Title:
A Fuzzy Number based Hierarchy Analytic Method and Applications in Design of Rehabilitation Devices

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Introduction:
Conceptual design develops design candidates based on product requirements. Product requirements may be decided from customer needs, benchmarks of competing products, and other criteria. These requirements are then translated into measurable technical attributes that can be used to evaluate design candidates. Criteria must be considered to choose a best design. Designers need to identify corresponding alternatives of product concepts using criteria and their relative importance levels. However, mapping from product requirements into design candidates is a daunting task. There may be a number of feasible design options. As a result, an evaluation model is required for the effective solution selection.

Traditional methods of the conceptual design evaluation include the expert rating, gray evaluation, quality function allocation, and fuzzy analytic hierarchy process (FAHP) methods [5]. These methods have some shortcomings, such as the lack of an objective evaluation, complex analysis process, and hard to handle inaccurate information. This research proposes an efficient method for analyzing, prioritizing, and ranking design solutions. A multi-criteria decision-making method is proposed for the evaluation of design solutions to reduce the subjective preference of decision-makers and influence of uncertain factors in the decision-making. The FAHP analyzes factors that affect the competitiveness of products using a hierarchy of the design scheme evaluation. Using triangular fuzzy numbers instead of scales in the conventional analytic hierarchy process, the method can fully consider views of experts in various fields to reduce risks of product development. Three selected rehabilitation products are used as examples to verify feasibility and effectiveness of the proposed method. An improved design of the exoskeleton device is proposed for the upper extremity exercise rehabilitation.

Main Idea:
Fuzzy multi-criteria model
Table 1 shows membership functions of triangular fuzzy numbers in the form of triples (a1, aM, a2).

Triangular fuzzy number	*Membership function*
$\tilde{1}$	(1, 1, 3)
$\tilde{x}$	（x-1, x, x+1）for x=2, 3, 4, 5, 6, 7, 8
$\tilde{9}$	(8, 9, 9)

Tab. 1: Triangular fuzzy numbers and their corresponding membership functions.

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According to the Dempster-Shafer theory of evidence [3], the upper and lower means of fuzzy number A are deduced as follows.

$$E_*(\tilde{A}) = \int_{-\infty}^{+\infty} x\, dF^*(x) \ , \ E^*(\tilde{A}) = \int_{-\infty}^{+\infty} x\, dF_*(x) \qquad (1)$$

In Eqn. (1), $F^*(x)$ and $F_*(x)$ are right continuous functions that describe the upper bound and lower bound distribution functions of fuzzy number A, respectively. The average value of A is a closed interval composed of expected values calculated by the upper and lower distribution functions, which is $E(\tilde{A})=|E^*(\tilde{A}), E_*(\tilde{A})|$.

When optimism q∈ [0, 1], attitude $E_q(\tilde{A})$ for fuzzy number A mapped to the real number field can be defined as a combination of $E_*(\tilde{A})$ and $E^*(\tilde{A})$ as follows.

$$E_q(\tilde{A})= qE_*(\tilde{A})+(1-q)E^*(\tilde{A}) \qquad (2)$$

In Eqn. (2), $E_q(\tilde{A})$ indicates that fuzzy number $\tilde{A}$ is evaluated under optimism q. The larger the q, the more important upper mean $E_*(\tilde{A})$ of fuzzy number A is. Therefore, q is used to represent the optimism of decision makers. q = 0 corresponds to the least optimistic, q = 1 means the most optimistic. Considering triangular fuzzy number $\tilde{A}$ = (a1, aM, a2), we have follows.

$$E_q(\tilde{A}) = (1-q)(a1+aM)/2+q(aM+a2)/2 \qquad (3)$$

FFAHP method

For given products A, B, C, a fuzzy number based fuzzy hierarchy ranking process (FFAHP) includes following steps.

Step 1. Identify the hierarchical model of products' performance evaluation.

Step 2. Calculate fuzzy evaluation vectors at different levels, respectively. Fuzzy evaluation vector $\tilde{A}_i$ at different levels is calculated using Eqn. (4).

$$\tilde{A}_i = \tilde{C}_i \otimes \tilde{w}_i^T = \begin{bmatrix} \tilde{C}_{i,11} & \tilde{C}_{i,12} & \cdots & \tilde{C}_{i,1n} \\ \tilde{C}_{i,21} & \tilde{C}_{i,22} & & \tilde{C}_{i,2n} \\ \vdots & & \ddots & \vdots \\ \tilde{C}_{i,m1} & \tilde{C}_{i,m2} & \cdots & \tilde{C}_{i,mn} \end{bmatrix} \otimes \begin{bmatrix} \tilde{w}_{i,1} \\ \tilde{w}_{i,2} \\ \vdots \\ \tilde{w}_{i,n} \end{bmatrix} = \begin{bmatrix} \tilde{C}_{i,11}\otimes\tilde{w}_{i,1} \oplus \tilde{C}_{i,12}\otimes\tilde{w}_{i,2} \oplus \dots \oplus \tilde{C}_{i,1n}\otimes\tilde{w}_{i,n} \\ \tilde{C}_{i,21}\otimes\tilde{w}_{i,1} \oplus \tilde{C}_{i,22}\otimes\tilde{w}_{i,2} \oplus \dots \oplus \tilde{C}_{i,2n}\otimes\tilde{w}_{i,n} \\ \vdots \\ \tilde{C}_{i,m1}\otimes\tilde{w}_{i,1} \oplus \tilde{C}_{i,m2}\otimes\tilde{w}_{i,2} \oplus \dots \oplus \tilde{C}_{i,mn}\otimes\tilde{w}_{i,n} \end{bmatrix} \qquad (4)$$

Where $\tilde{C}_i$ is the fuzzy judgment matrix of each index in each level; $\tilde{w}_i$ is the fuzzy weight vector of each index in the level corresponding to $\tilde{C}_i$; $\tilde{w}_{i,j}=\tilde{1},\tilde{2},\tilde{3},\tilde{4},\tilde{5},\tilde{6},\tilde{7},\tilde{8},\tilde{9}$; $\tilde{C}_{i,kj}=\tilde{1},\tilde{2},\tilde{3},\tilde{4},\tilde{5},\tilde{6},\tilde{7},\tilde{8},\tilde{9}$; k=1,2 … m; j=1,2 … n; i represents the i th criterion.

Step 3. Establish a general fuzzy rating vector $\tilde{R}$, the total level of the evaluation vector $\tilde{R}$ is obtained from Eqn. (5).

$$\tilde{R}=\tilde{A}\otimes\tilde{O}^T=[\tilde{A}_1, \tilde{A}_2, \dots \tilde{A}_n]\otimes[\tilde{O}_1, \tilde{O}_2, \dots \tilde{O}_n]^T=[\tilde{r}_1, \tilde{r}_2, \dots \tilde{r}_m]^T \qquad (5)$$

Where $\tilde{A}$ is the fuzzy judgment matrix in the general level. It is composed of fuzzy vectors of different levels in the previous step. $\tilde{O}$ is a weight vector of each criterion.

Step 4. Calculate the average of fuzzy numbers of the overall fuzzy evaluation vector $\tilde{R}$, and the fuzzy mean $E_q(\tilde{r}_1)$ of the optimistic degree of reaction for decision makers under the optimistic degree q using Eqn. (3).

Step 5. Normalize $E_q(\tilde{r}_1)$ using Eqn. (6).

$$N_q(\tilde{r}_i) = E_q(\tilde{r}_i)/[E_q(\tilde{r}_1) + E_q(\tilde{r}_2) + \dots + E_q(\tilde{r}_m)] \qquad (6)$$

The largest $N_q(\tilde{r}_i)$ of the product concept will be the best design.

Case study

Design of an upper limb rehabilitation device is a complex process with multi-criteria. A variety of limb rehabilitation devices has been proposed for different needs of the upper limb rehabilitation. Following design requirements are identified [1].

(1) Safety: This is a primary need to avoid the secondary injury in affected limbs during rehabilitation. The device operation should be within the range of physical activities of a normal person.

(2) Economy: Affordability should be considered in the design.

(3) User friendly: A good interactive ability is required between the device and users, such as easy to wear, lightweight, interest and comfortable experience.

(4) Adaptability: This is to meet different rehabilitation needs. The size of the device should be adjustable to meet different users in height and limb size, such as a changeable length of forearm and upper arm.

Three upper limb rehabilitation products that meet above needs are selected as benchmark products analyzed in this research including A) CADEN-7 exoskeleton robot [6], b) ARMin exoskeleton upper limb rehabilitation training robot [2], and C) EXO-UL7 dual arm exoskeleton robot [4]. An evaluation system is established using the proposed method. It measures the four design requirements. The hierarchical structure of the design evaluation is shown in Figure 1.

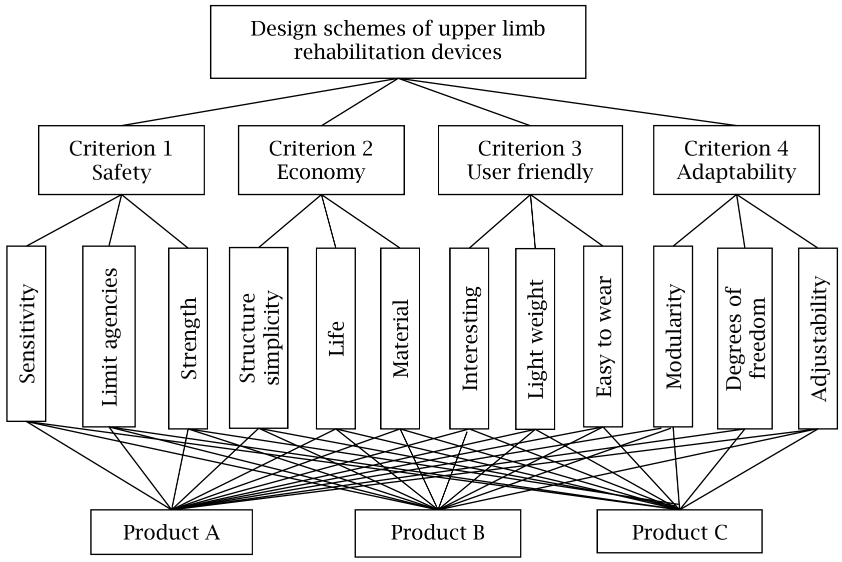

Fig. 1: Hierarchical structure of the design evaluation.

According to Table 1, triangular fuzzy numbers $\tilde{1}, \tilde{2}, \tilde{3}, \tilde{4}, \tilde{5}, \tilde{6}, \tilde{7}, \tilde{8}, \tilde{9}$ are assigned to related contributions of various indicators. The fuzzy index scores and weights of each index are shown in Table 2.

Functional indicators	Product demand	$\widetilde{\omega_i}$	A	B	C
Safety $\widetilde{\omega_1}$	Sensitivity	$\tilde{7}$	$\tilde{2}$	$\tilde{1}$	$\tilde{3}$
	Limit agencies	$\tilde{5}$	$\tilde{3}$	$\tilde{2}$	$\tilde{2}$
	Strength	$\tilde{5}$	$\tilde{2}$	$\tilde{1}$	$\tilde{3}$
Economy	Structure simplicity	$\tilde{4}$	$\tilde{3}$	$\tilde{2}$	$\tilde{1}$

$\widetilde{\omega_2}$	Life	$\tilde{6}$	$\tilde{2}$	$\tilde{1}$	$\tilde{2}$
	Material	$\tilde{3}$	$\tilde{2}$	$\tilde{2}$	$\tilde{2}$
User friendly $\widetilde{\omega_3}$	Interesting	$\tilde{6}$	$\tilde{1}$	$\tilde{3}$	$\tilde{3}$
	Light weight	$\tilde{5}$	$\tilde{2}$	$\tilde{2}$	$\tilde{1}$
	Easy to wear	$\tilde{3}$	$\tilde{3}$	$\tilde{3}$	$\tilde{1}$
Adaptability $\widetilde{\omega_4}$	Modularity	$\tilde{5}$	$\tilde{1}$	$\tilde{1}$	$\tilde{3}$
	Degrees of freedom	$\tilde{9}$	$\tilde{2}$	$\tilde{2}$	$\tilde{3}$
	Adjustability	$\tilde{5}$	$\tilde{3}$	$\tilde{3}$	$\tilde{2}$

Tab. 2: Fuzzy index scores and weights.

Using Eqns. (1-6), results of the design evaluation can be calculated are as follows:

$$\text{Nq(A)} = \frac{E_{q[\widetilde{r_1}]}}{E_{q[\widetilde{r_1}]}+E_{q[\widetilde{r_2}]}+E_{q[\widetilde{r_3}]}} = \frac{448.5+471q}{1485.5+1437q} \quad (7)$$

$$\text{Nq(B)} = \frac{E_{q[\widetilde{r_2}]}}{E_{q[\widetilde{r_1}]}+E_{q[\widetilde{r_2}]}+E_{q[\widetilde{r_3}]}} = \frac{494+463.5q}{1485.5+1437q} \quad (8)$$

$$\text{Nq(C)} = \frac{E_{q[\widetilde{r_3}]}}{E_{q[\widetilde{r_1}]}+E_{q[\widetilde{r_2}]}+E_{q[\widetilde{r_3}]}} = \frac{563.0+522.5q}{1485.5+1437q} \quad (9)$$

Where $q \in [0,1]$, $Nq(A) \in [0,1]$, $Nq(B) \in [0,1]$, $Nq(C) \in [0,1]$, values of Nq(A), Nq(B) and Nq(C) represent the optimization of products A, B and C, respectively, under the optimist q of the decision-maker. The bigger the value of Nq is, the more possible it is to be selected. From the results, it is found that Nq(A) <Nq(B) < Nq(C). Therefore, product C is a preferred design.

Design improvement
Based on the design evaluation, it is found that product A has the simplest structure, product B uses the most portable material, and has the best performance of human-machine interactions, product C has the maximum degrees of freedom, it can also be used in both left and right arms. Considering anatomy of the human upper limb, the motion range of each joint, and evaluation results, a new design of five degrees of freedom for an upper limb rehabilitation device is proposed by combining advantages of the three benchmark products as shown in Figures 2 and 3.

The improved performance of proposed device has a lightweight with ensured safety. The aluminum alloy is used to have the lightweight and high strength properties. Aluminum alloy #6610 is selected for the main structure of the exoskeleton device based on the comparison of mechanical properties of different grades of aluminum alloy.

In Figure 2, J1, J2, J3 are for rotations of shoulder adduction and abduction, flexion and extension, internal and external turn, respectively. J4 is for elbow flexion and extension. J5 provides rotation of the forearm. In order to ensure the safety, motion limits are set at each joint. For the adaptability, the device is designed as a detachable connection at A to achieve dual-arm versatility by changing orientation of the forearm. Virtual reality-based games are introduced in the rehabilitation process. The motion sensor is used for the user-device interaction to improve patient's interest and initiative action.

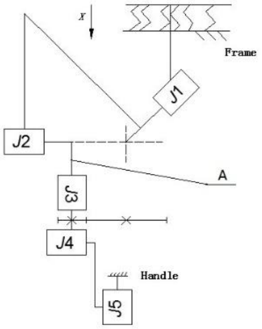

Fig. 2: Robot arm structure diagram.

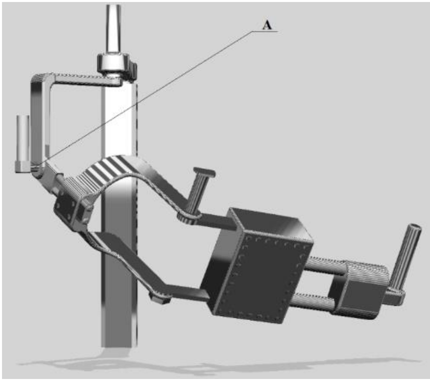

Fig. 3: 3D model of the proposed rehabilitation device.

Conclusions:

Design evaluation is a key process in product development for decision-making. Due to the complexity of evaluation requirements and difficulty of quantification, the design information itself may have the characteristics of vagueness, uncertainty and incompleteness, which increases the difficulty of decision-making. The FFAHP method provides a simple and efficient process in the design evaluation for complex products with multi-criteria of performance measures. This paper presents a fuzzy multi-criteria decision-making model for the evaluation of design alternatives. Taking the upper limb rehabilitation products as example, the fuzzy set theory, analytic hierarchy process and multi-criteria decision theory are comprehensively applied to improve the design solution for diversified needs of the product.

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