Title:
Meshfree CAD-CAE Integration through Immersed B-rep model and Enriched Iso-geometric Analysis

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Introduction:
The goal of iso-geometric analysis techniques (IGA, [1,2]) is to seamlessly integrate CAD with CAE. However, arguably, much work remains to integrate IGA analysis tools with commercial CAD tools. Currently, most CAD systems utilize boundary representation (B-rep) models constructed from trimmed Non-Uniform Rational B-Splines (NURBS) patches. The IGA [1] models, on the other hand, commonly rely on a tri-variate NURBS mesh. Thus, during IGA mesh generation, the irregular domain is meshed into several tensor-product NURBS entities that do not directly utilize the original B-rep CAD model. An alternative approach is to build an analysis approximation by immersing the B-rep model within a regular mesh in space containing analysis unknowns. In such an event, the mesh does not conform to the B-rep model boundary.

When the analysis grid does not conform to the B-rep model boundary, no behavioral degree of freedom exists directly on the essential boundary of the physical domain for one to apply the boundary condition directly. This challenge, common to nearly all meshfree approximations (e.g., moving least square or reproducing kernel method), necessitates weak imposition of boundary condition through one of many possible techniques including the penalty functions, Lagrange multipliers, Augmented Lagrangian, or Nitsche's method.

The challenge of applying boundary conditions is not restricted to immersed boundaries, but also exists when tri-variate NURBS entities are used for analysis since the degrees of freedom are associated with the control points and not the geometrical boundary. Similar to other mesh free method shape functions, the NURBS basis does not interpolate the control or nodal points. The non-interpolatory nature of the approximation necessitates special treatment of the essential boundary conditions.

In the present paper, we propose an approach to analyze complex B-rep model immersed in regular grids using Enriched IGA (EIGA, [3]). In EIGA, the boundaries as well interfaces are explicitly represented by a lower-dimensional NURBS entity with additional degrees of freedom directly specified on the control points of the interface. In the EIGA frame work, the field approximation on the continuous domain is enriched with an approximation with known characteristics defined on the enriching boundaries. For instance, the enriching boundary may correspond to a crack, in which case, the enrichment must possess the known physical behavior such as displacement discontinuity on the boundary. The composition of the enriching boundary is restricted to a local region as dictated by a blending function that is monotonic with respect to distance. In our prior work, a monotonic measure of approximate distance was constructed using the algebraic level sets [5]. Furthermore, the sign of the algebraic distance field constructed on bounded solids enables the point membership query in CAD/CAE applications.

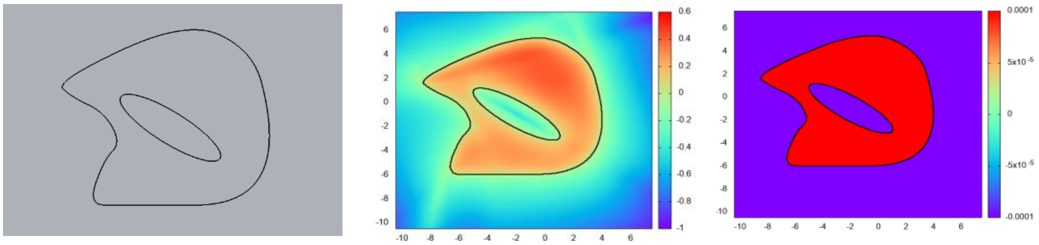
In the present paper, using algebraic level sets, EIGA is implemented in a parallelized Fortran code termed OOF-HiDAC (Object-Oriented Fortran based Hierarchical Design and Analysis Code) that can import geometric models as IGES files from commercial CAD software including Rhino and Solidworks. The analysis is carried out directly on the B-rep surfaces created by the CAD software without needing volumetric discretization (i.e., mesh). The present framework includes modules to build the approximation as well as compose the approximations to build more complex ones, a matrix solver and a post-processor to visualize results. Several validation problems are solved to demonstrate the framework.

Algebraic Distance Field

As mentioned before, the influence of an enriched field must vanish with distance away from the enriching entity. This is done through convex blending of the enriching field with the underlying field using weights that vanish with distance. Thus, the weight functions represent a monotonic measure of distance from the boundaries. Distances to parametric boundaries are usually estimated using the Newton Raphson method. However, the numerical procedure is not robust and often yields either non-unique values of distance or non-reachable points on the boundary. An alternative approach is to build distance using a polyhedral approximation to the bounding surface, which sacrifices the exactness of the geometry to the CAD model. Upreti et al [4] described a procedure to construct algebraic distance measures from low-degree NURBS entities. The main idea behind the method is to implicitize the parametric entity and use the level sets of the implicitized function as a measure of distance with the following properties: (1) Exact locally near the surface (2) Monotonic function of exact distance (3) Sufficiently smooth for engineering applications and (4) Efficiently obtained without numerical iterations.

