Title:
The Bézier Segmentation of T-spline Solid in Parametric Domain

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Introduction:
Isogeometric analysis, a numerical method based on spline function proposed in 2005[4], uses the same geometric representation for both engineering design model and analysis model, thus eliminates the huge model difference between design and analysis, and improves the efficiency and accuracy of the overall product design process [9]. However, the traditional geometry model still has limitations in expressing three-dimensional objects. This is due to the fact that all the current CAD systems use NURBS or T-spline bounding surface set to represent solid objects, which lack the numerical parameterization of internal domain. This kind of defect has become the bottleneck of the integration of design and analysis in many engineering fields such as acoustics, electromagnetics [2] and hydrodynamics [1][3].

T-spline solid inherits the advantages of T-spline surface, which makes up for the shortcomings of non-uniform rational B-Splines (NURBS) in expressing complex surfaces, such as the need for trimming, and the inability of local refinement [7-8]. T-spline solid construction is one of the key technologies to support isogeometric analysis technology to achieve practical level. A few works have been devoted to construct T-spline solid and apply T-spline solid in geometric analysis. Zhang et al. proposed a method of transforming genus-zero boundary mesh model into T-spline solid model [11]. Wang et al. proposed the method of transforming the arbitrary genus boundary mesh model into the T-spline solid model and carried out isogeometric analysis of the obtained T-spline solid [9]. Zhang et al. completed the work of transforming special T-spline surfaces into T-spline solid [10]. The above research on T-spline solid mainly focuses on the construction process of T-spline solid. In [9], isogeometric analysis of T-spline solid is carried out, but the emphasis is on displaying the analysis results, and the detailed steps in the analysis process are not introduced.

As a key algorithm for T-spline solid applied to isogeometric analysis, Bézier extraction was proposed by Scott et al. [6]. When this algorithm is applied to the isogeometric analysis of T-spline solid, there are two main steps: (1) Bézier segmentation of three-dimensional T-mesh in parametric domain. (2) Get the Bézier extraction matrix corresponding to each blending function of three-dimensional T-spline.

Literature [6] gives a detailed description of the implementation process of step (2), but does not give the specific implementation process of step (1). The other literatures on T-spline solid also lack specific algorithms. Although there is some research on the algorithm of Bézier segmentation of two-dimensional T-mesh, due to the intrinsic complexity of three-dimensional T-mesh, the algorithm cannot be directly extended to the Bézier extraction of T-spline solid. In this paper, the problem of Bézier segmentation of the T-spline solid is studied, and a parametric domain Bézier segmentation algorithm of T-spline solid was proposed. Some examples are illustrated to verify the algorithm.

Main Idea:
In order to ensure the isogeometric analysis of the T-spline solid, the smallest hexahedron elements that make up T-spline solid must be rational Bézier solids, but not each hexahedron on parametric T-mesh can be represented by a rational Bézier solid. This phenomenon is illustrated by a common example, as shown in Fig. 1. Figure 1 is a parametric domain T-mesh of a cubic T-spline solid. V is a parametric vertex, C is a hexahedron in the parametric domain T-mesh, and the red region is the definition domain of the blending function corresponding to the parametric vertex V. According to the differentiability of B-spline basis function, the internal C^∞ continuity of T-spline solid corresponding to hexahedron C cannot be achieved, while C^∞ continuity of rational Bézier solid can be achieved. Therefore, the T-spline solid corresponding to hexahedron C cannot be represented by a rational Bézier solid. In order to implement Bézier extraction of T-spline solid, the T-spline solid must be segmented first.

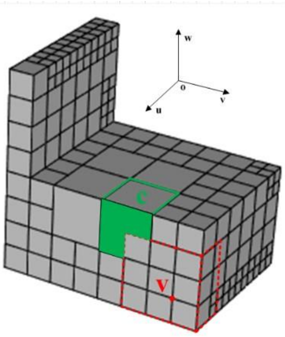


Fig. 1: L-shaped region generated by segmentation.

If the brutal force algorithm is used to traverse every vertex for Bézier segmentation, when the vertex V is reached, the problem that the green L-shaped area after segmentation is difficult to store effectively will be encountered. In addition, irregular shapes like L-shaped regions can cause repeated calculations. Therefore, in order to efficiently implement Bézier segmentation of the parametric domain T-mesh and avoid the difficulty of storing the intermediate segmentation results, this paper presents a Bézier segmentation algorithm for T-spline solid parametric domain T-mesh.
Algorithms 1: T-spline Solid Parametric Domain T-mesh Bézier Segmentation

STEP1: Build a stack to store all hexahedral elements in the parametric domain T-mesh.

