Title:
A Topology Optimization Approach Using Explicit Stress Tensor Analysis and NURBS Curves

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Introduction:
Topology optimization (TO) aims at defining the ideal layout of structures in order to minimize the use of material within a given design space. Such goal is pursued by applying an optimization strategy, where the possible solutions are the feasible geometries, and the functional to minimize is the mass structure.

Obtaining high performance in terms of mass reduction depends on many factors [6]. The most well-known and widespread topological optimization approaches can be divided into two main categories: microscopic and macroscopic.

In microscopic approaches, such as SIMP and BESO, the feasible domain available for the solid material is divided in a finite number of discrete elements (FEM). The density of the material of each element represents the design variables, which may vary with continuity in SIMP method (density approach) or being binary in BESO Method (evolutionary approach) [6].

In macroscopic approaches, such as Level Set [1], the parameters describing the evolution of the boundaries of the solid domain of the structure are the design variables [3]. Again, a discretization of the design domain is required, even if the implementation of this strategy in the optimization algorithm doesn't intrinsically need the decomposition in a finite number of elements.

The discussion regarding the choice of the best strategy to achieve a better performance is still an open research topic. We opted for an optimization process in two stages [2], preferring the BESO method for implementing in the first optimization stage. This choice was made due to the fact that TO results vary due to use of different methodologies. Moreover, they are highly sensitive to the choice of mesh resolution adopted to discretize the variable domain.

This work aims at overcoming this sensitivity problem and also reducing undesired "checkerboard pattern". We introduce a novel approach using explicit stress analysis to perform TO in the first stage and extend it by refining the results with parametrizing the boundaries using NURBS curves.

Main Idea:
In this work, an inedited method for mechanical structures optimization is proposed and is constructed by the following stages:

- Discretize the work-space defining a starting mesh of elements
- Carry out a raw BESO optimization, obtaining a new set of elements
- Re-arrange and connect the new elements according to their mechanical properties using tensor analysis
- Apply NURBS curves to the resulting TO in order to achieve continuous elements

The main advantage of the method is that modifying the orientation of the resulting elements creates the final layout, so that the discretization of the Area of Interest (AoI) is more accurately represented by the distribution of tension.

This is important because, in microstructural approaches, the reliability of the solution is dependent on the definition of the mesh, and the results have the typical fuzzy aspect. On the other hand, macroscopic approaches are more rigid in generating new topologies. For the above depicted reasons, we present a method that has the flexibility of the finite elements approach and is independent using a particular parametrization.

In order to realize this purpose, a procedure of re-meshing of the design space may be applied. The reason why this procedure is so important is explained analyzing Fig.1. Fig. 1(a). depicts the result of the optimization of a truss, obtained implementing the BESO algorithm which was implemented by using Abaqus script [7]. To avoid tessellation of the resulting structure, the basic algorithm was integrated with a sensitivity filter. The result of this combination is well-defined solid/void zones, which are identified. As seen in Fig. 1(a)., some sub-structures are highlighted with red rectangles, which may be interpreted as “beams”.

