Title:
A New Formulation of the Minimum Variation Log-aesthetic Surface for Scale-invariance and Parameterization-independence

Authors:
Sho Suzuki, suzuki.sho@shizuoka.ac.jp, Shizuoka University
R.U. Gobithaasan, gr@umt.edu.my, Universiti Malaysia Terengganu
Shin Usuki, dsusuki@ipc.shizuoka.ac.jp, Shizuoka University
Kenjiro T. Miura, miura.kenjiro@shizuoka.ac.jp, Shizuoka University

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Introduction:
Recently, aesthetic design which takes account of designability has become popular. In the aesthetic design, the creation of High-quality curve and surface models is demanded. However, on current CAD systems, the operator must move control points by trial and error to obtain high-quality curves and surfaces. This incurs high costs and requires a great deal of expertise. Therefore, an efficient method to generate fair curves and surfaces is desirable to achieve high quality that will satisfy customers' aesthetic requirements.
The log-aesthetic curve was proposed as a curve which satisfy these quality requirements. Harada et al. [1] defined "Aesthetic curves" as curves whose logarithmic distribution diagram of curvature (LDDC) can be approximated by a straight line. In response to this research, Miura et al. [3] derived analytical solutions of the curves whose logarithmic curvature graph (LCG) as an analytical version of the LDDC is strictly given by a straight line and defined the curve as the log-aesthetic curve. For a given curve, the arc length of the curve and the radius of curvature are denoted by s and ρ, respectively. The log-aesthetic curve satisfies the following equation.

$$\rho^{\alpha} = cs + d \tag{1.1}$$

Here, α, c, and d are constants. In particular, α is the slope of LCG and a parameter for controlling the impression of the curve. Fig.1 illustrates log-aesthetic curves for various α values. Since the log-aesthetic curve defined by use of curvature as above equation, its curvature distribution is smooth. In addition, it includes logarithmic (equiangular) spiral, clothoid, and circular involute as well as Nielsen's spiral. For these reason, it is expected to be utilized in the field of aesthetic design [8].
Although the log-aesthetic curve has a number of good properties, it is difficult to extend it to the surfaces because of complexity of its general equation. As a solution of this problem, the minimum variation log-aesthetic surface [7] was proposed. This surface is defined as the surface which minimize an objective function and allows the usage of arbitrary boundary curves. However, the value of the objective function depends on the scale of model and parameterization.
In this research, we derive a new formulation of the minimum variation log-aesthetic surface for Scale-invariance and Parameterization-independence.

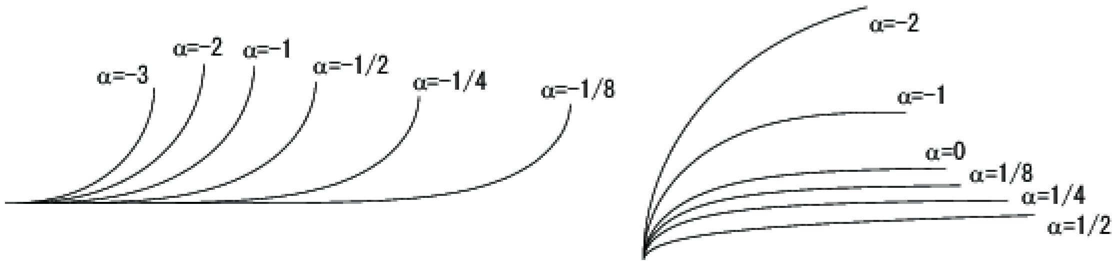

Fig. 1: Log-aesthetic curves with various ▯ values.

