



**Title:**

**A Genetic Algorithm Approach for Boundary Optimization in Converting Trimmed NURBS to Untrimmed T-spline**

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**Introduction:**

T-splines enjoy several advantages over NURBS, including local refinement, seamless stitching and data compression [7-8]. Trimmed NURBS surface can be approximately expressed by untrimmed T-spline surface, which will bring much convenience for surface representation and editing, and isogeometric analysis (IGA) scenarios [1]. In the process of converting a trimmed NURBS to an untrimmed T-spline, B-spline curves with the prescribed number of control points are expected to approximate the trimming curves defined in the trimmed NURBS [7]. Then the obtained control points of the fitting curve and the vertices of the predefined enclosed polygon are connected by a middle layer of control points. However, the edges of T-mesh generated in this step are likely to be distorted and intersect with each other in the narrow neighborhood of the trimming curve, which will result in bad parameterization as shown in Fig. 1 and affect the simulation results in the context of isogeometric analysis [11]. The reason for this phenomenon is the disconnection between the trimming curve approximation and the predefined enclosed polygon.

This paper is to solve the problem of the distortion and intersection that arise when converting a trimmed NURBS into an untrimmed T-spline, which is an unstructured T-spline including multiple extraordinary points and can be further used for isogeometric analysis [6]. Genetic algorithm is employed to optimize the distance between the control polygon of the fitting curve and the predefined enclosed polygon. Several examples are investigated to verify the validity of the proposed method.

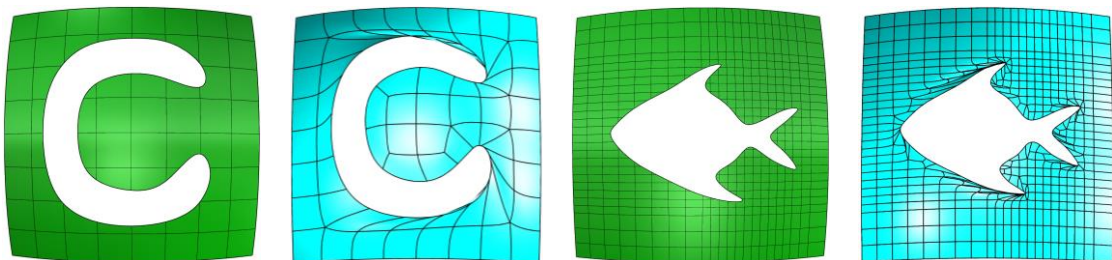


Fig. 1: Two examples of trimmed NURBS surfaces and the corresponding converted T-spline surfaces: C-trimmed and fish-trimmed surfaces.

### Trimmed-NURBS to Untrimmed T-Splines:

In this section, the general process of converting a trimmed NURBS surface to an untrimmed T-spline surface is described as shown in Fig. 2. Subdivision method could be used to narrow the approximated error. Both the trimmed NURBS surface and untrimmed T-spline surface are bi-cubic in this paper. The whole process can be divided into four steps:

#### Step 1: Reversing trimming curve.

According to the relationship between the trimming curve and NURBS surface in the physical domain, it is easy to find the corresponding trimming curve  $C$  in the NURBS surface's parametric domain through inverse computation engineering as presented in Fig. 2(a). The trimming curves in the parametric domain corresponding to the physical trimming curves already exist in IGES files but not in STEP files.

#### Step 2: Finding an axis-aligned polygon.

Finding the smallest enclosed-polygon  $A$  (denoted by the blue polygon) of the trimming curve in the parametric grid as presented in Fig. 2(b). The sides of the polygon  $A$  are axis-aligned and coincide with the iso-parametric lines of the NURBS surface. Knot refinement algorithm can be used to refine the NURBS parametric grid to obtain a closer polygon to the trimming curve and narrow the approximation error. Another more efficient way is to first convert the untrimmed NURBS surface to T-spline and realizes local refinement of the T-spline. Nevertheless, how to efficiently realize the local refinement of T-splines is still an open question full of challenges [3, 5].