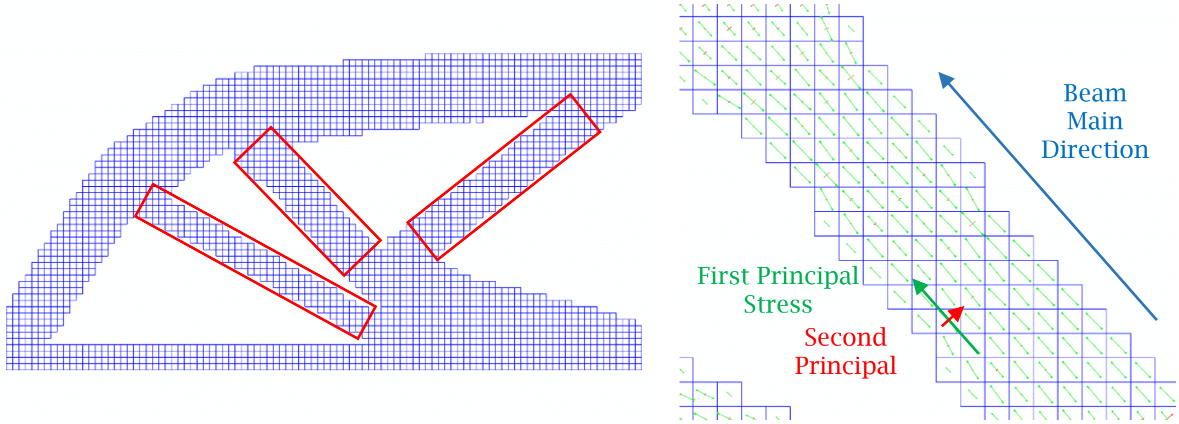

Fig. 1: Result of the process of topological optimization using a sensitivity filter: (a) the whole resulting topology, (b) a particular of a sub-structure, and principal stress tensor.

It can be noted that depending on the initial definition of the mesh, the beam sub-structures are not disposed in the same direction of their elements. In addition, analyzing the state of tension of the elements, shown in Fig. 1(b)., the first principal tension is bigger than the second one, and it has the same direction of the beam. This stress configuration may be interpreted as a “pure” compression for the beam.

In the proposed approach, coherently with the previous empirical observations, some information about the stress configuration is incorporated in the definition of the mesh. Based on the flux of tensions inside the optimized workspace a re-meshing of the design space is performed. In fact, an enhanced distribution of material occurs inside the work space if the elements are purely compressed or in traction.

The optimization process is composed of the following phases:

1. Carry out a first raw BESO optimization: the result of such TO consists in the set of elements with high elastic energy. Low energy elements are eliminated by the initial mesh.
2. Obtain the state of tension for every element of the mesh resulting from the previous step. The analysis of all structure is involved, and every element is considered.
3. Re-arrange the elements coherently with the results of the stress analysis, in particular the principal stresses and the slope of the principal reference system.
4. Provide a connection between the reoriented elements, and the boundaries for the solid material domain in order to provide the continuity of the structure.
5. Carry out a further fine shape or size optimization

6. Use NURBS in order to obtain a continuous boundary.
7. Eventually perform a further shape optimization using a gradient-based method.

Phases 1 and 2 are depicted in Fig. 2. The result of a very raw TO of the truss, based on the previous BESO method [7] is shown in Fig. 2(a). This is the initial analysis of the work space (AoI), highlighting the principal stresses, σ_I and σ_{II} in correspondence to the elements. These quantities are already expressed in the principal (local) system of reference. In addition to the original reference system of the element, the state of tension is described by all the components of the stress tensor σ_x, σ_y, and τ_{xy}.

The result of the TO is shown in Fig. 2(b)., where the sub-structures of the beams are identified. While investigating the nature of the stresses in such sub-structures, it is highlighted that the major principal tensions are oriented in the same direction of the elements (as indicated by the red rectangles)..

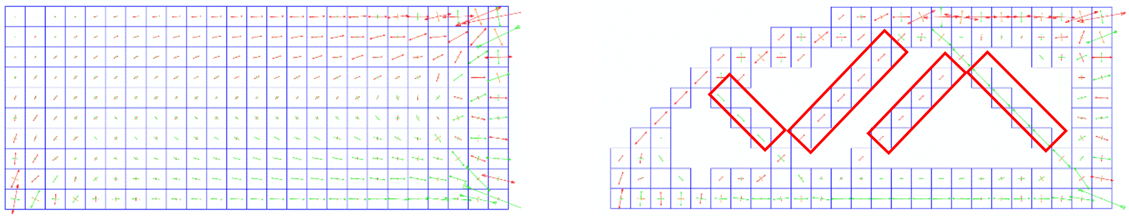

Fig. 2: Stress fields: (a) in original structure, (b) in optimized structure.

