Title:
Tolerance Analysis Based on Skin Model Shapes and Polytopes

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Keywords:
Tolerance Analysis, Skin Model Shapes, Polytopes

DOI: 10.14733/cadconfP.2018.123-127

Introduction:
The objective of tolerance analysis is to check the feasibility and quality of assemblies or parts for a given GD&T scheme. The dimensional and geometric variations of each part in an assembly have to be limited by tolerances to ensure not only a standardized production but also the conformance of the functional requirements assigned on the whole assembly. Tolerance analysis process involves the representation and propagation of tolerances from part to part [2], which requires an effective method that considers both these two aspects. Research from the tolerancing domain has proposed numbers of models describing the allowed variations of the tolerance limits and zones, i.e., TTRS model [3], Vector loop [1] and Jacobian-tosor model [4]. However, these models are either inapplicable for geometric tolerances denotation, especially form defects, or ineffective for deviation transfer calculation.

Given the limitations of the former, the non-ideal model called skin model shapes [5] is proposed to provide a global representation of the parts' surfaces and acts to express all kinds of geometric specifications and verification in design, manufacturing and inspection. However, this method requires the generation of skin models shapes for all surfaces, assembling of parts bit by bit and measure of distances for all points between functional features which restricts its use on complex assemblies containing a large number of parts and joints. For the latter problem, the Polytope model developed by Teissandier [7] is suitable for tolerance stack-up computation by the use of Minkowski sum and intersection on polytopes, but this method is based on ideal features. The tradeoff between strengths and weaknesses of these two models motivates the integration of skin model shapes into the polytopes model which consequently combines the advantages in representing the deviation propagation from part to part in assembly as well as available incorporation of all kinds of geometric tolerances to establish more realistic production models. Therefore, in this paper, a novel method by combining of skin model shapes and polytopes is presented for tolerance analysis. The combined skin model shapes and polytopes is firstly introduced. A detailed description of modelling polytopes with constraints is then depicted. The analysis on functional requirement is presented afterwards. Conclusions are finally drawn.

Combining skin model shapes and polytopes:
In traditional polytope modelling process, each real surface is replaced by a substitute surface which is a perfect surface where the form defects are neglected. Since form errors influence various stages of manufacturing and assembly, this method based on simplified variations (orientation and location) of the nominal model cannot represent the real product. Therefore, the skin model shape which is a non-ideal model representing the actual shape of real parts is adopted to generate the polytopes, as shown

in Fig. 1. The framework for tolerance analysis by the combination of skin model shapes and polytopes covers four steps:

- Identification of all the toleranced features
- Generation of skin model shapes for features [5]
- Modification of the key parameters, i.e., two deviations d_i^{inf} and d_i^{sup} in constraints equations considering the real position of each point
- Recomputation the Minkowski sum or intersection between polytopes

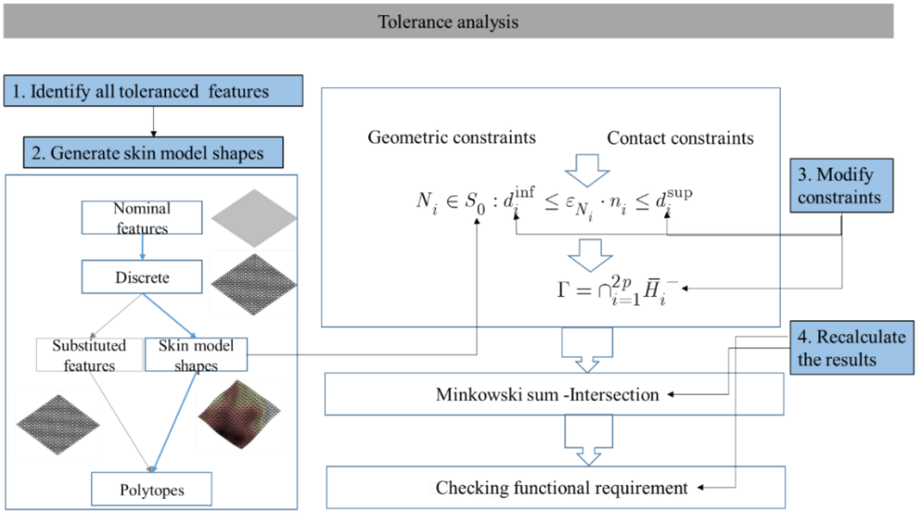

Fig. 1: Framework for tolerance analysis.

