

### <u>Title:</u> Visualization of Folding Motion of Rotationally Symmetric Curved Folding

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#### Introduction:

We propose a GUI system to generate and visualize the folding motion of rotationally symmetric curved folding with some user manipulation. The rotationally symmetric curved folding is a type of origami with curved creases placed in rotational symmetry, and the folded sheet of paper forms a three-dimensional shape (Fig. 1). Various paper arts of this type have been created by many artists and hobbyists, and a software for designing them was developed [4]. While the crease patterns and their final folded 3D shapes are well known, the folding motion, or the paper shapes in between the flat unfolded state and the final folded state, is not clear. To help understand the folding motion of such paper artworks, our system generates and visualizes the paper shape in the intermediate state, with some user manipulation and the key frame interpolation. As a result, it helps the users to learn how to fold the curved folding efficiently, which is sometimes difficult for beginners. It may also be useful in designing a new crease pattern by examining its foldability. Moreover, understanding the folding motion is essential for the automation of the folding, such as RoboFolds [1]. In some applications, the paper shape in the intermediate state is useful by itself, such as lamp shades and containers (Fig. 2).

Unlike the classical paper folding with straight crease lines, the curved folded paper consists of the rulings, the straight lines on the curved surface (Fig. 1, Fig. 3), which transits as the paper is being folded and the bending direction of the curved surface changes gradually. In our previous research, we proposed a method for an interactive design of a curved folding considering its folding motion [7]. In the work, the rulings and the paper shape are calculated according to the parameters of the crease curve: 2D or 3D curvature, the torsion, and the folding angle. The user can change the parameters and check the result instantly until having an intended shape, with no rulings intersecting on the paper nor a selfcollision which does not occur in the real world. However, we found that as the user add more creases, even a small displacement of the curve easily causes the intersection of the rulings. In response to such problems, we propose a new framework by narrowing down the target to a specific type of crease pattern with rotational symmetry. The paper is modeled as a group of identical segments placed in rotational symmetry, as shown in Fig. 1-middle row. By editing one segment, the rest of the paper is cloned according to the symmetric property. Consequently, the curved folding with several creases is modeled using our previous method, as each segment contains only a small number of creases. Moreover, by allowing small gaps between the segments, the method can model the curved folding including straight creases, which is mathematically impossible. Hereafter, we will explain the whole process using the origami-sphere with 6 segments, a typical example of the rotationally symmetric curved folding. It is composed of one curved crease on a segment divided by straight radial line creases, as shown in Fig. 1. The system may also simulate curved folds with five or more segments and creases in different shapes.

In recent studies on modeling developable surface with curved folding, Tang et al. defined a curved surface by a pair of spline curves and solved the constraints for its global developability through

iterative processes [5]. Kilian et al. modeled the 3D shape of a paper in string actuated folding motion by re-meshing the triangular mesh according to the local surface curvatures [3]. Rabinovich et al. introduced discrete orthogonal geodesic nets, which approximates a developable surface by optimizing the corner angles of the quad mesh to be orthogonal [6]. Partly similar and different from their works, our system focuses on generating and visualizing the folding motion with the user manipulation, helping them to understand the movement of the paper, not by a physical simulation nor a numerical calculation. Putting more weight on visual simplicity than robustness, our method models a developable surface with quad mesh based on the rulings, which transits as the 3D shape of the paper changes.



Fig. 1: Folding and unfolding motion of origami-sphere. Top: rendered 3D models. Middle: wireframe models with one segment rendered with rulings. Bottom: photos.



Fig. 2: Example applications. Left: paper clip container. Center: lamp shade. Right: un-foldable cup.

# <u>Main Idea:</u>

Our system contains a 3D model and a crease pattern. The 3D model changes gradually during the folding motion, while the crease pattern is a set of creases in a flat state and stays unchanged. To generate the folding motion, our system starts with the two key frames: the flat state and the final folded state. Initially, the intermediate frames are generated by the linear interpolation of the two key frames (Fig. 3-top row). As this generally causes large gaps between the segments, the user picks one frame, modifies it, and adds it as a new key frame. The rest of the non-key frames are then re-interpolated (Fig.

3-bottom row). This process is carried out until the user is sufficient with the output, having the segments placed adjacently in all frames with no large gaps nor major self-collision.



Fig. 3: Overall process of folding motion generation. Top: initial state. Bottom: after one modification.

