Title:
Optimal Path Smoothing with Log-Aesthetic Curves based on Shortest Distance, Minimum Bending Energy and Curvature Variation Energy

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Introduction:

Path planning is a process of constructing a desired movement from an initial position into discrete motions which satisfy given constraints before reaching to the final location. In practice, there are two common formulation used in path planning; either parametric representation (Beziers and BSplines) or nonlinear representation (clothoid generation with Fresnal Integral). Even though cubic Bezier/Bpline parametric representation has been widely used in CAD/CAM practices, it imposes several unwanted characteristics, e.g. complicated curvature/arc-length computation and curvature extrema. However, natural spirals do not have these problems except for the generation of the spiral itself which involves integration. With the advancement of computers, numerical integration can be carried out with minimal effort while preserving high precision. Runge-Kutta methods can also be to generate spiral [3] which greatly reduces computation time. There are also attempts to represent spirals by means of Beziers [11], however these curves lose their degree of freedom while satisfying curvature monotonicity to mimic spiral.

Typical curves used to replace polyline path include Dubins path; the combination of line segments and circular arcs. It is one of the most popular choice in path smoothing [1]. Dubin paths are widely used for path planning but it only satisfies G^1 continuity. Highway design and railyway route planning has somewhat similar design procedure as compared to path planning. The underlying property is to satisfy given G^2 data which comprises of position, tangent and curvature at the endpoints. Hickerson [5] stated that highway designers must avoid sudden changes between curves with different curvatures or long tangents. He proposed using gradual increase/decrease types of curvatures which is in fact the main feature of spirals. Baass [2] simplified G^2 highway design into five cases using clothoid; (1) Line to circle with a single spiral, (2) Circle to circle forming S-curve with a pair of spirals, (3) Circle to circle forming C-curve with a pair of spirals, (4) Circle to circle with a single spiral, and (5) Line to line with a pair of spiral. These cases are the building block to design highway/railway design.

There are many attempts to solve these cases using various types of curves, e.g., Bezier spiral, Pythagorean Hodograph spirals etc. The solution are either unique for given G^2 data or no solution since the proposed method is curve centric. Hence, designers have no option on finding optimum path which minimizes arc-length, bending energy or curvature variation.

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In highway/railway design, there may exist numerous obstacles which need to be avoided, e.g, rock hills, historical buildings etc. Given numerous obstacles, it is rather straightforward to draw guiding polyline path avoiding the obstacles to reach to the final destination. The process of finding a smooth obstacle-avoiding path to replace polyline is called path smoothing [7]. Hence, the solution of the fifth case stated by Baass comes in handy, however the proposed solution is rather limited since the spiral which was considered is a clothoid. The solution region can be expanded if we consider a bigger family of spirals. In this paper, we propose Log-aesthetic curves (LACs) as an alternative for path smoothing due to its monotonic curvature property, hence free from local curvature extrema [8]. Let S be a LAC space representing spirals with 3 shape variables, α, c_1, c_2. Various family of spirals can be generated by giving specific α values, $S = \{s_\alpha | \alpha \in \mathbb{R}\}$. A collection of natural spirals $s_N \subset S$ is obtained when $\alpha = \{-1, 0, 1, 2\}$ where the spiral families are {clothoid, Neilsen's spiral, Logarithmic spiral, circle involute}. The rest two free variables can be utilized for satisfying design constraints. In 2006, an algorithm which fits LACs to a set of G^1 Hermite data proposed in [12]. Later, Miura et al. [9] developed an algorithm to implement tri-LAC segments to solve G^2 Hermite interpolation problem. According to Levien and Sequin [6], LACs is a promising curve for industrial and CAD applications due to its self-affinity property. This elegant feature has made it attractive for applications in various aesthetic related fields such as automobile design [9], shape completion in archeology [4] and architectural design [10].

The first part of this paper introduces a bi-LAC segment to solve the fifth Baass's case; connecting two lines with a G^2 LAC which offers multiple solutions. The second part of the paper proposes a four-step algorithm on constructing an obstacle avoiding path optimized for three types of constraints: shortest distance, minimum bending energy or minimum curvature variation energy. Numerous simulation results are presented to show the validity of the proposed algorithm.