<u>Modelling polytopes with constraints:</u>

Geometric constraints

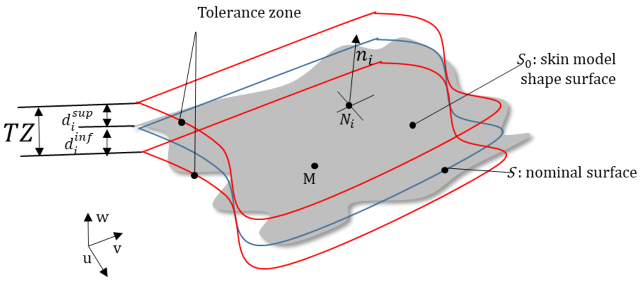

Fig. 2: Tolerance feature and tolerance zone.

Let S_1 be a surface related to a skin model shape feature S_0 (see Fig. 2). The tolerances applied will define a tolerance zone TZ, offsetting S_0 from d_i^{inf} to d_i^{sup}. This tolerance zone implies a restriction on the position of the points $N_i \in S_0$:

$$S_1 \in TZ \Leftrightarrow N_i \in S_0 : d_i^{\text{inf}} \le \varepsilon_{N_i} \cdot n_i \le d_i^{\text{sup}} \tag{3.1}$$

where ε_{N_i} is the translation displacement of S_1 in relation to S_0 at point N_i, n_i is the unit outward pointing vector normal to the nominal surface S in N_i. The skin model shape S_0 is a discrete surface which has p points N_i that will give p Eqn. (3.1) with $1 \le i \le p$.

By small displacement theory, Eqn. (3.1) can be written for any point M (which is assumed to be rigidly linked with the tolerance feature) in the Euclidean space:

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$$d_i^{\inf} \le \varepsilon_M + N_iM \times r \cdot n_i \le d_i^{\sup} \quad (3.2)$$

where r is the rotation vector of S_1 in relation to S_0. It should be noted that n_i is normal vector for nominal surface.

If we define the vectors $r = [\alpha, \beta, \gamma]^T$, ε_M=$[u, v, w]$, $N_iM = \left[d_{iu,}d_{iv,}d_{iw,}\right]^T$, $n_i = \left[n_{i\alpha}, n_{i\beta}, n_{i\gamma}\right]$, and setting $[\alpha, \beta, \gamma, u, v, w] = \left[x_{1,}x_{2,}x_{3,}x_{4,}x_{5,}x_{6,}\right]$, then we can obtain Eqn. (3.3) by expanding the vectorial and scalar products in Eqn. (3.2):

$$d_i^{\inf} \le\ n_{i\beta}d_{iw} - n_{ir}d_{iv}\ \ x_1 +\ n_{ir}d_{iu} - n_{i\alpha}d_{iw}\ \ x_2 +\ n_{ia}d_{iv} - n_{i\beta}d_{iu}\ \ x_3 + n_{i\alpha}x_4 + n_{i\beta}x_5 + n_{i\gamma}x_6 \le d_i^{\sup} \quad (3.3)$$

These constraints can be modeled with half-spaces of R^6. For each point $N_i \in S_0$, two parallel half-spaces are obtained. Therefore, for all the p points of the skin model shape, there are $2p$ constraints are derived. The intersection of these constraints defines a convex H-polyhedron in R^6.

$$\Gamma = \cap_{i=1}^{2p} \bar{H}_i^{-} \ \text{ with }\ \bar{H}^{-} = \ x \in R^6 : \mathrm{b}_i + a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + a_{i4}x_4 + a_{i5}x_5 + a_{i6}x_6 \ge 0,\ i=1,...2p \quad (3.4)$$

where a_{ij} $(1 \le j \le 6)$ are scalar parameters that corresponds to parameters before x_j in Eqn. (3.3).

