineering, 194(39), 4135–4195, 2005. http://doi.org/10.1016/j.cma.2004.10.008.

[4] Hughes, T.J.; Taylor, R.L.: Unconditionally stable algorithms for quasi-static elasto/visco-plastic finite element analysis. Computers & Structures, 8(2), 169–173, 1978. http://doi.org/10.1016/0045-7949(78)90019-6.

[5] Lubliner, J.: Plasticity theory. Courier Corporation, 2008.

[6] Neto; EduardodeSouza: Computational methods for plasticity:theory and applications, 2008. http://doi.org/10.1002/9780470694626.fmatter.

[7] Owen, D.R.J.; Hinton, E.: Finite elements in plasticity: theory and practice, 1980.

[8] Zienkiewicz, O.C.; Taylor, R.L.: The Finite Element Method, 2005. http://doi.org/10.1007/0-387-26160-5_6.