Resulting from the optimization process, material may be "disposed" along the stress flux, in order to reach an optimal distribution of matter, and ensure constrained structural performances. This means that it is possible to formulate a rule in order to modify the starting mesh of the finite element analysis, to better locally represent the behavior of the material, "aligning" the elements to the principal directions.

Phases from 3 to 5 are depicted in Fig. 3. The re-mesh procedure is performed after the BESO algorithm, and can be resumed as follows:

- Fig. 3(a).: the stress tensor corresponding to every single element is analyzed
- Fig. 3(b).: evaluating a Mohr circle in the barycenter point of each element, which is defined by the traction/compression and shear tensions. Due to such analysis, we identify the rotation angle needed to impose onto the finite element in order to orient it
- Fig. 3(c).: the definition of the rotation angle is obtained by the use of known relations which describe the Mohr circle for a plane state of tension:

$$\theta = \frac{1}{2} arctg\left(\frac{2\tau_{xy}}{\sigma_x + \sigma_y}\right) \tag{2.1}$$

Eqn. (2.1) expresses the passage from the original local system (σ_x, σ_y, τ_{xy}) to the principal local system (σ_I, σ_{II}). In this way, the new disposition of the element is obtained by a rigid rotation of the element itself by an angle θ

- Fig. 3(d).: implementation for all the elements in the AoI, resulting a loss of continuity, which demands a new definition of the mesh
- Fig. 3(e).: re-connecting the adjacent elements
- Fig. 3(f).: size optimization is performed, which means that the section of the re-oriented elements depends on the module of the first principal stress. This is done and for every specific element, in a unique manner

Evolutionary methods, such as BESO, are iterative processes, and generate different "intermediate" layouts. For every iteration step, the method recalculates the stress configuration of the remaining structure, and eventually carries out the re-meshing process.

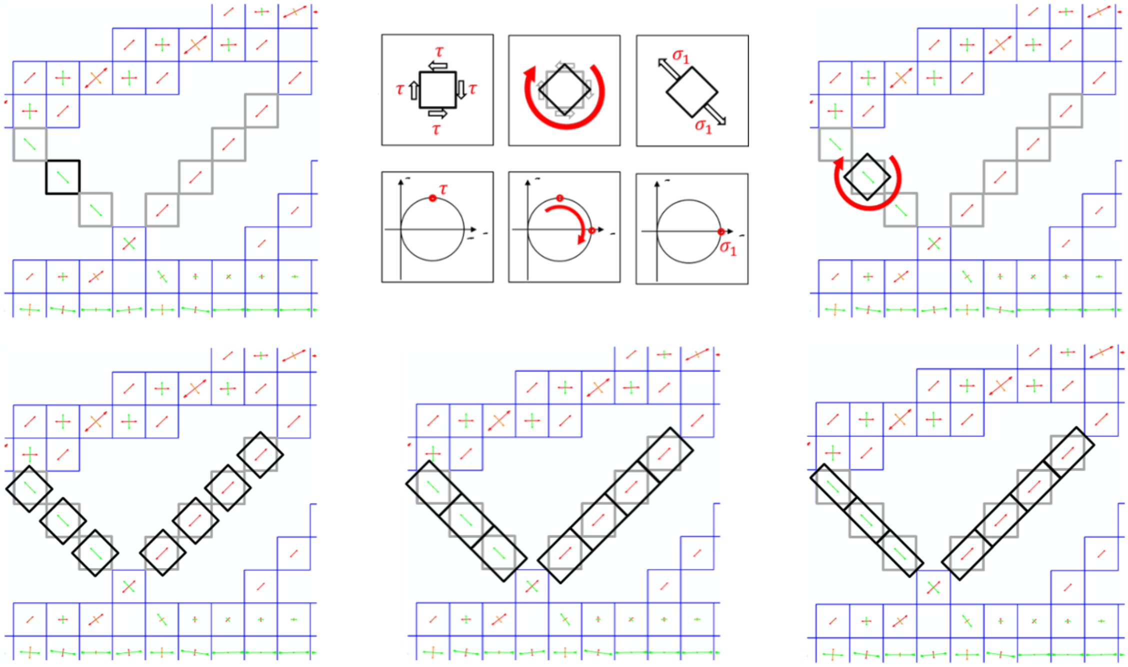

Fig. 3: The re-mesh procedure: (a) stress tensor of the elements, (b) calculation of the angle between original and principal reference system using the Mohr circle, (c) rotation of a single element, (d) rotation of many adjacent elements, (e) connection of separated adjacent elements, (f) size optimization of connected elements.