For illustration purpose, we take a planar surface as an example, shown in Fig. 3(a). The skin model shape with consideration of form defects can be generated by adding systematic and random deviations to the nominal model, as shown in Fig. 3(d). Two boundary constraints are set to restrict each point N_i on the skin model shape within the tolerance zone which can be defined by Eqn. (3.5). The set of constraints for all points form the polytopes. The 3D projection $(r_x, r_{y,}t_z)$ of the operands which consider the form defects are comparatively shown in Fig. 3(c) and Fig. 3(e). The difference between two polytopes shows the effects of form defects. The operands are created at the central point M of the plane.

$$d_i^{\inf} \le \varepsilon_{N_i} \cdot n_i \le d_i^{\sup} \ \text{ with }\ d_i^{\inf} = N_iN_{oi} \cdot n_i - \frac{T}{2},\ \ d_i^{\sup} = \frac{T}{2} - N_iN_{oi} \cdot n_i \quad (3.5)$$

where T is the specified tolerance. N_{0i} is the correspondence of N_i on nominal model.

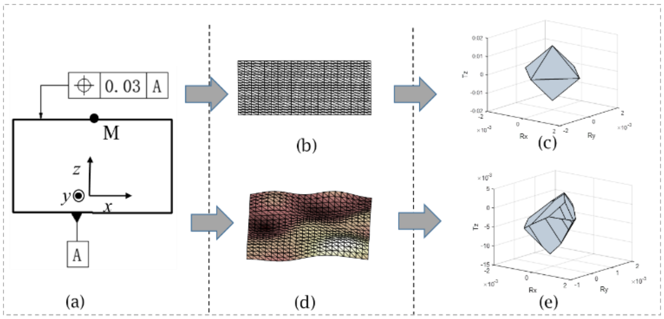

Fig. 3: Operand polytopes for planar surface. (a) CAD Model, (b) Nominal feature, (c) A general polytope in 3D, (d) Skin model shape, and (e) Adjusted polytope in 3D.

Contact constraints

Similar to geometric polytope, the contact constraints define the allowable displacement of features potentially in contact. The toleranced feature is not defined in the nominal case of permanent contact but considers the form defects. We assume the two surfaces A and B of joint are skin model shapes. The signed distance of pairs of corresponding points between surfaces in contact is employed to identify and avoid interpenetrations. If n_i is the outward pointing vector normal to the surface A at any point N_i, then the signed distance is defined by the scalar product as shown in Eqn. (3.6):

$$d_i = N_iN_{oi} \cdot n_i,\ N_i \in A,\ N_{oi} \in B \quad (3.6)$$

The nearest neighbour N_{0i} (correspondence) for every point of the skin model shape N_i in A is computed by minimizing an adapted projected point-to-point distance [6], as expressed in Eqn. (3.7), in which B_j denotes the corresponding point identified in feature B for N_i.

$$N_{oi} = B_j;\ \ j = \arg\min \left\| N_i - B_j \right\| \quad (3.7)$$

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It is obvious that the displacements of points on skin model shape A are bounded by the non-interference requirement with skin model shape B and the allowed fit clearance of the joint J. The constraints associated to the joint can be defined as follows:

$$-d_i \leq \varepsilon_{N_i} \cdot n_i \leq J - d_i \tag{3.8}$$

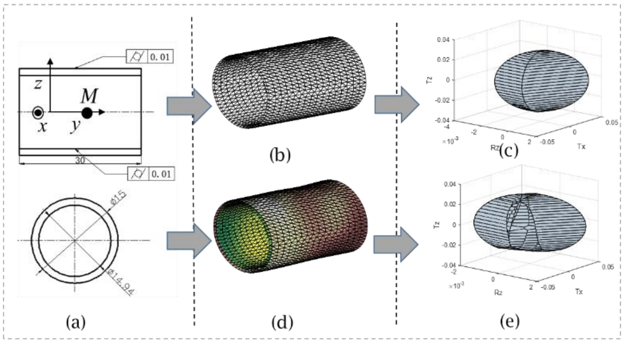

Fig. 4: Operand polytopes for cylindrical contact. (a) CAD model, (b) Nominal features, (c) A general polytope in 3D, (d) Skin model shapes, and (e) Adjusted polytope in 3D.

An example of pin-hole joint is presented to illustrate the modelling of contact polytope. By generating skin model shapes for both pin and hole surfaces, the parameters in Eqn. (3.8) for each points can be determined, as a consequence, the adjusted polytope can be derived (see Fig. 4(e)) which is different from the normal polytope (see Fig. 4(c)).

