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# Idealizing Quasi-Axisymmetric 3D Geometries to 2D-Axisymmetric Finite Element Models 

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## Introduction:

The desire to utilise 3D Computer-Aided Design (CAD) models to represent components earlier in the design process is hindered by the need for the supported analysis process to be efficient, using models of the appropriate cost and complexity. For instance, during the design phase it could be of interest to carry out analyses of different versions of the product using simplified FE models. Then, the analyst can develop progressively more detailed representations of the model if required. However, the CAD models provided to the analyst may be very complex and idealisation steps are required to make them suitable for analysis $[7,12]$. The main operations for geometry preparation are: i) defeaturing [8], ii) geometry clean-up [3], iii) partitioning [11], iv) decomposition [9] and v) dimensional reduction [4]. These are used to create simplified, computationally efficient versions of the geometry for simulation $[1,5]$.

Usually, idealising a complex CAD model will require more than one of these operations, and much research has been devoted to managing and streamlining the process, e.g. [10]. The dimensional reduction step is at the core of this work. Specifically, the automatic creation of a 2D-axisymmetric FE model from the 3D CAD model of a quasi-axisymmetric component. Such components have an obvious axis about which much of their geometry is axisymmetric, but include features which are not complete revolutions around the axis. Some geometry clean-up operations are also required during the dimensional reduction process.

## Main Idea: Geometric recreation

The idealization methodology has been implemented as: i) a software capability for the creation of the 2D-axisymmetric profile and ii) a Python script which exploits the Abaqus Scripting Interface to calculate the shape coefficient. The overall methodology consists of six steps. The input is a triangular facetted representation of the CAD geometry. A facetted representation was selected because while it is possible to analytically compute an analytic description of an analytic shape on a plane, it is not possible to do this for NURBS geometry which is common in CAD. A Facetted model can be created for all common engineering geometry types (e.g. NURBS geometry, analytic CAD geometry, meshed geometry and point clouds). In CAD the facetted representation is created by replacing each of the faces of a model with a set of triangular facets. The quickest way to achieve this is to export the geometry from the CAD system in a facetted format. VRML was used in this work but the approach would work equally well with any

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triangular surface mesh/facet. The mesh coarseness should be selected to reflect the size of the features in the part, but increasing the number of elements will impact the processing time. The different steps implemented in the process are reported below with the geometry at different stages shown in Fig. 1

Step 1: Projecting face facets circumferentially onto a plane. The axisymmetric model exists on a plane; consequently, the first step in the idealization process is to circumferentially project the facets representing each face onto a plane (the $r-Z$ plane where $r$ is the radial direction and $Z$ the axial).

Step 2: Classifying faces. Once the facets representing the model faces have been projected onto the $r-Z$ plane it is necessary to classify them. The classifications are:

- Case 1: After the projection, the 3D face becomes a 2D degenerate face of zero area (referred to as a 'mapping face' in this work). This occurs when the area of each facet representing the face becomes zero after the projection.
- Case 2: These are faces that include some facets that have non-zero area when projected circumferentially onto the $r-Z$ plane.

Step 3: Identification of mapped edges. The facets identified in step 2 as case 1 are used to define the axisymmetric profile of the part. These projected facets have three collinear vertices. The associated mapped edges are bounded by the extreme points. The mapped edges are then stored in a list called SHAPE-EDGES for use in step 5 .

Step 4: Identification of silhouettes edges. Silhouette edges define other edges in the axisymmetric profile. Silhouette edges are identified as those bounding adjacent facets whose normals point in opposite directions after the projection to the $r-Z$ plane. Identified silhouette edges are added to the SHAPEEDGES list which now contains all the necessary edges to form the 2D-axisymmetric profile.

Step 5: Obtaining a raster representation of the 2D-axisymmetric profile. The set of SHAPE-EDGES is sufficient to define the overall boundaries of the 2D-axisymmetric model. However, at this stage the shape edges are not topologically connected and do not represent the topology of the 2D-axisymmetric model. The consequence is that many of the edges will be redundant (i.e. have been projected onto the same locations).

Rasterization strategies are used to deal with the edge redundancy issue and to generate an appropriate data representation for topology construction via contour recognition techniques (addressed in step 6). The rasterization approach consists of discretizing the $r-Z$ plane to a square grid (i.e. a raster formed by square cells) which the shape edges are mapped to. If a cell in the raster is intersected by one or more edges, then the cell is stored. In this work Bresenham's line algorithm [2] was used. The 2D-axisymmetric model profile is reduced to a binary structure from which the regions that form the 2D-axisymmetric model can be extracted.