<u>Minimum Variation Log-aesthetic Surface:</u>

As mentioned in the introduction, it is very difficult to extend the log-aesthetic curve to the surface that have similar good properties to the log-aesthetic curve. To solve this problem, two surface formulas besides the minimum variation log-aesthetic surface have been proposed that generate free-form surfaces by sweeping the log-aesthetic surface [2,6]. The log-aesthetic curved surface [2] is defined as a sweeping surface using two profile curves, which are composed of log-aesthetic curves, and one guide line composed of a non-log-aesthetic curve. The surface guarantees the isoparametric curves parallel to two profile curves become the log-aesthetic curve and the quality along the isoparametric curve is guaranteed. In contrast, the isoparametric curves parallel to the guide line do not become log-aesthetic curves and high quality in this direction cannot be guaranteed. As a solution to this problem, Saito et al. proposed the complete log-aesthetic surface [6]. The complete log-aesthetic curve described is defined as a pure sweeping surface with two log-aesthetic curves. This formulation also uses the log-aesthetic curve as the guide line and guarantees that all parametric curves are log-aesthetic curves. However, for these two formulations, at least one boundary curve cannot be specified. Consequently, the situations where these formulations can be used are severely restricted.

In contrast, the minimum variation log-aesthetic surface can be used for bounded by arbitrary four curves. Minimum variation log-aesthetic surface is defined by reformulating the log-aesthetic curve with the variational principle and extending it to surfaces. From Eqn. (1.1), when we assume $\sigma = \rho^{\alpha}$ the log-aesthetic curve is given by a straight line connecting two given points (s_1, σ_1) and (s_2, σ_2) in the s-σ plane (aesthetic space) as shown in Fig.2 where the horizontal and vertical axes are the arc length s and σ, respectively. Therefore, from the variational principle, the log-aesthetic curve is reformulated as a curve that minimizes the following energy J_{LAC}.

$$J_{LAC} = \int_{s_1}^{s_2} (1 + \sigma_s^2) ds \tag{2.1}$$

The Euler equation of Eqn. (2) is as follows.

$$\sigma_{ss} = 0 \tag{2.2}$$

Obviously, Eqn. (2.1) is equivalent to the second derivative of Eqn. (1.1). Furthermore, the Euler equation of Eqn. (2.1) is equivalents to that of the following equation K_{LAC}.

$$K_{LAC} = \int_{s_1}^{s_2} \sigma_s^2 ds \tag{2.3}$$

Finally, Eqn. (2.3) is represented by the arc length parameter s, and we rewrite Eqn. (2.3) using a general parameter t and obtain the following expression:

$$K_{LAC} = \int_{t_1}^{t_2} \frac{1}{\left\| \frac{dC}{dt} \right\|} \alpha^2 \rho^{2\alpha-2} \rho_t^2 dt \tag{2.4}$$

Here, $\|dC/dt\|$ represents the norm of the first derivative of curve C with general parameter t. We use Eqn. (2.4) as the objective function of the log-aesthetic curve.

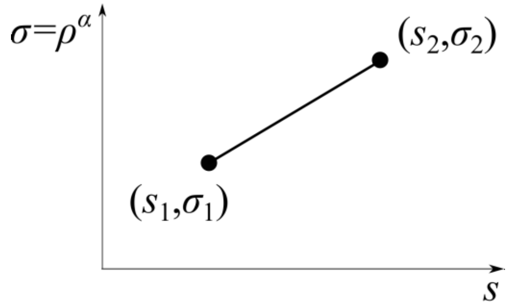

Fig. 2: Straight line connecting two given points (s_1, σ_1) and (s_2, σ_2) in the s-σ plane (aesthetic space).

The objective function of the log-aesthetic surface is derived by extending the objective function of the log-aesthetic curve K_{LAC} to surfaces. The objective function is defined so that minimizing the objective function transforms isoparametric curves into log-aesthetic curves. We obtain the following objective function J_{LAS} by applying Eqn. (2.4) to both the direction of surface and define the minimum variation surface as minimizing this function.

$$J_{LAS} = \int_{u_1}^{u_2}\int_{v_1}^{v_2}\left\{\frac{1}{\sqrt{E}}\alpha^2(\rho^u)^{2\alpha-2}(\rho_u^u)^2 + \frac{1}{\sqrt{G}}\beta^2(\rho^v)^{2\beta-2}(\rho_v^v)^2\right\}dvdu \tag{2.5}$$

Here, E and G are elements of the first fundamental and are given by $E=\partial S/\partial u\cdot\partial S/\partial u$ and $G=\partial S/\partial v\cdot\partial S/\partial v$, respectively. ρ^u and ρ^v are the radii of curvature of isoparametric curves with u and v direction, respectively. In the integral of Eqn. (2.5), the first term is the optimization term of the isoparametric curve with u direction and the second term is the optimization term of the isoparametric curve with v direction.