<u>On analysis of functional requirement:</u>
As demonstrated in Fig. 1, the first three steps are aimed at generating adjusted polytopes with consideration of form defects, the final step focuses on operating on these polytopes, i.e., Minkowski sum is performed if the features are assembled in a serial configuration while intersection is applied to the parallel configuration. Thereafter, by means of summing and intersections of polytopes, the variations between any features of functional requirement can be characterized, for example, the two parts assembled through a serial cylinder contact, the variation of surface 2.1 in relation to surface 1.1 at point O (see Fig. 5(a)) can be yielded as the Minkowski sum of two polytopes, as shown in Fig. 5(b). The specified tolerances are $t_{1,2} = 0.02, t_{2,1} = 0.01$ and the nominal clearance between two parts is 0.05. For clearer illustration purpose, an extra example with planar features is provided. The skin model shape for parts (see Fig. 6(a)) and operand polytopes (see Fig. 6(b) and Fig. 6(c)) are generated with parameters $t_1 = 0.02$ and $t_2 = 0.1$. It is obvious that the obtained functional requirement is narrower than the original calculation approach, since the defected surface restricts the allowed variation of each feature.

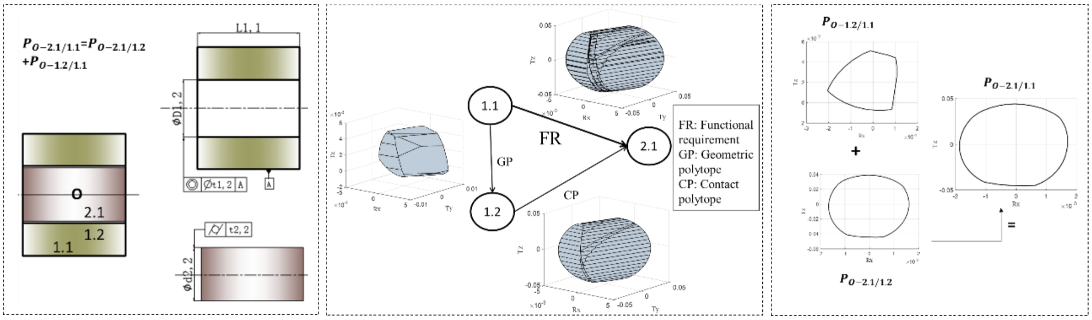

Fig. 5: Example assembly. (a) Detailed drawing of assembly, (b) Assembly graph and operand polytopes in 3D, and (c) operand polytopes in 2D.

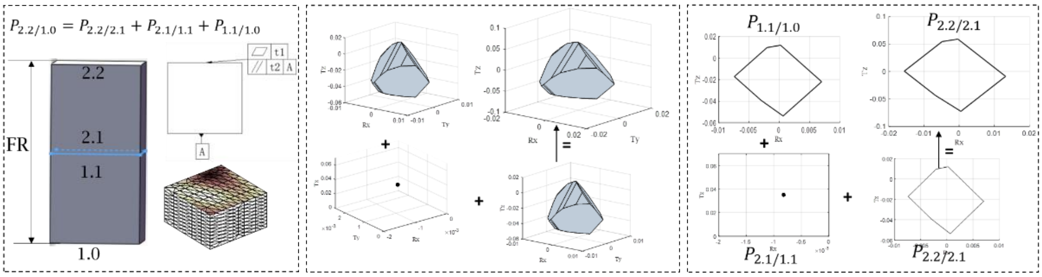

Fig. 6: Example assembly. (a) Detailed drawing of assembly, (b) Assembly graph and operand polytopes in 3D, and (c) operand polytopes in 2D.

Conclusions:

In this paper, a method for assembly analysis is developed which combines the benefits of the skin model shape and the polytope. By taking the actual toleranced surfaces into consideration, the newly built polytope is smaller than the traditional substitute-based polytope, which is due to the fact that the previous modelling method reduces geometric deviations to translational and rotational feature defects without considering form deviations. This proposed method, to some extent, demonstrates the applicability of the polytopes in tolerance analysis by integrating form defects. Thanks to the application of skin model shapes to represent the actual features in modelling polytopes, the accuracy and reliability of the analysis result for assessment of assembly functional requirement can be significantly improved.

Acknowledgements:

This research was supported by the National Natural Science Foundation of China (No. 51575484 and U1501248), Science Fund for Creative Research Groups of National Natural Science Foundation of China (No. 51521064) and the China Scholarship Council (CSC).

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