Title:
Experiments with T-Mesh for Constructing Bifurcation and Multi-furcation using Periodic Knot Vectors

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Introduction:
In surface modeling technology, application of branched surface construction is widely applied in various fields that include biomedical and automobile industry. These applications make use of scanned data set of the original object and reconstruct a new or modified version of the same object as per the design requirement.

The construction of the branched surface faces complexities against best topological relation between adjacent contours, to maintain the geometrical continuity at junction [6]. Some initial attempts for the construction of bifurcating surface include the method of triangulation [5], joining of right circular cylindrical surfaces [4] and merging of half tubular Bezier patches [13]. These methods could not meet easily with the desired requirements of continuity and control over the surfaces. Although B-spline offers additional control over the surface through knot vectors unlike Bezier surface but the involvement of T-spline that allows a row of control point to terminate at an appropriate position, increases the flexibility of the surface with relatively fewer number of control points. One possible solution to achieve the same quality of the surface is by merging of two or more B-spline surfaces of different knot vectors through T-spline [7]. These methods are good in the sense that smooth surface could be constructed However they lack the flexibility and speed of a single T-spline equation to construct such surfaces.

The concept of the disjoint surface for generating branched object was explained by Bhatt et al. [3]. They used B-spline surface representation to developed bifurcating surfaces. A similar method for constructing disjoint surfaces was developed by Asthana et al. [1]. They constructed an iterative procedure to develop multiple bifurcating surface models. The model developed by [1] was capable of handling any order of B-spline surfaces in both of the defining directions along with G1 continuity, which was maintained by aligning the control polygon segment in a straight line. In case of T-spline surfaces, continuity can be achieved by allowing local refinement in the corresponding T-mesh hence the absence of local refinement is considered as a major constraint while working with the B-spline based surface model. The present work therefore is an effort towards describing a complex surface with the help of a single T-spline surface equation.

Overview of this Paper:
This work illustrates generation of bi-furcating and multi-furcating surface models by performing some experiments on T-mesh templates through local refinement at the required positions of the corresponding T-mesh. These experiments are performed to develop open and close branching surfaces. Periodic T-splines and disjoint surfaces are used along with a presumed set of coordinates which acts as control points for the obtained T-spline surface. The discussed surfaces can be extended to any polynomial degree but most of the surfaces are generated through series of experiments performed on T-mesh having order k=4 in both of the parametric direction.

Definition of T-mesh and T-spline Surface
To generate a T-spline surface, T-mesh is used as a two-dimensional pre-image of control polyhedron. It is a rectangular grid where each edge of every rectangle represents a positive integer value termed as knot value. For a given T-mesh of order k and each control point P_i having associated weight w_i,, the points on the T-spline surface S(u,v) can be evaluated by the equation:

$$S(u,v) = \frac{\sum_{i=1}^{N} w_i P_i B_i(u,v)}{\sum_{i=1}^{N} w_i \, B_i(u,v)} \quad (1)$$

Where B_i (u,v) are tenser products of univariate B-spline basis function given by:

$$B_i(u,v) = \; N_{i,k}(u).N_{j,k}(v) \;\;,\quad (u,v) \in D \quad (2)$$

In a T-mesh, (u, v) represent knot coordinates for the anchor assigned as control points and D is the effective parametric domain to define a surface [7]. Evaluation of the set of local knot vectors in both directions, for odd and even degree polynomial is explained by Bazilevs et al. [2].

<u>Control Polyhedron Representation for Closed Surface:</u>
The construction of control polyhedron is based on the shape of the desired surface. The control point layout presented by Fig. 1, gives the geometrical arrangement of control points in parallel horizontal planes along the y-axis, forming rhombus shapes (either one or more) is referred to as a control polygon for that plane. These number of plane of control polygons, are equal to the number of rows in the corresponding T-mesh. These layouts have been utilized and discussed further to construct T-meshes and corresponding surfaces.