Point Classification

Point membership classification is traditionally performed using a raytracing approach. The basic logic is that for a closed curve, any point inside the solid model intersects the surface of the model an odd number of times. The operation is computationally intensive on parametric surfaces. In the present paper, the point classification problem is solved using signed algebraic distance fields. Upreti and Subbarayan [5] constructed the signed distance field by extending the algebraic level sets mentioned above. The distance to the boundary of the model is defined as positive inside and negative outside. Thus, point containment queries are equivalent to getting the sign of distance (Figure 1).



(a) (b) (c)

Fig. 1: Examples of signed distance field (a) B-rep model (b) signed distance field (c) sign of distance.

Enriched Iso-geometric analysis

The notion of enriched field approximations is enabled by Partition of Unity Finite Element Method (PUFEM) and Generalized Finite Element Method (GFEM) [6], in which the underlying finite element approximation is generalized by adding degrees of freedom representing complex local behavior.

Convergence of the approximations is ensured by the partition of unity property of the finite element shape functions. In other words, the FE approximation space is "enriched" by the known local behavior. Tambat and Subbarayan [3] proposed the so called enriched iso-geometric analysis wherein they enriched known behavior on explicitly defined lower-dimensional geometric features. The base approximations are "enriched" iso-geometrically on parametrically defined lower-dimensional geometrical features and by constructing distance fields from them. Any field representing the behavior is approximated as:

$$f(x) = (1 - w(d))f_{\Omega}(x) + w(d)\, f_{\Gamma}\big(P(x)\big) \tag{1}$$

where, $w(d)$ is the weight, which represents the contribution of enrichment and vanishes as distance d from the enrichment increases, Ω is the underlying domain, Γ is the enriching curve, and P is the foot point of any point projected onto the enrichment.

Boundary Conditions as Enrichments

To accurately capture the behavior near the boundary, an iso-geometric approximation with hybrid function/derivative enrichment is proposed in the present paper. This enriched approximation is a smooth blending of C^1 or higher order continuous iso-geometric approximation of underlying domain enriched with a C^0 continuous local approximation. Eq. (2) demonstrates how this method is implemented for a single enrichment with extra degrees of freedom u_{Γ_e} and G_{Γ_e}.

$$u(x) = \big(1 - w_e(d)\big)u_{\Omega}(x) + w_e(d)\big(u_{\Gamma_e}(P(x)) + d * G_{\Gamma_e}(P(x))\big) \tag{2}$$

The field corresponding to the enrichment contains two parts: u_{Γ_e} is the displacement of control points on the enrichment, which represents the rigid body motion of the boundary under load, and $G_{\Gamma_e}$is the associated derivative of the displacement in the direction normal to the enriching curve, so that $d * G_{\Gamma_e}$ represents the local deformation in the normal direction.

Since the enriching field is defined on the enriching entity directly, there is no necessity for weak imposition of boundary conditions. The essential boundary condition can be applied directly on the corresponding control points of the NURBS boundary curve or surface. Neumann Boundary can be applied by integration on the enrichment to get the corresponding nodal force on the enrichment. The blending procedure is pictorially illustrated in Fig. 2. The problem domain can be decomposed into the underlying domain Ω, Dirichlet boundary Γ_d and Neumann boundary Γn. By blending underlying domain and each boundary condition according to Eq. (2), the behavior on the entire domain can be obtained.

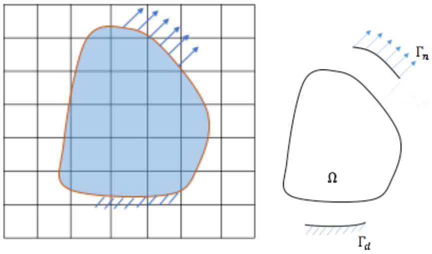


Fig. 2: Illustration of enriched approximations for applying boundary conditions.

Patch Tests

To better illustrate how this blending procedure works, a one-dimensional bar example is described in detail. The procedure is next extended to two-dimensional and three-dimensional examples. Patch tests for square and cube under uniform tension are demonstrated to study convergence. Also, to better capture the behavior near the boundary of the blending region, different integration schemes are studied. Patch test demonstrates good convergence with high accuracy.

Numerical examples

Several cases, such as plate with a hole under tension, plate with multiple holes, arch-shaped structure, a curved T structure are studied using the procedure described in Fig. 3.

