Title:
Interactive Modelling of Curved Folds with Multiple Creases Considering Folding Motions

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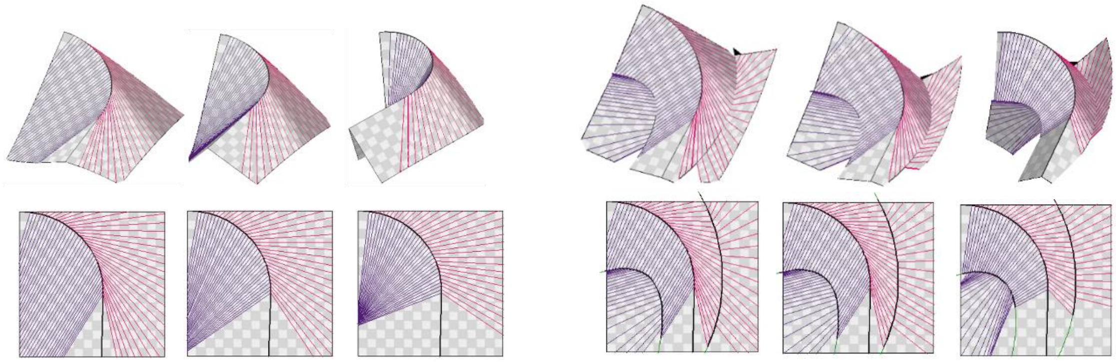
Introduction:
We propose a framework of a user interface for modeling a curved folding and visualizing its folding motions. In the proposed framework, the transition of rulings during fold or twist motion, as shown in Fig. 1, is supported on the 3D model of a sheet of paper with multiple crease curves.

In the area of paper modelling and simulation, 3D curved surfaces generated by curved folding are often represented by static quad strips. Kilian proposed a method to approximate a 3D shape by developable surface composed of quad strips, by initially fitting planer polygons on the 3D shape and then optimizing their shapes and orientations [2]. Their method is successful in static shape modelling but does not design nor consider its folding motion. Tachi proposed flat-foldable vault structures composed of tubes with curved creases assembled by welding two sheets [3]. His study simulates the folding motion of the structure, but, by restricting the folding angles to be constant throughout the curve, the rulings are fixed and the model is folded rigidly. In recent studies, Tang et al. introduced an interactive design system of a curved-creased origami, which solves the constraints for its global developability through iterative process [4].

In our system, the paper shape is represented as quad strips adjacent via curved creases as in the previous works, but allows the rulings to transit during folding motion by updating the geometry of the curved surface while the connectivity between quads is consistent. The 3D shape of the paper is calculated instantly with no iterative process. The geometry of the folded paper is calculated according to the shape of the folding curves, defined by the curvature and the torsion, and the folding angle, designed by the user. The rulings are derived from the three parameters, which are curvature, torsion, and folding angle, using the formula induced by Fuch and Tabachnikov [1] shown in the subsequent section. In this method, it is sometimes difficult for a user to give appropriate parameter values which generates existable rulings as in real paper, with no crossings. Drawing a free-form curve manually is an intuitive method to design a crease curve, but it often end up in severe crossings of rulings (Fig. 2). It is especially difficult to add a new crease to a curved surface without having the rulings crossing (Fig. 2 (b)). This is because the ruling directions are easily affected by a small change in parameter values. Furthermore, the values of different parameters or parameter values in different points does affect each other. A small unintended shift on a curve affects the curvature, torsion, and the folding angles, may result in a large difference in ruling directions. To support the task of designing the shape of the curved folds, our system allows the user to add and adjust the crease curve while observing the calculated rulings instantly. The system also provides cost functions to restrict the movement of the user-adjusted curve, or optimize the shape of the curve, according to the cost functions, to reach user intended shape effectively.



(i) (ii) (iii) (i) (ii) (iii)

(a) Paper with one crease (b) Paper with multiple creases

Fig. 1: Transition of rulings during fold and twist motion. (Top: 3D space, Bottom: 2D space).

(a) Crossing caused by one crease (b) Crossing caused by the added crease

Fig. 2: Crossings of rulings. (Left: 3D space, Right: 2D space).