Step 6: Building the 2D-axisymmetric FE model. The OpenCV image processing library was used to identify contours in the binary structure resulting from step 5 . The contours detected using this function are stored as arrays of points defining polygons. The resultant polygons can be unnecessarily complex from a FE perspective. To simplify the polygons, the Douglas-Peucker algorithm [6] (function ApproxPoly in EmguCV) was used. These polygons are then converted into a usable CAD model by writing them in a specific format (e.g. STEP). The automated idealisation result of some simple concepts are shown in Fig. 2

## FE model creation:

The 2D-axisymmetric FE model (Fig. $1(\mathrm{~g})$ ) is obtained by meshing the CAD model resulting from the six steps described. In this work the creation of the FE model was automated using the ABAQUS python API. The main steps in the FE model creation were the creation of an axisymmetric profile and appropriate boundary conditions (e.g. $U_{y}=0, R_{x}=0$ and $R_{z}=0$, where $x$ is the axial direction and $z$ is


Fig. 1: (a) Overlapping edges in a 2D-axisymmetric profile; (b) Overlapping edges reduced to a set of unique cells; (c) Contours detected in the binary profile; (d) Simplified contours using Douglas-Peucker algorithm; (e) Stitching of the simplified contours; (f) 2D-axisymmetric CAD created; (g) 2D-axisymmetric FE model


Fig. 2: Automated rebuild of axisymmetric shape
the radial) and the application of 2D axisymmetric elements (e.g. CAX6M) in the different sub-regions of the model. The conversion of a 3D quasi-axisymmetric model to its equivalent 2D-axisymmetric version results in a loss of geometric information in the circumferential dimension. In order to ensure the physical relevance of the 2D-axisymmetric model with respect to the original 3D model this loss must be compensated. A physically straightforward way to account for this, is to modify the 2D-axisymmetric FE model with a shape coefficient applied to the material properties and to the loading. For this purpose, the 'Volume Fraction' coefficient $\left(K_{v}\right)$ is defined. This coefficient establishes for each region in the model the fraction of the volume around the axis of revolution the 3D model actually occupies. This coefficient is equal to 1 for an axisymmetric region.

To validate the proposed methodology a rotating blade of tapered profile is analysed. The radial stresses and displacement resultant from both the 3D model (Set-up A), the generated 2D-axisymmetric model with corrected material properties (Set-up B corr) and the analytical model are compared. The 3D model, depicted in figure 3, consists of a shaft with 20 tapered blades occupying $2 \%$ of the volume at all radii, consequently the volume fraction $K_{V}$ is equal to 0.02 for entire non-axisymmetric region. Stresses and displacements acting along the radial direction caused by a centrifugal body force are evaluated in this example. It is assumed that the blade is rotating with an angular velocity $\omega=250 \mathrm{rad} / \mathrm{s}$. The analytical expressions are given in Fig. 3 (right).

The contour plots of the radial stress for the 3D section and the corrected 2 D axisymmetric are shown in figure 4 left. From the figure it is clear that the results obtained using the two approaches are similar. For a quantitative analysis, 35 points radially distributed along the midline of the blade have been compared for both models. Their values have been plotted in figure 4 right and compared with the analytical solution of the model (green line). Excellent matching is shown. The maximum radial stress along the midline of the blade is at the interface between the disk and the blade. At this point Abaqus calculated a stress of 11320 MPa for Set-up A and 11450 MPa for Set-up B. From the interface on, the radial stress decreases progressively along the length of the blade.

Analytical solution for cylindrical sector:


$$
\sigma_{r_{d}}=A+\frac{3+\nu}{8} \rho \omega^{2}\left(r_{1}^{2}-r^{2}\right)
$$

with A the residual stress at the interface between the blade and the disk

$$
A=\frac{m_{\text {blade }} \cdot \omega^{2} \cdot C O G}{\pi r_{1}^{2} t}
$$

Where $\sigma, r, \omega$ have the usual meaning and $t$ is the thickness of the blade.
Analytical solution for tapered blade along the midline $\left(r_{1} \leq r \leq r_{2}\right)$ :

$$
\sigma_{r_{b}}=\rho \omega^{2}\left(\frac{r_{2}^{3}}{3 r}-\frac{r^{2}}{3}\right)
$$

Fig. 3: Left: Shaft with 20 tapered blades uniformly distributed along the circumference; Right: Analytical solution of the model


Fig. 4: Left: Stress contour plots; Right: Stress along the radial direction

## Discussion:

The automated idealisation process took less than two minutes on a standard desktop workstation (Intel i7 with 16GB Ram). Factors which impact the process time include:

- The complexity of the component model shape. A more complex component model, with more faces and of greater curvature, will be represented by a larger number of facets.
- The calculation of $K_{v}$, along with the associated revolve and Boolean operations, is required for each separate quasi-axisymmetric region in the model.

Over the course of this work, the idealisation of even seemingly complex models has required less than 5
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minutes to process. In the process described above, one $K_{v}$ value is calculated for each quasi-axisymmetric region in the model. This was fine for the example model because the tapered blade profile meant that the $K_{v}$ value was constant across the blade, and the calculate $K_{v}$ was accurate at all positions. Should the thickness of the geometric features in the axisymmetric regions not exhibit this property, then the $K_{v}$ value calculated will be the average for the region. Should the average not be sufficient then the quasiaxisymmetric profiles in the 2D model can be divided into smaller regions. This will produce smaller sections, each with $K_{v}$ averaged for the region they cover. At the highest level of granularity, the quasiaxisymmetric regions can be meshed in the 2D model, and each element teared as a separate region. This will provide the highest possible accuracy, but there will be a cost associated with its computation.

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