#### Step 3: Removing the invalid control points and fitting the trimming curve.

As shown in Fig. 2(c), removing the control points located in the enclosed polygon  $A$  and approximating the trimming curve in physical domain with a B-spline curve whose control polygon is denoted by  $B$ . The number of control points in  $B$  is equal to the number of vertices of polygon  $A$ . Assuming that the polygon  $D$  denotes the corresponding polygon of  $A$  in physical space, the ideal control polygon  $B$  should possess a 'similar' shape of the polygon  $D$  to obtain a good parameterization. In consequence, we have to coordinate the curve approximation and the shape consistency, which is a nonlinear optimization problem. A genetic algorithm is introduced to minimize the Euclidean distance between the polygon  $D$  and the control polygon  $B$ .

#### Step 4: Completing the T-mesh.

Inserting a middle layer of purple control points between polygon  $D$  and the control polygon  $B$  as depicted in Fig. 2(d). The choice of purple control points should minimize the orthogonal distance between the trimmed NURBS surface and the converted T-spline surface. The knot interval between the purple control points and the corresponding black control points are set to zero to satisfy the Bézier boundary conditions. It should be noticed that the choice of purple control points is also an optimization problem and will not be discussed in this paper.

### B-spline Curve Approximation:

Automatic Curve approximation is a long-standing problem that has been widely used in various fields. The classical curve approximation method is the least squares curve approximation [4], which mainly includes data parameterization, knots placement and least squares minimization. Squared distance minimization (SDM) method could also be used to approximate data points and considered to be a stable and efficient approach [10]. In this task, B-spline curves are employed to approximate NURBS curves with least squares approximation method. The detailed process is not described here.

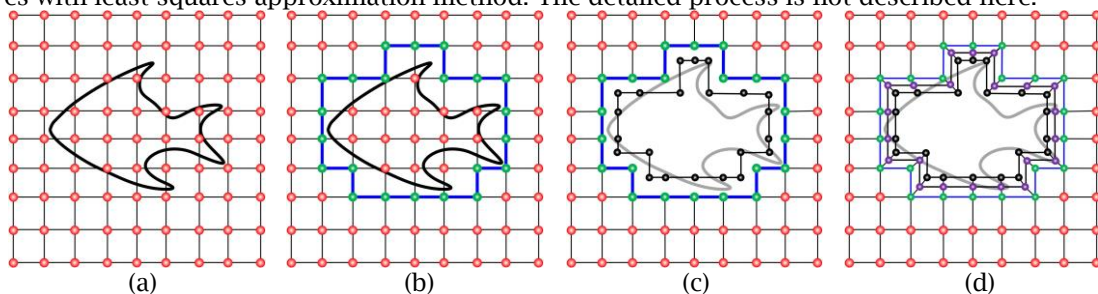


Fig. 2: Process of converting a trimmed NURBS surface to an untrimmed T-spline surface.

### Optimization with Genetic Algorithm:

The problem of finding a 'similar' shape between the control polygon of the fitting curve  $\mathbf{B}$  and the predefined enclosed polygon  $\mathbf{D}$  is simplified to the problem of minimizing the distance between  $\mathbf{B}$  and  $\mathbf{D}$  as illustrated in Fig. 2. Meanwhile, it is expected that the fitting error is as small as possible, which is also an optimization problem. Therefore, it is a multi-objective optimization problem and can be solved by using a genetic algorithm which has been successfully applied in numerous fields including curve fitting [12].

#### Initial Individuals and Population

According to the definition of T-mesh, the parametric length of the opposite sides in each element in T-mesh should be equal. Therefore, the knot vector of the fitting curve with control polygon  $\mathbf{B}$  can be defined according to the parametric length (knot intervals) of the enclosed polygon  $\mathbf{D}$ . For simple trimming shape, we can use a single curve for fitting while several fitting curves should be used for complicated trimming shape. Note that knot vectors, number of control points, and degree depend on the enclosed polygon and initial NURBS surface. Therefore, the parameters for data points are the only variables for optimization.