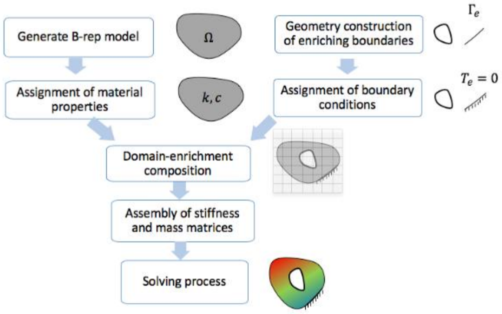


Fig. 3: CAD-EIGA integration Analysis flow.

The first example (Fig. 4) is of a plate with a single hole under uniaxial tension. The whole geometry is immersed in the regular background NURBS mesh. The resulting stress concentration value agrees well with theoretical solution of 3. Example 2 (Fig. 5) extends the implementation to plate with multiple holes. Example 3 (Fig. 6) is of a curved T-shaped structure under compression. The stress concentration is captured at the sharp corner. Example 4 (Fig. 7) is a loaded wheel, which becomes flat under compression.

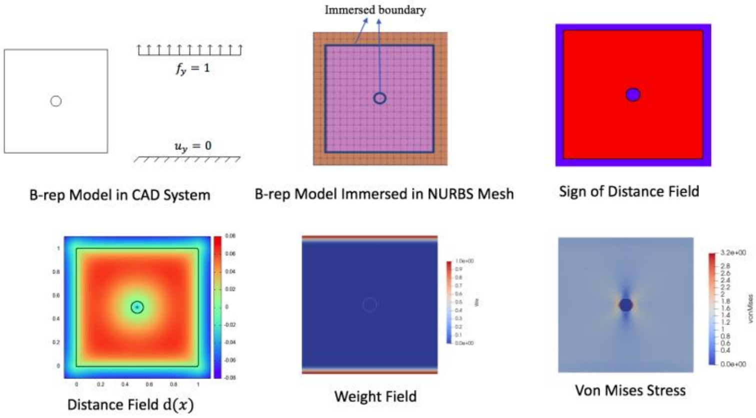


Fig. 4: Example 1: Plate with a Hole.

Summary

In the present paper, we describe a CAD/CAE integration technique that enables direct analysis of B-rep CAD models without meshing using exact (to CAD) geometry to capture known behavior on boundaries. The signed algebraic level sets are utilized for point classification query as well as to estimate distance from the boundary. Using Enriched iso-geometric analysis (EIGA), an approach is proposed to apply boundary condition for general immersed boundaries. The boundary conditions are treated as enrichments with associated degree of freedom. This strategy avoids weak imposition of boundary conditions as is commonly done in meshfree methods.

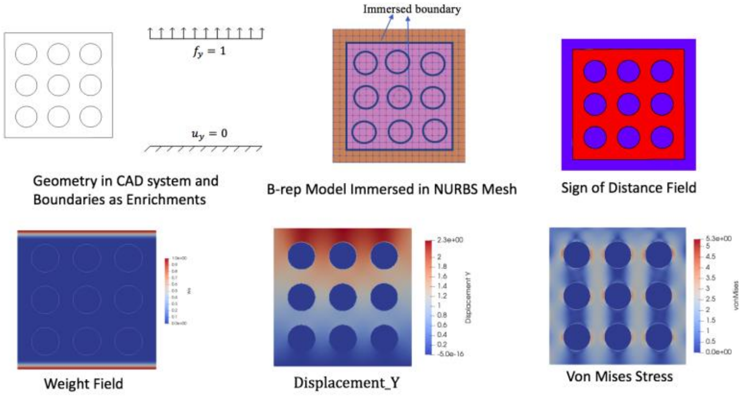


Fig. 5: Example 2: Plate with Multiple Holes.

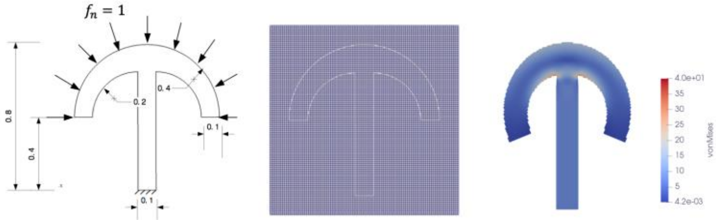


Fig. 6: Example 3: Curved T-structure.

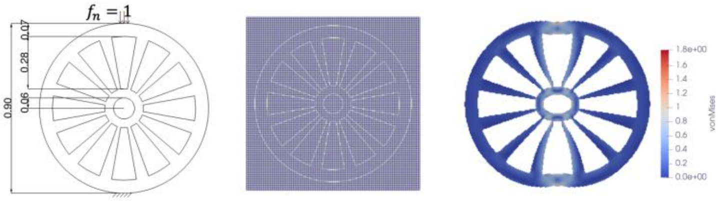


Fig. 7: Example 4: Loaded wheel.

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