Single & Bi-LAC:

The equations of LACs are intrinsically simple and it was coined by Miura [8]. For the sake of simplicity, we assume that the curvature of LAC $\kappa \geq 0$ and its shape parameter $\alpha \neq 0, 1$. Eq.(1) below is the curvature function $\kappa(\tilde{s})$ of a single LAC segment:

$$\kappa(\tilde{s}) = \frac{(c_0\tilde{s}+c_1)^{-1/\alpha}}{L}, \quad 0 \leq \tilde{s} \leq 1 \tag{1}$$

where $\tilde{s}$ represents normalized arc length, L be the total length of curve and c_j is constant where $j = 0, 1$. Let $\theta(\tilde{s})$ be the tangent angle of the curve. By using the definition of $\theta'(\tilde{s}) = \kappa(\tilde{s})$, the following equation is obtained:

$$\theta(\tilde{s}) = \frac{\alpha}{(\alpha-1)c_0}(c_0\tilde{s} + c_1)^{(\alpha-1)/\alpha} + c_2, \quad 0 \leq \tilde{s} \leq 1 \tag{2}$$

where c_2 is a constant. Eq.(2) is used to define parametric expressions for LACs coordinates as shown below where $\{x_0, y_0\}$ represents start point of the curve:

$$\{x_s(\tilde{s}), y_s(\tilde{s})\} = \{x_0 + L\int_0^1 \cos\theta(\tilde{s})d\tilde{s}, y_0 + L\int_0^1 \sin\theta(\tilde{s})d\tilde{s}\} \tag{3}$$

One of the interesting feature of LAC is its curvature is either monotonically increasing or decreasing. Therefore, in the case of joining two line segments where the initial and final curvatures are both 0, we require at least two piece of LAC segments to ensure a smooth transition between them. The following shows the curvature function $\kappa(\tilde{s})$, tangent angle $\theta(\tilde{s})$ and parametric expressions of a bi-LAC segment.

$$\kappa(\tilde{s}) = \begin{cases} \frac{(c_{10}\tilde{s}+c_{11})^{-1/\alpha}}{L}, & 0 \leq \tilde{s} \leq s \\ \frac{(c_{20}\tilde{s}+c_{21})^{-1/\alpha}}{L}, & s < \tilde{s} \leq 1 \end{cases} \tag{4}$$

where s be the arc length at the joint and c_{ij} is constant where $i = 1, 2$ and $j = 0, 1$. To note, i represents number of LAC segments and j is parameter number.

$$\theta(\tilde{s}) = \begin{cases} \frac{\alpha}{(\alpha-1)c_{10}}(c_{10}\tilde{s} + c_{11})^{(\alpha-1)/\alpha} + c_{12}, & 0 \leq \tilde{s} \leq s \\ \frac{\alpha}{(\alpha-1)c_{20}}(c_{20}\tilde{s} + c_{21})^{(\alpha-1)/\alpha} + c_{22}, & s < \tilde{s} \leq 1 \end{cases} \tag{5}$$

$$\{x_{LAC}(\tilde{s}), y_{LAC}(\tilde{s})\} = \{x_0 + L(\int_0^s \cos\theta(\tilde{s})d\tilde{s} + \int_s^1 \cos\theta(\tilde{s})d\tilde{s}), y_0 + L(\int_0^s \sin\theta(\tilde{s})d\tilde{s} + \int_s^1 \sin\theta(\tilde{s})d\tilde{s})\} \tag{6}$$

Geometric Constraints of LACs with G^2 Continuity:

A curve is G^2 continuous if and only if it satisfies given positional, tangent as well as curvature at end-points. Let the initial and final endpoints, tangent and their corresponding curvature denoted as $\{x_0, y_0\}$, $\{X, Y\}$, $\{\theta_0, \theta_1\}$ and $\{\kappa_0, \kappa_1\}$, respectively. The following shows the conditions on curve segment(s) once we impose the relevant G^2 constraints on its endpoints.