Main Idea:

Our system generates the 3D shape of the curved folded paper from one primary crease curve and some additional crease curves (Fig. 3). First, the shape of the primary crease curve is defined by the user by giving the values of curvature in 2D or 3D, torsion, and the folding angles, from which a 3D developable surface with a single curved fold is generated. Secondly, additional curves are drawn by the user to add new folds on the curved surface. The geometry of curved folds and surfaces are calculated based on the following formula [1]:

$$k_{2D}(s) = k(s)\cos\alpha(s) \tag{1}$$

$$\cot\beta_L(s) = \frac{-\alpha(s)' + \tau(s)}{k(s)\ \sin\alpha(s)} \tag{2}$$

$$\cot\beta_R(s) = \frac{\alpha(s)' + \tau(s)}{k(s)\ \sin\alpha(s)} \tag{3}$$

where, as shown in Fig. 4, $\alpha(s)$ is the folding angle and $\beta_L(s)$, $\beta_R(s)$ are the angles between the tangent vector of the crease curve and the rulings in 2D on the left and the right side of the curve. $k(s)$ and $\tau(s)$ are the curvature and the torsion of the 3D curve and $k_{2D}(s)$ is the curvature of the 2D crease curve, with s indicating the arc-length parameter. Given two of the three elements, the 2D crease curve, the 3D crease curve, and the folding angle $\alpha(s)$, the other element and the rulings are calculated. The shape of the folded paper is represented as quad strips whose edges are the rulings, segments of crease curve, and segments of paper edges. Note that intersection of crease curves is not allowed.

The procedures of designing curved folds

This subsection explains the procedures of designing curved folds.

1. The primary crease curve is defined by the user by giving the values of curvature $k(s)$, $k_{2D}(s)$, torsion $\tau(s)$ and/or folding angle $\alpha(s)$ at the control points evenly placed on the curve. The user can select a control point and adjust the value of one of the above parameters at that point. Then the parameter values at all the sample points are interpolated along the whole curve. Because these parameters are mutually dependent, the values of the other parameters are changed based on

equation (1). From these parameters, the ruling directions are obtained by equation (2) and (3). The 3D surfaces composed of the rulings are shown to the user as a prompt feedback. An example of the result are shown in Fig. 5.

2. Additional crease curves are drawn by the user as free-form curves on the 2D space. Each curve is then approximated by a B-spline curve to make the curve smooth, and then discretized by plotting the intersections of the rulings and the curve. By projecting the curve onto the 3D surface, a 3D space curve, its curvature, torsion, and folding angle are obtained. At last, the rulings of the new folded surface are calculated by equation (2) and (3). In this step, the new rulings are likely to have crossings (Fig. 6(a)), which are to be eliminated in the next step. If two or more additional curves are input, they are processed one by one starting from the closest one to the primary curve.
3. To resolve the crossings of the rulings, the shape of the additional curve is adjusted by moving the control points of the B-spline. The user can drag the control points on the screen, so that the crease curve and the rulings are in the intended shape. While the control points are moved, the parameters of the curve and the ruling directions are re-calculated and shown to the user promptly. The system provides the following three editing modes to help the user adjust the control points effectively.
   (a) No system support (No restriction in moving the control points).
   (b) With some restrictions on movement of the control points based on the cost functions explained in the next subsection.
   (c) With full support of optimization based on the cost function.

   The user can also see the values of the cost and the 3D paper shape while moving the control points. One may also check the folding motion by changing the folding angle $\alpha(s)$ of the primary curve. An example of the result are shown in Fig. 6(b).
4. The user may also trim the paper to eliminate crossing of rulings by drawing a curve in "trim" option instead of adding crease.

Cost functions

Our system provides three types of cost functions used in the procedure step 3(b) and 3(c). These cost functions are designed empirically and may be improved with other types of functions. The cost functions include the following values, with smaller cost value indicating the better 3D shape.

(a) The difference of torsion between the primary crease curve and the additional crease curve.
(b) The difference of left and right ruling angles.
(c) The area on 2D crease pattern where the projection to the 3D space is non-injective due to the rulings crossing, as shown in Fig. 2.