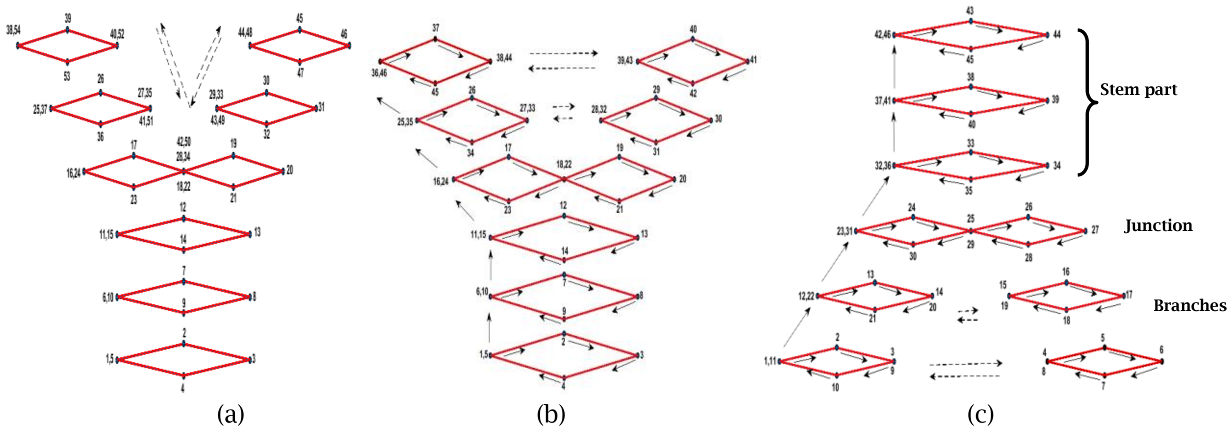
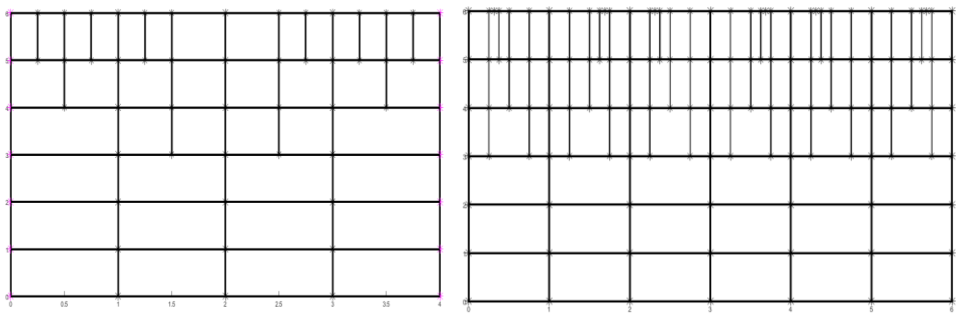


Fig. 1: Three Layouts of Control Polyhedron for Bifurcating Surface: (a) Separation of Two Branches without Disjoining Technique, (b) Separation of Two Branches with Disjoining Technique, (c) Inverted Arrangement of Control Points.

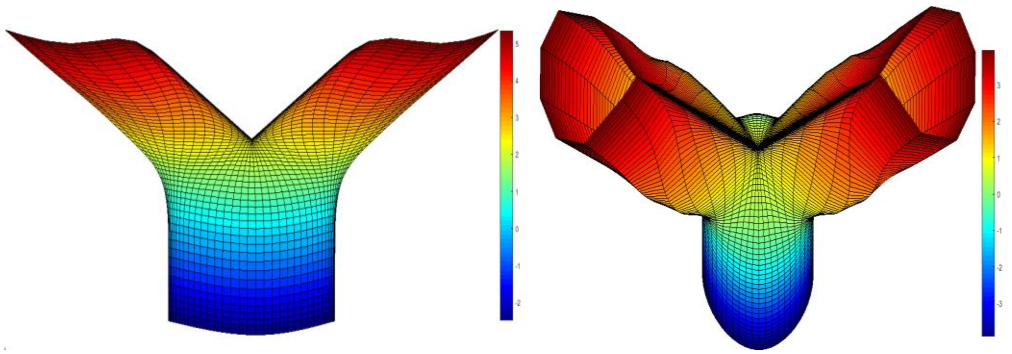
<u>Open and Closed Bifurcating T-spline Surface:</u>
This section shows the T-mesh and corresponding surfaces which follow the layouts of control polyhedron, displayed in Fig. 1.

Open and closed bifurcating surface using first layout
The surfaces shown in Fig. 2(b) have been obtained by considering the layout shown in Fig. 1(a). For open surface the control points have been removed from layout in negative z-direction. The layout has been constructed by to and fro repetition of the control point through the junction of the bifurcating surface. In order to create junction and branches, knots are inserted in the T-mesh shown in Fig. 2(a).

To achieve the closed branched surface, periodic knot vectors has been adopted in the definition of T-spline surface, given in Eqn. (1). Since the obtained closed surface, Fig. 2(b) has been dominated by back and forth repetition of the control points, to disjoint the branches from stem part. The repetition enables the surface to pass through the desired control point however this will degrade the continuity of the surface. The multiple parametric lines converged at the junction are visible in closed surface shown in Fig. 2(b).



(a)



(b)

Fig. 2: T-mesh (order k=4) and corresponding Open and Closed Bifurcating Surface using Layout Shown in Fig. 3(a): (a) T-meshs used for the Construction of Open and Closed Bifurcating Surfaces, (b) Resulting Open and Closed Bifurcating Surfaces.

Closed bifurcating surface using second layout

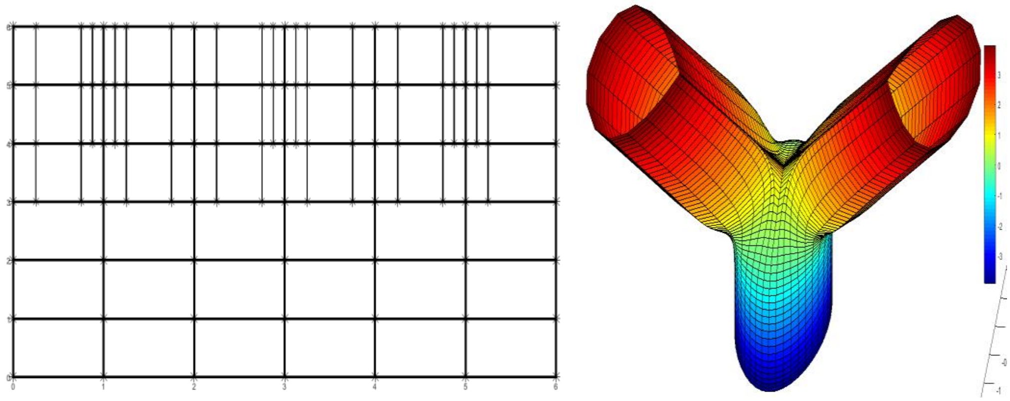
The design of the closed bifurcating surface using arrangements of control points, illustrated by Fig. 1(b) has been addressed in this part. The surface shown in Fig. 3(b) has been made to minimize the multiple parametric lines which are visible due to continuous repetition of control point as mentioned previously. In this approach, two parallel contours, in branched part are separated by disjoint surfaces as shown in Fig. 1(b). The minimum number of the control point has been utilized (started from stem part) to decide knot interval in the first three rows of the corresponding T-mesh shown in Fig. 3(a). After that knots have been inserted in order to create junction and branched part respectively. New acquired closed bifurcating surface as shown in Fig. 3(b), is smooth and continuous at the junction as compared to Fig. 2(b).

Use of the second layout, has been found simple for the construction of bifurcating surface through T-mesh. In the first row of T-mesh, knot values are placed according to the control point in stem part (minimum number of control points). As one move from stem to branched part knots are inserted in between the previously assigned knots to get the desired shape. In case of the multi-furcating surface when control point has been increased, it becomes difficult to insert more control points in between previously assigned knot values.

Closed bifurcating surface using final layout

This part introduced final layout shown in Fig. 1(c), which has been carried out to draw the T-mesh illustrated in Fig. 4(a). To achieve surface shown in Fig. 4(b), knot interval in the first three rows of the

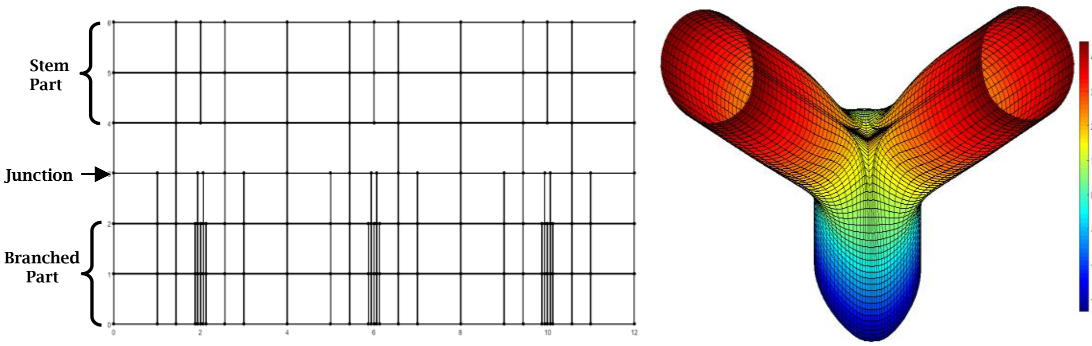
T-mesh Fig. 4(a), has been decided according to the branched part, unlike previous one in which knot interval was decided from stem part of the layout shown in Fig. 1(b).



(a) (b)

Fig. 3: T-mesh (order k=4) and corresponding Closed Bifurcating Surface using Control Polyhedron Shown in Fig. 1(b): (a) T- mesh used for Construction, (b) Resulting Closed Bifurcating Surface.

Even though bifurcating surfaces shown in Fig. 3(b) and Fig. 4(b) obtained by employing the layouts shown in Fig. 1(b) and Fig. 1(c) was quite similar, still inverted arrangement of control point, Fig. 1(c) has been found more convenient, as maximum number of control point can be utilize as knot values while drawing T-mesh for multi-furcating surfaces.



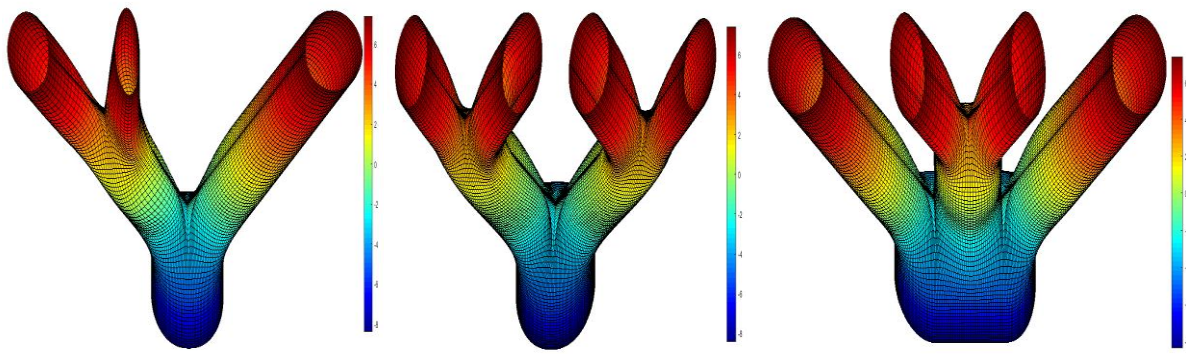
(a) (b)

Fig. 4: T-mesh (order k=4) and corresponding Closed Bifurcating Surface using Similar Arrangement of Control Points Shown in Fig. 1(c): (a) T- mesh used for Construction of the Surface, (b) Resulting Closed Bifurcating Surface.

<u>Closed Multi-furcating T-spline Surfaces</u>

The method for constructing bifurcations is further extended to multi-furcations. To design a multi-furcating surface, control polygon in the first plane may vary according to the branches required in the surface. In order to maintain the flexibility of T-mesh in multi-furcation, a similar pattern of the control points has been incorporated with the disjoining method employed to separate the branches of the surface.

It has been observed, while constructing asymmetrical branching, that use of uniform knot interval in s-direction will cause differences in knot interval of the same control point (different knot vectors) used to close the surface. To avoid this in case of asymmetrical branching non-uniform knot interval has been assigned to construct T-mesh.



(a) (b) (c)

Fig. 5: Multi-furcating Surfaces (order k=4): (a) Asymmetrical Closed Surface, (b) Symmetrical Closed Surface, (c) Tri-furcating Surface.

Conclusions:

In the present work, some experiments have been performed on T-mesh templates to construct bifurcations and multi-furcations. These experiments show that use of disjoint surface in periodic T-spline is an effective method to construct smooth and continuous branched surface. However, in case of multi-furcation, it is difficult to maintain the flexibility of the T-mesh for upper level branching with respect to the lower level bifurcation. This problem can be resolved by altering the T-mesh according to the pattern of control point shown in the final layout. These methods can be applied in applications where smooth reconstruction of bifurcating or multi-furcating part is required.

References:

[1] Asthana, V.; Bhatt, A.-D.: G1 Continuous bifurcating and multi-bifurcating surface generation with B-spline, Computer-Aided Design & Applications, 14(1), 2017, 95-106. https://doi.org/10.1080-/16864360.2016.1199759

[2] Bazilevs, Y.; Calo, V.-M.; Cottrell, J.-A.; Evans, J.-A.; Hughes, T.-J.-R.; Lipton, S.; Scott. M.-A.; Sederberg, T.-W.: Isogeometric analysis using T-splines, Computer Methods in Applied Mechanics and Engineering, 199(5-8), 2010, 229-263. https://doi.org/10.1016/j.cma.2009.02.036

[3] Bhatt, A.-D.; Goel, A.; Gupta, U.; Awasthi, S.: Reconstruction of Branched Surfaces: Experiments with Disjoint B-spline Surfaces, Computer-Aided Design & Applications, 11(1), 2014, 76-85. https://doi.org/10.14733/cadconfP.2014.14-16

[4] Felkel, P.; Wegenkittl, R; Buhler, K.: Surface models of tube trees, Computer Graphics international, 2004, 70-77.

[5] Klein, R.; Schilling, A.; Strasser, W.: Reconstruction and simplification of surfaces from contours, Graphical Models, 62(6), 2000, 429-443. https://doi.org/10.1006/gmod.2000.0530

[6] Meyers, D.; Skinner, S.; Sloan, K.: Surfaces from Contours, ACM Transactions on Graphics, 11(3), 1992, 228-258. https://doi.org/10.1145/130881.131213

[7] Sederberg, T.-W.; Zheng, J.; Bakenov, A.; Nasri, A.: N. L.: T-splines and T-NURCCs, ACM Transactions on Graphics, 22(3), 2003, 477-484. https://doi.org/10.1145/882262.882295

[8] Ye, X.; Cai, Y.-Y.; Chui, C.; Anderson, J.-H.: Constructive modeling of G1 bifurcation, Computer Aided Geometric Design, 19(7), 2002, 513-531. https://doi.org/10.1016/S0167-8396(02)00131-0

[9] Zheng, J.; Wang, Z.: Periodic T-splines and Tubular Surface Fitting, Curve and Surfaces-ICCS 2010, Springer, Avignon, France, 2010.