Assuming that the number of data points is  $m$  and knot values belong to  $[a, b]$ , the parameters for the first and last data points are chosen as  $a$ , and  $b$ , the parameters corresponding to the rest of the data points denoted by  $\xi_i, (i = 2, 3, \dots, m-1)$  are considered to be genes for optimization. These  $m-2$  genes for each individual are generated randomly in  $[a, b]$  and sorted with size. The initial population consists of  $N$  individuals and is denoted by  $\xi_N$ .

#### Fitness Function

Two objective functions are considered in this paper: one is the distance between the polygon  $\mathbf{D}$  and the polygon  $\mathbf{B}$ , the other is the fitting error. Assume that the total number of control points in the polygon  $\mathbf{D}$  and  $\mathbf{B}$  is  $n$ . The vertices are denoted by  $D_i$  and  $B_i$  with  $i = 1, 2, \dots, n$ . Then the first objective function  $d_1(\bar{\xi})$  can be written as

$$d_1(\bar{\xi}) = \sum_{i=1}^n \|D_i - B_i(\bar{\xi})\|. \quad (4.1)$$

Note that here  $B_i$  denotes the control point and can be obtained by approximation. The second objective function is used to minimize the fitting error and can be expressed as

$$d_2(\bar{\xi}) = \sum_{i=1}^n \|Q_i - C_i(\bar{\xi})\|. \quad (4.2)$$

Then a single-objective function can be written as

$$d(\bar{\xi}) = w_1 d_1(\bar{\xi}) + w_2 d_2(\bar{\xi}) \quad (4.3)$$

Where  $w_1$  and  $w_2$  denote the weights and are chosen as 0.5 in this paper. For better procedure implementation, this minimization problem is transformed into a maximization problem by introducing a series of fitness functions defined as

$$f_i(\bar{\xi}) = \max_{j=1,2,\dots,N} d^j(\bar{\xi}) - d^i(\bar{\xi}), \quad i = 1, 2, \dots, N \quad (4.4)$$

Where  $f_i(\bar{\xi})$  denotes the fitness value for the  $i$ -th individual and is abbreviated as  $f_i$  in the following expressions. A larger fitness value means a larger possibility of the individual to be chosen in the selection stage.

#### Selection Method

The roulette wheel method is employed to select special individuals for reproduction. Let  $f_i$  be the fitness values of individuals at a generation, the possibility of an individual being selected in the generation is computed as:

$$p_i = f_i / \sum_{i=1}^N f_i, \quad i = 1, 2, \dots, N \quad (4.5)$$

Given  $N$  random values belonging to  $[0, 1]$ , we can build a new selected generation consisting of all chosen individuals. Note that one individual may be chosen several times.

#### Crossover Method

Crossover is to produce a "child" which inherits a lot of characteristics from its parents. A possibility value  $p_c$  is given to determine which individuals are chosen to produce "children". The selected generation obtained after using the selection method discussed in the above subsection is divided into  $N/2$  pairs. Iterate each pair and input a random value of  $r$ , if  $r < p_c$ , this pair of individuals are chosen to produce two "children" to replace the parent individuals in the next generation. Otherwise, this pair of individuals have remained in the next generation. Let  $G_1$  and  $G_2$  be the selected pair of parent individuals, two children individuals  $G'_1$  and  $G'_2$  can be produced by:

$$\begin{aligned} G'_1 &= \alpha G_1 + (1 - \alpha) G_2 \\ G'_2 &= \alpha G_2 + (1 - \alpha) G_1 \end{aligned} \quad (4.6)$$

where  $\alpha$  is a random number between 0 and 1. A so-called simulated binary crossover (SBX) operator is also an appropriate choice for real-coded crossover [2].

#### Mutation Method

Mutation, analogous to biological mutation, is to maintain genetic diversity in the reproduction by altering one or more gene values in each individual. A predefined value  $p_m$  is given to determine the possibility of mutation. For each individual, if a random value  $r < p_m$ , a random gene value in this individual is replaced by a random value. The gene values should be reordered after mutation for curve fitting. Note that the possibility of mutation ( $p_m$ ) and crossover ( $p_c$ ) can be decided adaptively [9].