(a) Primary crease curve (b) Additional crease curve

Fig. 3: Primary and additional crease curves. (Left: 3D space, Right: 2D space)

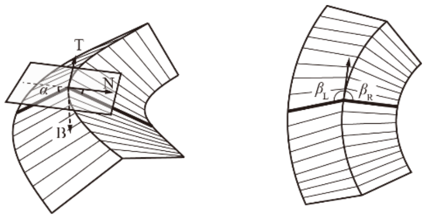


Fig. 4: The definition of folding angle $\alpha(s)$ and ruling directions $\beta_L(s)$, $\beta_R(s)$. T, N, B is the tangent vector, normal vector, and binormal vector of the curve. (Left: 3D space, Right: 2D space)

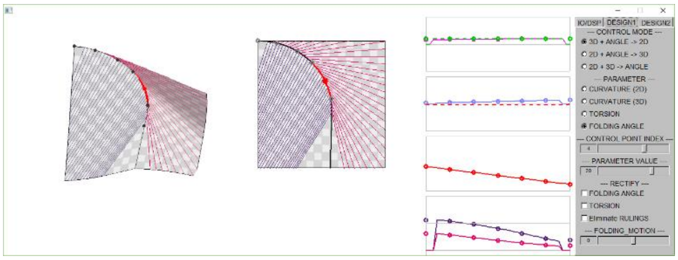
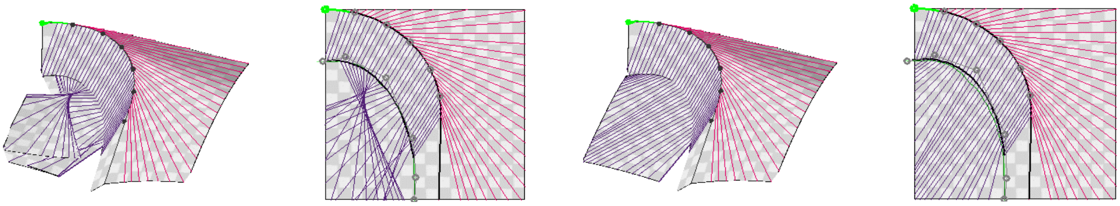


Fig. 5: Design of primary curve. (Left: 3D space, Center: 2D space, Right: parameter values on the control points and along the curve). The red dot in 3D space and crease pattern indicates the control point whose parameter value being adjusted, and gray dots shows the other control points.



(a) Additional curve in initial state (b) Additional curve in user adjusted state

Fig. 6: Design of additional curve. Control points of both primary crease curve and those of additional curve are depicted. (Left: 3D space, Right: 2D space)

Conclusion:

In this work, we proposed a system for designing a developable surface with multiple curved folds which allows the transition of rulings during the fold motion. Figure 1-(b) shows three states in the folding motion of a paper with an additional crease on each side of the primary crease curve. The folding angles of primary crease curve range in (i) 7 to 26 degrees, (ii) 11 to 40 degrees, and (iii) 38 to 57 degrees. Given appropriate additional creases by the user, the folding motion of the curved fold are simulated successfully, with ruling transition.

To evaluate the developability of the model, we calculated the sum of corner angles adjacent to each vertex on the curves and checked the difference between 360 degrees. We also investigated the flatness of each quads composing the curved surface by calculating the distance between one vertex, on the end of the ruling, and a plane passing the rest of the vertices. The results are shown in Tab. 1 and Tab. 2. With small errors, we could conclude that the model is developable and does represent a sheet of paper with curved folds.

As future work, there is room for improvement in user interface to help users design intended and consistent surface more easily and effectively. Better cost functions may lead to the improvement. Adding some flexibility in GUI system is also expected to help users, such as recovery from local minima.

Fold. Ang.	*(i) 7 to 26 degrees*		*(ii) 11 to 40 degrees*		*(iii) 38 to 57 degrees*	
Crease curve	*Left*	*Right*	*Left*	*Right*	*Left*	*Right*
Average	0.101	0.000782	0.00044	0.106	0.0398	0.000426
Maximum	2.12	0.0159	0.0012	2.11	1.14	0.00189

Tab. 1: The Difference between Sum of Corner Angles and 2π. (Units in Degrees)

Fold. Ang.	*(i) 7 to 26 degrees*	*(ii) 11 to 40 degrees*	*(iii) 38 to 57 degrees*

Quads\Crease curve		*Left*	*Right*	*Left*	*Right*	*Left*	*Right*
Left	Average	0.00657	0.00166	0.00888	0.00254	0.0393	0.00233
	Maximum	0.0268	0.00208	0.0366	0.00322	0.566	0.00383
Right	Average	0.000119	0.00545	0.00014	0.00542	0.00102	0.00934
	Maximum	0.000311	0.049	0.000318	0.0199	0.0194	0.0353

Tab. 2: Flatness of quads. (Unit in mm. Note that the length of the paper edge being 200mm)

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