- when $\tilde{s} = 0$:

$$\kappa_0 = \frac{(c_{11})^{-1/\alpha}}{L} \tag{7}$$

$$\theta_0 = \frac{\alpha}{(\alpha-1)c_{10}}(c_{11})^{(\alpha-1)/\alpha} + c_{12} \tag{8}$$

- when $\tilde{s} = s$:

$$(c_{10}s + c_{11}) = (c_{20}s + c_{21}) \tag{9}$$

$$\frac{\alpha}{(\alpha-1)c_{10}}(c_{10}s + c_{11})^{(\alpha-1)/\alpha} + c_{12} = \frac{\alpha}{(\alpha-1)c_{20}}(c_{20}s + c_{21})^{(\alpha-1)/\alpha} + c_{22} \tag{10}$$

- when $\tilde{s} = 1$:

$$\kappa_1 = \frac{(c_{20} + c_{21})^{-1/\alpha}}{L} \tag{11}$$

$$\theta_1 = \frac{\alpha}{(\alpha-1)c_{20}}(c_{20} + c_{21})^{(\alpha-1)/\alpha} + c_{22} \tag{12}$$

$$X = x_0 + L(\int_0^s \cos\theta(\tilde{s})d\tilde{s} + \int_s^1 \cos\theta(\tilde{s})d\tilde{s}) \tag{13}$$

$$Y = y_0 + L(\int_0^s \sin\theta(\tilde{s})d\tilde{s} + \int_s^1 \sin\theta(\tilde{s})d\tilde{s}) \tag{14}$$

Note that $\{x_0, y_0\}$, $\{X, Y\}$, $\{\theta_0, \theta_1\}$, $\{\kappa_0, \kappa_1\}$ and α are all user defined. Therefore, the remaining unknown variables s, L and c_{ij} where $i = 1, 2$ and $j = 0, 1, 2$ can be determined from Eq.(7) to Eq.(14). An example below shows the construction of a bi-LAC connecting two lines. Let d_1 and d_2 be the length of the corresponding line segments. Fig.1 shows that LACs offer numerous solutions with G^2 continuity depending on α values. Hence, we may choose appropriate α based on user defined metric.

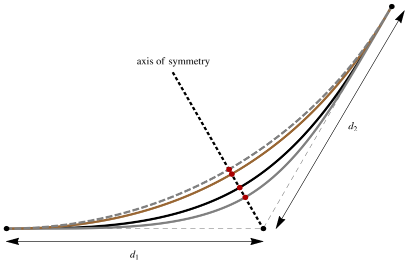


Fig. 1: A C-shaped line to line transition with a bi-LAC segment with $d_1 = d_2$ from top to down: $\{\alpha = -100, -5, -1 \text{ and } -1/2\}$.

Path Smoothing Algorithm:

Since LAC provides many feasible paths, an optimal path can be chosen based on three metrics denoted as M^*:

$$M^* = \begin{cases} L, & \text{L: } Arc-length \\ 1/L \times \int_0^1 \kappa(\tilde{s})^2, & \text{E: } Bending\ energy \\ 1/L \times \int_0^1 \kappa'(\tilde{s})^2 d\tilde{s}, & \text{V: } Curvature\ variation\ energy \end{cases}$$

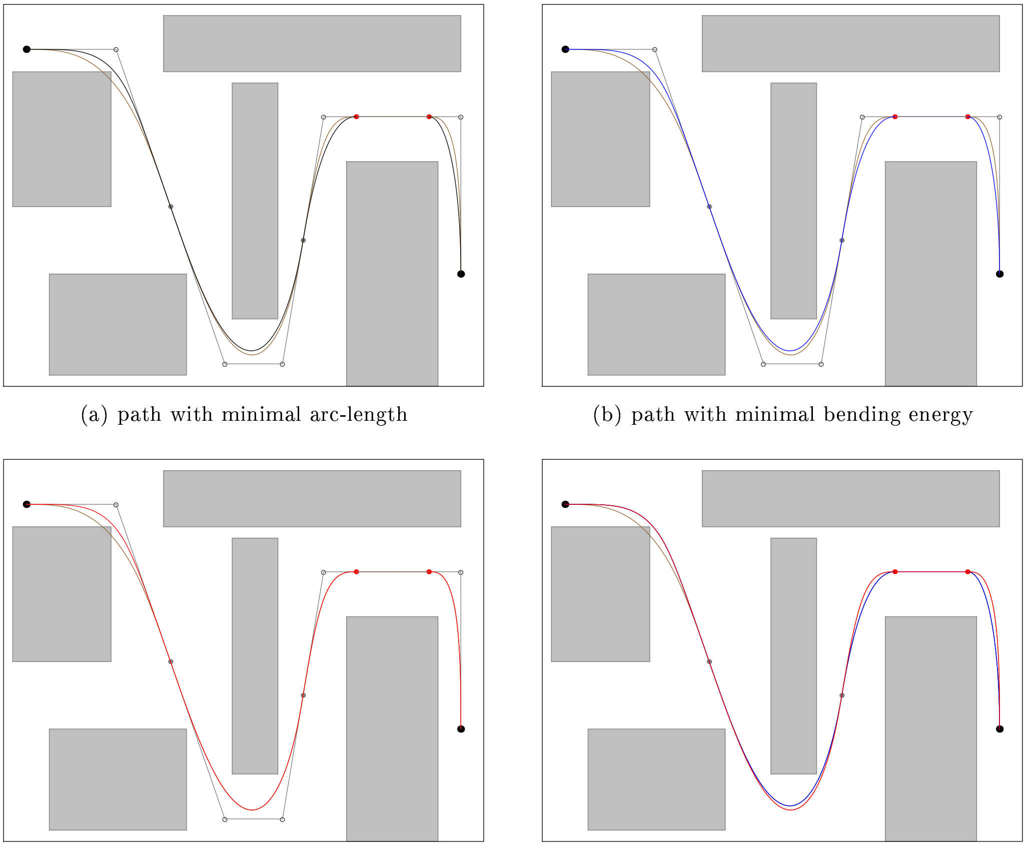
The shape parameter α is modified to obtain a curvature continuous collision free path satisfying chosen metric. The proposed path smoothing algorithm is divided into four steps:

1. partition given polyline into C-shaped sections,
2. identifying variable values to satisfy G^2 conditions,
3. smoothing polyline with clothoid, and finally
4. optimizing the shape parameter α based on chosen metric.

The first step is to partition the given polyline path into few C-shaped sections as proposed in [7]. Next, we impose the given G^2 constraints in Eq.(7) - Eq.(12) and solve the unknowns c_{ij} where $i = 1, 2$ and $j = 0, 1, 2$. The remaining unknowns s and L are determined from Eq.(13) - Eq.(14) by bisection method. The arc length of the first segment of a spiral path is denoted as s_l. For symmetric C-shapes, we set the value of $s_l = 0.5$, while in asymmetric cases, we may calculate s_l using bisection method. Then we construct a smooth reference path with $\alpha = -1$. There are three possibilities; (1) no solution, solution with obstacle collision and (3) solution with obstacle avoidance. In case there is no solution, polyline is further partitioned. Finally, suitable value of α for each C-shaped polyline is identified using bisection method. This step involves varying $\alpha \pm h$ and iteratively carrying out this process in the direction of minimizing M^*. There are two approaches proposed to detect collision based on distance threshold and point-region intersection.

Numerical Examples:

Fig.2 shows a polyline path is partitioned into three C-shaped sections. Since there is no solution available using clohoid for the last C-section, it is divided into two symmetric C-shapes with a line in between to generate a clothoid path (shown in brown). The rest of paths are generated using bi-LAC with various α values for each section with a desirable optimal path as shown in Table 1. The example clearly indicates that clothoid may fail to satisfy given polyline where it collides with given obstacle. With bi-LAC, the user has the option to choose their desired metric to smooth given polyline. The experimental results show that an obstacle-avoiding smooth path with minimal path length and bending energy can be obtained when $\alpha \to -\infty$. However, we found that $\alpha = -1$ is the optimal spiral to produce a smooth path with minimal curvature variation energy, however it does not guarantee obstacle avoidance. Thus, user may vary the paths by tuning α values to obtain next feasible obstacle-avoiding path with minimal curvature variation energy.



(a) path with minimal arc-length

(b) path with minimal bending energy

(c) path with minimal curvature variation energy

(d) Multiple paths

Fig. 2: Optimal LAC paths with minimal path length L (a), bending energy E (b) and curvature variation energy (c).

Table 1: Numerical results of Fig.2.

Fig.	α				M^*		
	seg 1	seg 2	seg 3	seg 4	L	E	V
2(a)	-0.4034	-1.106	-7	-11	8.92247	3.82781	14651.41
2(b)	-0.4034	-1.106	-7	-11	8.92247	3.82781	14651.41
2(c)	-0.4034	-1	-1	-1	9.03030	4.57958	122.28

Conclusions:

An alternative yet simple algorithm is proposed to utilize LACs in smoothing the given polyline with three metrics in which a family of clothoiud may fail. Two simple collision detection algorithm will also be presented in full paper to improve the efficiency of finding the feasible smooth path. Future work include developing an algorithm with tri-LAC in order to meet G^3 conditions.

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