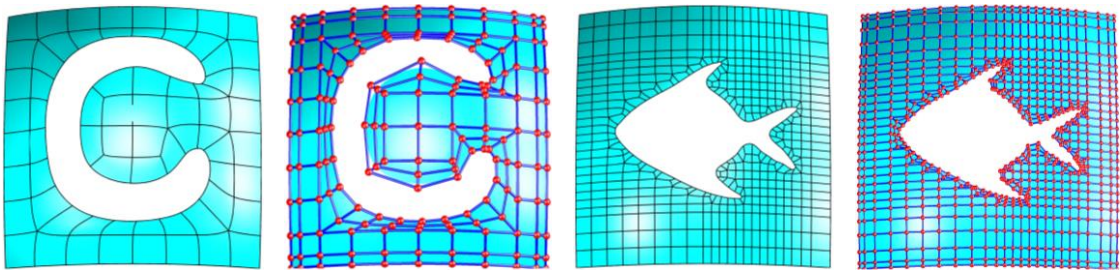


Fig. 3: Optimized T-spline surfaces and T-meshes converted from the trimmed NURBS.

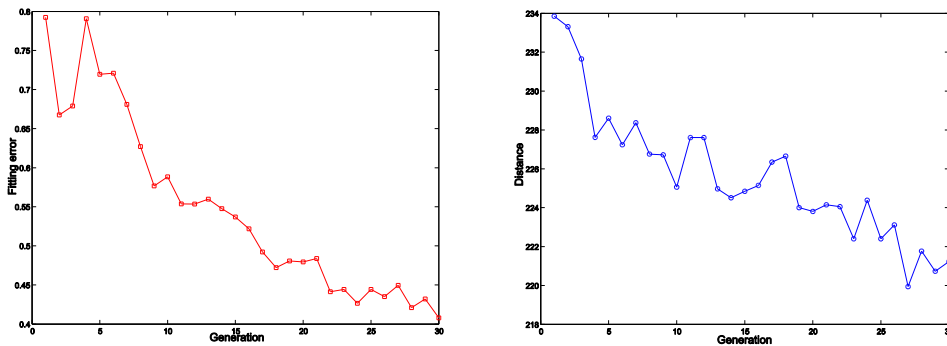


Fig. 4: The variation of the polygon distance error with respect to generations in the optimization of C-trimmed Model. Left: total polygon distance and Right: average fitting error.

As shown in Fig. 3, we present the results of the optimized T-splines converted from trimmed NURBS given in Fig. 1. Both the distance between the enclosed polygon  $D$  and control polygon  $B$  (see Sec. 3), and the fitting error are optimized by using a genetic algorithm. It can be found that the parameterization within the narrow neighborhood of the trimming curve has been significantly improved when compared with the one shown in Fig. 1. To obtain a better-optimized result, trimming

curve could be divided into several curves for approximation. The segment points can be chosen at the extreme values of the trimming curve's curvature. In addition, to better observing the variation of the objective functions, we plot the values of distance and fitting error with respect to the generation in the C-trimmed model's conversion as given in Fig. 4. Maximum generation is 30. Ignoring the slight fluctuation, both values of distance as plotted in Fig. 4a and fitting error as plotted in Fig. 4b gradually decrease with the evolution of each generation.

### Conclusion:

In the process of converting a trimmed NURBS to an untrimmed T-spline, we ought to strike a balance between the approximation error and the good parameterization. A multi-objective optimization problem is constructed to minimize the fitting error, as well as the Euclidean distance between the fitting curve of the trimming curve and the predefined enclosed polygon. A genetic algorithm is employed for optimization. The converted T-spline surfaces show that the proposed method could maintain a good parameterization and greatly alleviate the problem of intersection and distortion. Future works will focus on more complicated trimming cases and the conversion of multiple trimmed NURBS patches into a single T-spline patch.

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