#### *Crease pattern generation*

The crease pattern is the predefined data used in this system. The crease pattern of a segment consists of one curved crease in the center and two boundary creases on its sides, which are the boundaries between the adjacent segments. The curved crease is derived from the 2D curvatures on some evenly placed control points on the curve as in our previous work [7]. The two boundary creases are defined by choosing (i) the division number N of the rotational symmetric design, which is equal to the number of the segments, and (ii) the overall curvature  $k_{2D}$  (Fig. 4). The two congruent boundary creases are placed with a rotation of  $2\pi/N$  so that, in the flat state, the segments are placed with no gap. Fig. 1, 3, 5-7 shows an example with N = 6 and  $k_{2D} = 0.0$ , the straight boundary creases.



Fig. 4: Crease pattern of a segment. Left: boundary creases in different numbers of segments N. Right: boundary creases with different curvatures  $k_{2D}$ .

#### 3D model generation and interpolation

The 3D model generation and modification are applied to the key frames added by the user and to the final folded state if needed. It is composed of two steps: (i) generation of the 3D shape of a segment, and (ii) adjustment of the segment pose, or the 3D rotation of the segment. (i) The 3D shape is controlled by the folding angle and the torsion of the curved crease (Fig. 5), derived from the user-controlled parameter values of evenly placed control points on the crease curve. With our previous method [7] using Fuchs and Tabachnikov's equations [2], the 3D curvature is first calculated from the folding angle and the 2D curvature. Then the 3D shape of the curved crease is calculated from the 3D curvature and the torsion. Finally, the directions of the rulings are derived from the folding angle and the torsion, which defines the whole 3D shape of the segment. In our new system, the 3D shape is also derived by giving the ruling directions in 2D crease pattern. Alternately to (i), (ii) the segment pose is modified by an optimization process to minimize the gaps between the boundary creases, or by user adjustment through the mouse drag interface (Fig. 6). As the pose of one segment is adjusted, the other segments are also placed in rotational symmetry around the vertical axis. To integrate the segments into a sheet of paper, (i) the target segment shape must have the 3D shapes of the two boundary creases to be nealy congruent and (ii) the segment pose to be adjusted to minimize the gaps, without major self-collision. For the interpolated frames, the folding angles, the torsion, and the pose are linearly interpolated respectively between the key frames, followed by the same process as described above.



Fig. 5: Modification of 3D shape of a segment. Left: Initial shape with constant folding angle and zero torsion. Right: Modified shape with folding angle and torsion adjusted by the user.



Fig. 6: Adjustment of segment pose. Left: initial pose. Right: optimized pose.

#### GUI system

The proposed GUI system consists of four panes showing the 3D model, the 2D crease of a segment, the parameters of the curved crease shown in the graphs, and a control panel (Fig. 7).



Fig. 7: Proposed GUI system.

### Evaluation

As an evaluation of the generated folding motion of the origami-sphere, shown in Fig. 1, every frame was evaluated by (i) the developability of a segment and (ii) the connectivity of the 3D model. As to (i), the maximum error of the flatness of the quads was 0.1mm for a segment with length of one side being 200mm. The maximum error of the angle sum around each vertex on the curved crease was 0.02 degrees, which is sufficiently developable. (ii) The connectivity was evaluated by the average distance from the sample points on one boundary crease to the other boundary crease of the adjacent segment (Fig. 8). For the visual comparison, Fig. 1 shows that the generated 3D models with the rulings are in similar shapes to the photo of the real paper.



Fig. 8: Connectivity of the shapes in Fig. 1. Circles indicate key frames with both shape and pose modified. Squares indicate frames with only pose modified.

# Conclusion:

In this work, we proposed a GUI system to generate and visualize the folding motion of rotationally symmetric curved folding. The system models a sheet of paper with many curved creases, by having the user to edit one segment and cloning the rest according to the symmetricity of the crease pattern. The evaluation shows that segments are modeled as developable surfaces, while there are small gaps between the segments. The behavior of the paper shape and the rulings were visualized successfully, showing that the ruling angles are nearly right angle at final folded state and become acute in the intermediate state. This makes most of the rulings to end at the edge of the paper instead of the boundary crease, making the straight crease line to be folded with minimum curvature on the crease.

As a future work, the user interface to generate and modify the 3D shape of a segment should be improved, as well as adding optimization process, because this process is still difficult and requires some user experience. Also, being able to deal with discontinuous curved crease would expand the scope of the curved fold design.

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