Consequently, the new iteration is more accurate, and results a further sensitive to the stress configuration of the resulting structure. The proposed method is very useful to identify sub-structures as beams, but all these features require to be joined.

For this purpose, we propose the use of Non-Uniform Rational B-Spline (NURBS) curves. Such representation has been investigated in many researches [4,5] and NURBS curves offer continuity and flexibility for the analytic and parametric representation of TO results.

There are many other advantages in using NURBS curves for providing a feasible representation which includes:

- computational stability;
- the possibility to represent free forms in a unique and strict formulation;
- the ease to vary the NURBS curve shape varying control points and weights. Such feature ensures high flexibility and shape locality.

NURBS formalization provides three sets of parameters, which represent the three different degree of freedom, which rule a new curve: control vertices, weights corresponding to each control vertex and values of the knot vector.

Fig. 4 shows how the nodes of the rotated and re-joined elements are used to construct the NURBS curve. Nodes are used for defining the polygon vertices, being a partial input for calculating NURBS curves. The sequence of vertices is converted into the set of control points. The NURBS curves defined in this way may incorporate line and curves. In the present work we suppose to define the control points coordinates and eventually optimize the weights. Adopting this strategy, a good trade-off between complexity (number of parameters) and the possibility represent complex shapes may be achieved. This is done in order to define the boundaries of the domain of the solid material. If different beams are adjacent, the NURBS curves provide the necessary continuity.

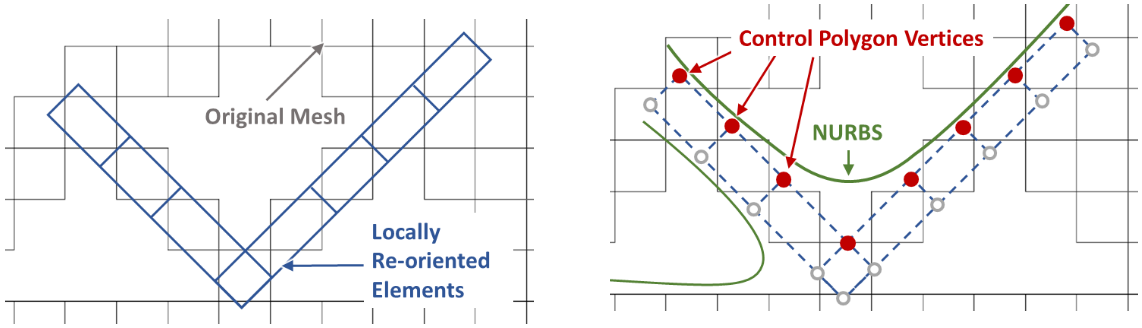

Fig. 4: NURBS application: (a) re-oriented elements, (b) definition of fitting NURBS.

Conclusions:

This work proposes a conceptual framework for a novel TO approach. It is performed by re-arranging the TO mesh obtained by BESO according to mechanical parameters. The principal stresses and the slope of the principal reference system, are calculated, rotated, and a process of joining and size-modifying elements is performed. The optimized elements are connected by NURBS curves in order to provide the continuity of the final topology. This new mathematical formulation is more suitable for a finer optimization which may be achieved by applying a gradient-based optimization to the defined boundaries of the solid domain. Until now, analysis optimization and elements rearrangement have been implemented in a MATLAB, application, and a first experimentation have been carried out. Moreover, mesh re-definition, NURBS definition and shape optimization are in a study stage.

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