Title:
Development of Multi-branch T-spline Templates and its Applications in Reverse Engineering

Authors:
Kritika Joshi, rme1615@mnnit.ac.in, MNNIT, Allahabad, India
Amba D. Bhatt, abhatt@mnnit.ac.in, MNNIT, Allahabad, India

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Introduction:
The term branch surface modeling has mainly come in picture from ongoing research in the field of biomedical industry. It can be applied for the study and treatment of internal branched structure like human vascular and bronchial system. The major domain area it covers includes engineering, medical and biological science. These applications require digital information (data acquisition) of the physical object. When a physical object is reconstructed and represented as a digital model, is known as reverse engineering of the shape of that object [12].

The initial step towards reverse engineering of shape involves the creation of triangular meshes from unorganized point cloud data [3]. While reconstruction of a branched surface, major complexity is to maintain geometrical continuity at the junction against the best topological relation between the two adjacent branches contour [11]. Some previous attempts have been done to simplified branching problem for the construction of bifurcating surface model [5],[9], which could not meet easily with the desired requirements of continuity and control over the surfaces.

Towards the development of free form shapes, sectional contours are used [10] through which parametric fitted surface can be created. The parametric surface offers flexibility and control over the desired shape of the model. Recently various parametric surfaces like B-spline, NURBS has been used for the design of branched surface models by using disjoint surface [1][4]. The involvement of T-spline surface allows local refinement, thus for the same shape relatively less number of control points has been used [2]. The absence of local refinement is considered as a major constraint while working with the B-spline based surface model.

Yang et al. [13] presented a T-spline surface skinning algorithm to overcome the knot compatibility problem of B-spline surface fitting. In order to manage large point cloud data, a fast fitting algorithm developed which adaptively divide the data in B-spline patch and connect them as a T-spline surface [6]. However, these fitting methods have not been found compatible of for reconstruction of surfaces having sharp changes in topology. The generation of one to many branches by interpolating data points in parallel cross section has been elaborated by using T-splines for smooth branching which are limited to parallel planes [7].

By considering the numerous applications of reverse engineering for designing of complex branch shape, present work comprises the method for the development of T-spline templates from sectional curves of the branched manifold. It uses the definition of T-spline surface and scanned data set of the real branch geometry for the purpose of reconstruction of single and multiple branches surface model.

Overview of this Paper:
This work demonstrates the generation of branch surface models by mapping of cross-sectional curves, into a T-spline template. In order to achieve the desired branched surface, the work has been divided into two phases. In phase one the cross-section curves have been extracted from the labeled point cloud...
data. The Second phase includes sequencing of control points to preserve the shape of the surface. The T-spline surface has been fitted into the branched shape (bifurcating and multi-furcating through serial bifurcating) control polyhedron. The set of periodic knot vectors along with the method of disjoining in the definition of T-spline surface are used to make closed branched shapes [8].

Fig. 1: Process diagram for phase 1: (a) Point cloud data, (b) Segmentation and labeling of data, (c) Cross-section curves of the data, (d) Cross-section curves with control points.

The process diagram for phase one is shown in Fig. 1. The extracted cross-sectional curves from segmented point cloud data has been used as an input for phase two. The number of rows used in T-mesh is corresponding to the number of sectional curves extracted from the scanned data. These templates have been used and discussed further to reconstruct the parametric surfaces model of the branch shape.

*Definition of T-mesh and T-spline Surface*

A T-spline surface is defined from a two-dimensional parametric template, called T-mesh. It allows local refinement with a smaller number of control points as compared to a non-uniform B-spline surface. Thus, reconstruction from a T-spline surface provides more flexible and realistic shape as compared to other mathematical surfaces (Bezier, B-splines, NURBS). The equation of a T-spline surface is given by:

\[
S(u,v) = \frac{\sum w_{ij} P_{ij} B_{ij}(u,v)}{\sum w_{ij} B_{ij}(u,v)}
\]  

(1)

Where each control point \(P_{ij}\) is associated with weight \(w_{ij}\) and a corresponding blending function is given by:

\[
B_{ij}(u,v) = N_i(u)N_j(v), \quad (u,v) \in D
\]

(2)

In the above equation, \(u\) and \(v\) are parameters defined by a domain \(D\) of horizontal and vertical knot vectors respectively. The values of knots in a knot vector can be assigned through T-mesh. The blending functions \(N_i(u)\) and \(N_j(v)\) at each control point affect the shape of the surface. Thus, the process of local refinement improves the blending functions to preserve the shape of the surface. In this method, all weights are assumed as 1 and the order of the T-spline surface has been taken as 4.
Reconstructed Surfaces:
This section shows the results obtained by T-spline surface fitting of stem part, single and multi-branch. All these surfaces utilize the cross-sections curves, displayed in Fig. 1.

Reconstruction of Stem Part
The surface shown in Fig. 2(b) has been obtained by mapping of control points on template shown in Fig. 2(a). The numbers of control points in each row of T-mesh are corresponding to points lies in each cross-sectional curve. In order to make closed surface, periodic knot vectors have been used in the definition of T-spline surface given in Eqn. 1.

To fit these control points in the template, two pseudo edges (green and blue) has been assumed which gives the idea of starting and end of the control points in a half control polygon. The points lie in green edge are considered as left starting point of each sectional curve and that lies in blue edge are assumed as right end edge of the curve. All other points positioned in between are repeated (to make periodic surface) and arranged as displayed in template.

The surface model is simple to construct and have desired shape accuracy corresponding to the input cross-section curves displayed in Fig. 2(b).

![Reconstructed Surfaces:](image)

Reconstruction of Single Branch
The construction of bifurcating surface model illustrated by Fig. 3 has been addressed in this part. The single branch consists stem part, junction and two branches which are separated through the junction. The midpoint approximation is used to evaluate junction point. For this two nearest point of the
junctional contours has been taken. The input curves are used to create the template for the parametric branch surface model. The obtained T-spline fitted surface is shown in Fig. 3(b) and Fig. 3(c). The final surface has topological similarity with respect to the point cloud data. The Small limbs on the branch surface have been clearly visible in the obtained surface model.

Reconstruction of Multi Branch
This part introduced the construction of multi-branch surface model shown in Fig. 4. The same method that has been used for single branch is applied for its construction also. However, the number of disjoints in T-mesh will be increased with the number of branches.

![Surfaces](image)

Fig. 3: Labelled point cloud data of a bifurcating wooden branch and resulting T-spline surface: (a) Point cloud data of the object, (b) Resulting parametric surface, (c) Surface with interpolated shading.

The total number of control points has been increased with the number of sectional curves. The multi branched parametric surface obtained from scanned data, is displayed in Fig. 4(b). The shape of the obtained surface matches with the input scanned data. The set of examples present here are the cases of serial bifurcation however, can be effectively applied for tri-furcation, quad-furcation also.

Conclusions:
The development of T-spline templates and its implementation on cross-sectional data of a wooden branch (bifurcates and multi-furcates) frames is presented in the proposed method. The approach used is simple for reconstruction of branch shape with a limited number of control points. The position of the control points can be modified to improve the topology of the surface, without adding extra knots to the template. These T-spline templates can be effectively applied to construct branch surface models for various reverse engineering-based applications.
Fig. 4: Point Cloud Data and Resulting Multi Branch Surface Model.

References: