Title:
**New Simplified Inverse Kinematics Method for 5-Axis Machine Tools**

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Introduction:
The 5-axis machine is viewed as two cooperating robots, one carrying the tool and one carrying the workpiece. The motion of the two robots must be such that the two robots "handshake". The "handshake" is defined as coincidence of the tool vector and tooltip coordinate with the CL vector and CL point coordinate. Many studies classify 5-axis milling machines into three basic categories [7]. By embedding a coordinate frame, using homogenous transformations matrix (HTM) and D-H convention, we can describe the relative position and orientation between these coordinate frames. D-H [3] proposed a method which reduces the number of basic HTM transformations. The DH method [3] [13] is a very general method for the kinematic analysis of a robot or mechanism. D-H parameters to procure the direct kinematics are discussed by [4]. D-H method applied on the 5-axis machine tool is presented by [1]. [8] Implemented the D-H convention to describe the geometry of the milling machine. The normal D-H notation uses only four parameters [9]. A modified D-H notation using five parameters has been discussed by [6]. The machine kinematics of four-half axis systems was presented using the modified D-H notation [11]. Author [12] modified the D-H notation to enhance the flexibility of the location of basic joint types, links, and cutting devices. A geometric post-processing method that does not require the forward and inverse kinematics equations has been discussed by [5]. Author [10] presents the forward kinematics and closed form solution. A cutter kinematic chain using D-H conventions is proposed by [14]. Author [2] classified the possible kinematics structure of five-axis machine tools.

In this paper, we propose a simplified inverse kinematics method to physically understand the inverse kinematics of a 5-axis machine tool, based on visual observation and manually jogging the machine to the handshake position where the tool and a real material CL vector merge. The first method is based on orienting the required coordinate systems in the same direction as the machine tool axes and minimizing the numbers of required coordinate frame to compute the inverse kinematics. The second method simplifies further the computations of the machine rotations based on the geometrical approach.

New Conceptual Interpretation of Inverse Kinematics of Machine Tool:
Tab. 1 gives a review of the existing kinematic models. From Fig. 1 & Fig. 2, the machine tool can be seen as two cooperating robots. Following the DH method, we start numbering the links from the frame onwards. The frame is numbered 0 the next links in sequence 1, 2… n. Fig. 3 shows the numbered links in the two kinematic chains. The Fig. 4 presents the values of the parameters for the first joint.

Simplified Inverse Kinematics Methods:
To avoid above complications, we propose two new ways to compute the inverse kinematic relations of a 5-axis machine tool. The first method is based on orienting the required coordinate systems in the

same direction as the machine tool axes and minimizing the numbers of required coordinate frame to compute the inverse kinematics. The second method simplifies further the computations of the machine rotations based on the geometrical approach.

| Kinematic Model developed by Authors Ref [ ] | Type of 5-axis machine Tool used | No of Frames applied for transformation using D-H notation (4-parameters) | No of Frames applied for transformation using D-H notation (5-parameters) | No of Frames applied for transformation without using D-H notation | Number of Transformation Matrix | Total number of Parameters | Approach for Post- processing | Outcome of Kinematic Model |
|---|---|---|---|---|---|---|---|---|
| [1] | Mikron UCP 600 (XYZAC) | 6 | - | - | 5 | 20 | Inverse Kinematics | A general postprocessor has been implemented using the D-H method and used for position milling. |
| [8] | Huron KX8 (XYZAC) | 6 | - | - | 5 | 20 | Forward Kinematics | An optimal post-processing module has been tested for validation on many different complex surfaces. |
| [6] | 5-Axis CNC Mill (XYZAC) | - | 6 | - | 5 | 25 | Inverse Kinematics | The generalized kinematics model is implemented for hybrid parallel serial 5-axis postprocessor. |
| Proposed method by authors | Maho600E (XYZAB) | - | - | 4 | 3 | 5 | Inverse Kinematics | A novel method to physically understand, to minimize the number of required frames and to compute the machine rotations. |

Tab. 1: A list of the existing kinematic models using D-H convention.

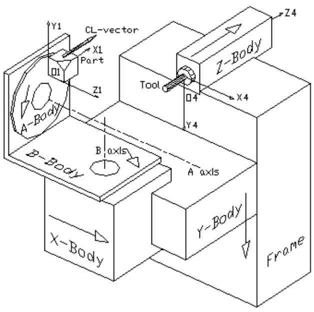


Fig. 1: Workpiece Coordinate System $O_1$ and Machine Coordinate System $O_4$ in the Reference Position.

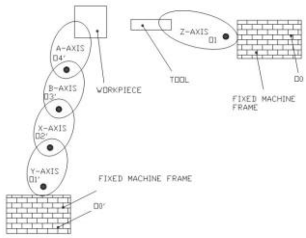


Fig. 2: Kinematic diagram.

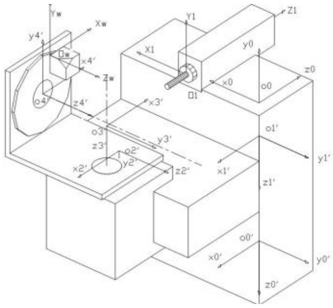

Fig. 3: DH coordinate systems.

Machine Axes Oriented Handshake (MAO-Handshake) Method:

As can be seen from the above application of the DH method it is possible to select the Cartesian coordinate systems origin in such a way that the origin of the DH coordinate systems are located on the rotary axes where they cross. Because the DH method aligns the z axis along the link direction these coordinate systems are not having the same orientations (they will be oriented along the X, Y, Z motion directions of the machine and this is very confusing). We propose now to orientate the joint axes of the two rotary axes along the centerlines of these rotations following the rule: The coordinate axis along the centerline of the rotations is having the same name as the machine tool axis that is parallel with it. The origin of these axes is chosen at the endpoints of the normal between the two centerlines. The other axes are oriented in the same direction as the machine tool coordinate system In general this rule will result in two orthogonal Cartesian coordinate systems with the origins on the rotation axis centerline. For the case of the machine in Fig. 1 we obtain the Cartesian coordinate systems $O_2$ on the A-axis centerline and $O_3$ on the B-axis as shown in the Fig. 7. The workpiece coordinate system can have any orientation but we make the coordinate axes planes parallel to the machine axes planes. In the origin of the workpiece coordinate system, we introduce a coordinate system $O_1$. The origin $O_1$ coincides with the workpiece coordinate system origin, however the $O_1$ axes are oriented along the machine tool coordinate system. At this stage we have 4 coordinate systems $O_1$, $O_2$, $O_3$ and $O_4$. The last coordinate system $O_4$ is the machine coordinate system $O_4$ that we can fix anywhere in space but fixed to the machine frame. A good choice is to locate the origin at point $O_3$ when the machine is in the reference position (another choice is on the reference surface of the spindle and on the centerline of the spindle when the machine is in the reference position) When the machine slides move all the coordinate systems will move except the machine coordinate system $O_4$.

With above coordinate systems is possible to derive the inverse kinematic equations of the machine. Example, the case of the machine in the Fig. 1, Based on the kinematic link diagram in Fig. 2, the set of axes that moves the workpiece and the tool can be determined. We compute the position of the CL Point and CL vector after the rotations A and B and the translations X and Y. We also compute the position of the tooltip coordinate and the tool vector after the translation Z. After this the tool tip, tool vector, CL point and CL vector must coincide (see Fig. 8). This will give us 6 equations that allow us to compute the required slide motions. The three relations between the tool vector and CL vector are independent of the linear motions. Solving this equation first will give us all the solutions for the machine tool rotations. Rotation A around X1 axis of the CL vector

$i_1(A) = i_1$ , $j_1(A) = j_1 \,Cos[A] - k_1 \,Sin[A]$ and $k_1(A) = k_1 \,Cos[A] + j_1 \,Sin[A]$ (1)

Coordinate transform from O1 to O2 (as this is a translation it has no effect on the vector)

$i_2 = i_1(A)$ , $j_2 = j_1(A)$ and $k_2 = k_1(A)$ (2)

Rotations B around $Y_2$ of the CL vector

$i_2(B) = i_2 \,Cos[B] + k_2 \,Sin[B]$, $j_2(B) = j_2$ and $k_2(B) = k_2 \,Cos[B] - i_2 \,Sin[B]$ (3)

As the tool kinematic links do not include rotation in this case the tool vector is constant.

$i_T = 0$ , $j_T = 0$ and $k_T = 1$ (4)

As the tool vector and CL vector must coincide after A and B rotation we obtain

$i_T = 0 = i_2(B) = i_2 \,Cos[B] + k_2 \,Sin[B]$ , $j_T = 0 = j_2(B) = j_2$ , $k_T = 1 = k_2(B) = k_2 \,Cos[B] - i_2 \,Sin[B]$ (5)

After substituting

$i_1 \,Cos[B] + (k_1 \,Cos[A] + j_1 \,Sin[A]) \,Sin[B] = 0$ , $j_1 \,Cos[A] - k_1 \,Sin[A] = 0$ and
$k_1 \,Cos[A] + j_1 \,Sin[A]) \,Cos[B] - i_1 \,Sin[B] = 1$ (6)

We get by solving above three equations for A and B

$A = ArcTan[j_1/k_1] + k\pi$ and $B = -ArcSin[i_1] + 2k\pi$ (7)

<u>Machine Axes Oriented Geometric Handshake (MAO-GEO Handshake) Method:</u>

The above results can also be obtained in a more straightforward geometric way. As the rotations in the case of the machine tool (Fig. 1) are orthogonal and aligned with the machine coordinate axis we can obtain the same solutions as above by: Rotate the tool vector in the horizontal plane X3/Z3. If we look in the vertical plane Y3/Z3 it can be observe that is can be done in 4 different ways with A smaller than $2\pi$ as shown (Fig. 5) Solutions A1 and A2 align the CL vector in the Z3 direction. Solutions A3 and A4 align the CL vector with pointing in the opposite direction of the Z3 axis. Once the CL vector is rotated in the horizontal plane by one of the above solution we can rotate over B to align the CL vector with the tool vector. The solutions are shown in the Fig. 6. The rotation B1 corresponds to the solutions A1, A2 and B2 corresponds to the solutions A3, A4. This geometric interpretation is very straight forward and useful. In some case it is however not possible to use this geometric way to find the possible solutions for the inverse kinematics rotations. In the case of the machine tool with non-orthogonal axes by experiment it is not possible to use the geometric solution. The three equations for the rotation angles must be solved analytically. Once the solutions for the rotations have been found it

is possible to find the corresponding linear slide motions. In this case we need also to take the translations between the coordinate systems into account.
In the case of the machine in Fig. 1 we obtain:
Coordinate transform from $O_1$ to $O_2$ -> $x_2 = x_1 + x_{o1o2}$ , $y_2 = y_1 + y_{o1o2}$ and $z_2 = z_1 + z_{o1o2}$ (8)
Rotation A around $X_1$ axis of the CL Point, $x_2(A) = x_2$ , $y_2(A) = y_2$ Cos [A] - $z_2$ Sin [A] and
$z_2(A) = z_2$ Cos [A] + $y_2$ Sin [A] (9)
Coordinate transform from O2 to O3 (as this is a translation it has no effect on the vector)
$x_3 = x_2(A) + x_{o2o3}$ , $y_3 = y_2(A) + y_{o2o3}$ and $z_3 = z_2(A) + z_{o2o3}$ (10)
Rotations B around $Y_3$ of the CL vector, $x_3(B) = x_3$ Cos [B] + $z_3$Sin [B] , $y_3(B) = y_3$ and
$z_3(B) = z_3$ Cos [B] - $x_3$ Sin [B] (12)
The CL point has now been rotated over the angles A and B. The translations Y and X also move the CL point. The resulting CL point position is:
$x_{4w}(Y,X,B,A) = x_3(B) + X$ , $y_{4w}(Y,X,B,A) = y_3(B) + Y$ and $z_{4w}(Y,X,B,A) = z_3(B) + z_{03O4}$ (13)
The tooltip will translate by Z -> $x_{4T}(Z) = x_{4Tref}$ , $y_{4T}(Z) = y_{4Tref}$ and $Z_{4T}(Z) = z_{4Tref} + Z$ (14)
With our choice of the Machine tool coordinate system we have
$x_{4Tref} = y_{4Tref} = 0$, $z_{4Tref}$ = Tooltip z coordinate (tool length) in $O_4$ (15)
The values of the machine translations X, Y and Z are found by solving the following equations:
$x_{4w} = x_{4T}$ ; $y_{4w} = y_{4T}$ ; $z_{4w} = z_{4T}$ (16)

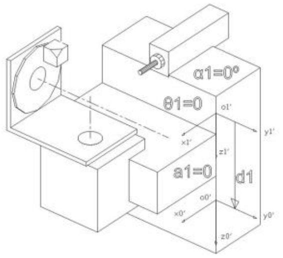

Fig. 4: DH Joint Parameters Joint.

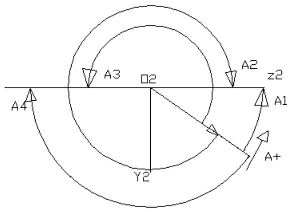

Fig. 5: The Possible A-axis rotations to rotate the CL vector in horizontal plane.

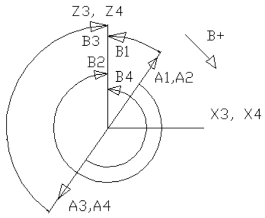

Fig. 6: Possible B solutions corresponding to the A rotations.

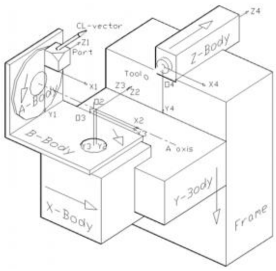

Fig. 7: Coordinate Systems $O_1$, $O_2$, $O_3$ and $O_4$ Oriented As Machine Axes Directions.

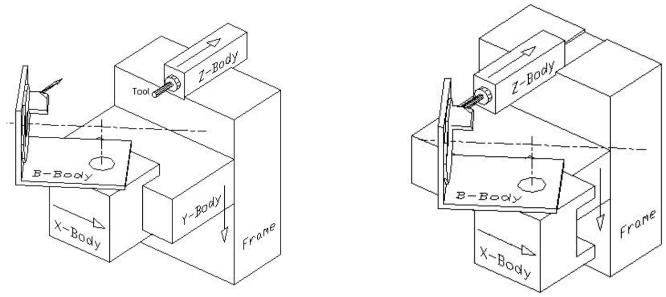

Fig. 8: Handshake Position after moving in A, B, and X, Y, Z.

From the resulting equations we can conclude some important results. The tooltip coordinate $z_{T4ref}$ is additive to the Z coordinate. So when the tool length is changed we can compensate this by an additional translation along the Z axis of the machine. When the workpiece origin ow is changed however we cannot compensate this by translations of the machine slides. We need to compute the new values of X, Y, Z with the equations because xowo1, yowo1, zowo1 are not just additive terms in the inverse kinematic equations. If the rotations would move the tool then a change in workpiece location can be adjusted by the slides but a tool length change will require repost-processing. If there is a rotary axis on the tool side and one on the workpiece side the whenever we change the tool length or the workpiece set up location we need repost-processing the CL point. The CL vector will however not change so the angles of the rotation do not need to be recomputed for each CL point. Some machine tools have rotary axes that are not perpendicular. In this case we still orient the joint coordinate systems with one coordinate axis along the centerline of rotation and the origin at the endpoints of the normal between the two centerlines.

## Conclusion:

A novel method to physically understand the inverse kinematics of a 5-axis machine tool was presented based on visual observation and manually jogging the machine to the handshake position where the tool and a real material CL vector merge. The 5-axis machine is viewed as two cooperating robots, one carrying the tool and one carrying the workpiece. The manual experimental method not only creates a better understanding but it is also used to verify the exact computation in the discussed methods above. After introducing the DH coordinate systems and minimizing the number of non-zero parameter the inverse kinematics solution was outlined and the analyzed. Finally, we proposed two new methods (MAO & MAO-GEO Handshake) that make the inverse kinematics as simple as possible improving the understanding and reducing to complexity.

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