STEP2: If the stack is not empty, the cube on the top of the stack goes out of the stack, traverses the vertexes those affect it, and executes STEP3 for each vertex.

STEP3: Calculate the knot vector on the current vertex. If a single interval of the knot vector completely covers the cube, calculate the next vertex. Otherwise, execute STEP4.

STEP4: Perform the segmentation of the current hexahedron element (using the *Algorithm2*), all the newly segmented hexahedron elements are pushed in the stack and return to STEP2.
Algorithm2: Parametric domain T-mesh hexahedron segmentation algorithm

STEP1: Segmentation of hexahedral elements by the vertex is decomposed into three directions: u, v and w. STEP2 is executed in three directions: u, v and w.

STEP2: Taking the u-direction as an example, the u-direction of hexahedron element is segmented by the u-direction knot vector (u_knot) of the vertex. There're three possible situations as shown in Fig. 2 (u_min, u_max are the minimum and maximum values of the hexahedron element of T-spline solid in u-direction).

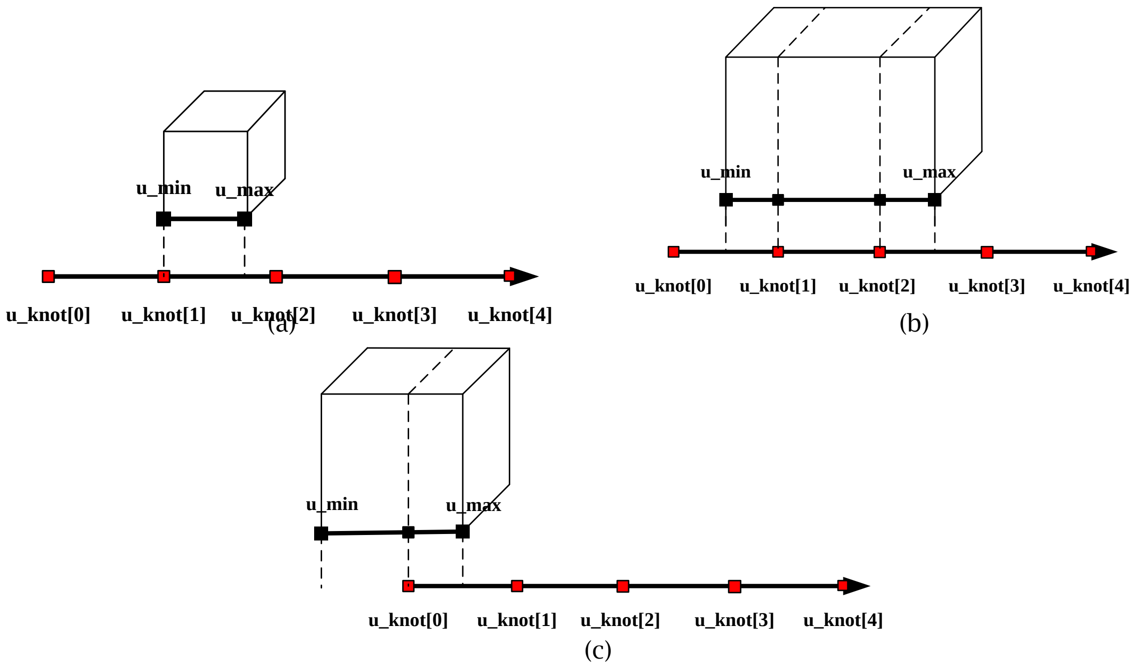


Fig. 2: (a) Hexahedron segmentation in u-direction (remain unchanged), (b) Hexahedron segmentation in u-direction (segmented by u_knot[1] and u_knot[2]), (c) Hexahedron segmentation in u-direction (segmented by u_knot[0])

STEP3: The segmentation in u, v and w directions is superimposed to obtain the segmentation results.

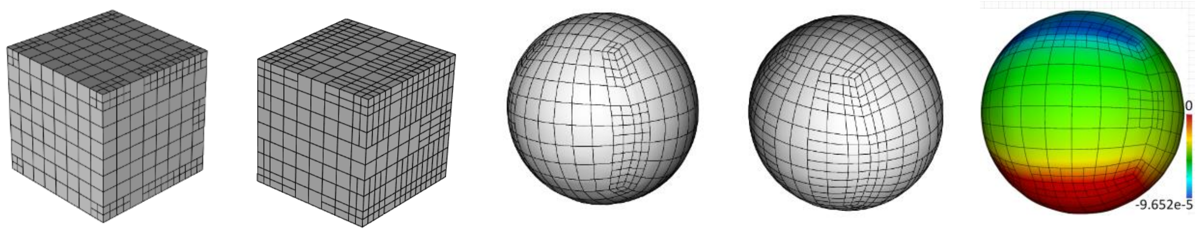
Algorithm2 solves the problem of producing L-shaped or more complex regions which are difficult to store in the process of segmentation. In addition, this process only segment each hexahedron element once, and can complete Bézier segmentation of T-mesh in parametric domain without repetition or omission.

<u>Conclusions:</u>
In order to verify the validity of the method proposed in this paper, the Bézier extraction algorithm of T-spline solid is implemented on Visual Studio 2010 software development platform. Firstly, the parametric domain T-mesh of T-spline model is processed by the Bézier segmentation algorithm proposed in this paper. After that, combined with the algorithm of extracting Bézier operator proposed in reference [6], the Bézier extraction of T-spline solid model is completed. Finally, the isogeometric analysis of T-spline solid can be performed. Statistics for all the tested models in the Bézier segmentation process are shown in Tab. 1. Basically, the time consumed in Bézier segmentation is dependent on the scale of the model, but as shown here, the result is highly related with the complexity of the model's geometry, as in complex situation, more irregular-shaped T-mesh cubes may be encountered.

Model	*T-mesh vertexes*	*T-mesh cubes*	*Bézier elements*	*Time of segmentation (s)*
sphere	1229	793	1324	5.28
bunny	3433	1105	1788	21.37
head	1624	977	3415	19.25

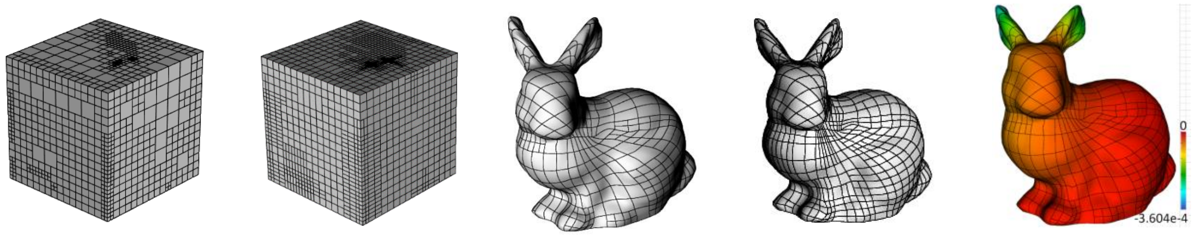
Tab. 1: Statistics of the tested models.

We have developed a 3D isogeometric analysis solver for static mechanics analysis, which uses rational T-splines as the basis, and we used it to test the Bézier segmentation results of T-spline solid. For all the models, we fix all the control points on the bottom and apply uniformly distributed oad on the top. The Young's modulus $E = 200GPa$, and the Poisson's ratio $\nu = 0.3$. Bézier segmentation results and displacement results of w-direction from the isogeometric analysis are given in Figs 3-5.



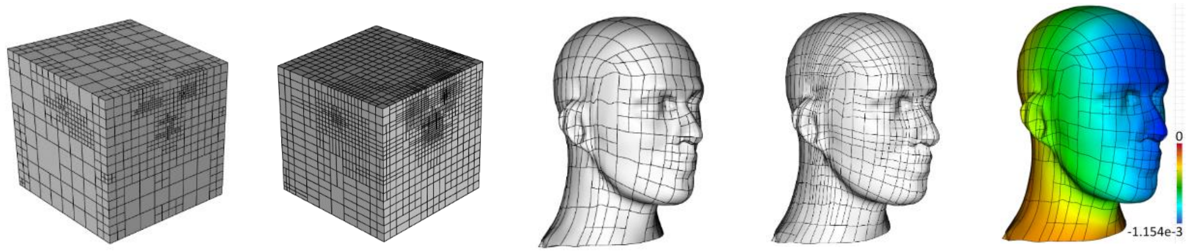
(a) (b) (c) (d) (e)

Fig. 3: (a) Parametric domain T-mesh of sphere model, (b) Bézier segmentation of sphere model in parametric domain, (c) T-spline solid of sphere model, (d) Bézier segmentation of sphere model, (e) Isogeometric analysis result (displacement of w-direction) of sphere model.



(a) (b) (c) (d) (e)

Fig. 4: (a) Parametric domain T-mesh of bunny model, (b) Bézier segmentation of bunny model in parametric domain, (c) T-spline solid of bunny model, (d) Bézier segmentation of bunny model, (e) Isogeometric analysis result (displacement of w-direction) of bunny model.



(a) (b) (c) (d) (e)

Fig. 5: (a) Parametric domain T-mesh of head model, (b) Bézier segmentation of head model in parametric domain, (c) T-spline solid of head model, (d) Bézier segmentation of head model, (e) Isogeometric analysis result (displacement of w-direction) of head model.

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