As isoparametric curves become log-aesthetic curves, this minimum variation surface is equivalent to the complete log-aesthetic surface. However, as this formulation defines the surface by minimizing the objective function, the minimum variation surface is markedly different from the complete log-aesthetic surface. That is, the minimum variation surface can specify an arbitrary boundary curve, and hence the objective function can be used for generation of the surface. In many boundary cases, surfaces in which the isoparametric curves completely become log-aesthetic curves cannot be generated.

<u>Scale-invariance:</u>
Moreton and Sequin [5] introduced the minimum variation surface (MVS) functional that measures curvature variation by integrating the principal curvature's squares of derivatives in its principal directions. They derived the its scale invariance [4]. Multiplication of the area term is used for scale invariance and scale invariance of the MVS functional is given by the following:

$$E_{MVS} = \int\left(\frac{d\kappa_{\max}^2}{de_{\max}} + \frac{d\kappa_{\min}^2}{de_{\min}}\right)dA\int dA \tag{3.1}$$

where $\kappa_{\max}$ and $\kappa_{\min}$ are principal curvatures. $e_{\max}$ and $e_{\min}$ are principal curvature directions.
In this section, we will perform a similar modification for K_{LAC} expressed in Eqn. (2.3) to make it scale-invariant and extend it to surfaces. First, we consider a curve whose arc length is equal to 1. If the curve is log-aesthetic, i.e. $\sigma = \rho^\alpha$ is a linear function of arc length s, there is a constant c such that

$$c = \sigma_s = \sigma_{end} - \sigma_{str} \tag{3.2}$$

Here, σ_{end} and σ_{str} are the value of σ at both end points.
Then, Eqn. (2.3) become

$$K_{LAC} = \int_{s_1}^{s_2} c^2 ds = c^2 = (\sigma_{end} - \sigma_{str})^2 \tag{3.3}$$

On the other hand, if we consider to introduce scale factor r and a curve which is scaled by that scale factor r. Then, arc length of the curve is equal to r and $\sigma = \rho^{\alpha}$ becomes $\sigma' = (r\rho)^{\alpha} = r^{\alpha}\rho^{\alpha}$.
There is a constant c' similarly to Eqn. (3.2) such that

$$c' = \sigma'_s = \frac{r^{\alpha}(\sigma_{end} - \sigma_{str})}{r} = r^{\alpha-1}(\sigma_{end} - \sigma_{str}) \tag{3.4}$$

And the value of the objective function Eqn. (2.3) become

$$K'_{LAC} = \int_{s_1}^{s_2} c'^2 ds = c'^2 r = r^{2\alpha-1}(\sigma_{end} - \sigma_{str})^2 \tag{3.5}$$

Therefore, by replacing scale factor r with arc length of curve h, the scale-invariant objective function of Eqn. (2.3) is given by

$$K_{LAC-SI} = \frac{K_{LAC}}{h^{2\alpha-1}} \tag{3.6}$$

Based on the curve case, we define the scale-invariance objective function of surface. First, as in the curve case, we consider a surface whose area is equal to 1. we separate the two terms in Eqn. (2.5) into two integrations with u and v directions as follows

$$J_{LAS} = \int K_{LAC\_u} dv + \int K_{LAC\_v} du \tag{3.7}$$

Here,

$$K_{LAC\_u} = \int_{u_1}^{u_2} \frac{1}{\sqrt{E}} \alpha^2 (\rho^u)^{2\alpha-2} (\rho_u^u)^2 du \tag{3.8}$$

$$K_{LAC\_v} = \int_{v_1}^{v_2} \frac{1}{\sqrt{G}} \beta^2 (\rho^v)^{2\beta-2} (\rho_v^v)^2 dv \tag{3.9}$$

Note that K_{LAC_u} and K_{LAC_v} indicate the objective function of log-aesthetic curve Eqn. (2.4) with respect to iso-parametric curves of parameter direction u and v respectively.
Next, we consider the case that the surface is scaled by scale factor r (such that the area of surface become r^2). Then, from the discussion of the curve case, K_{LAC_u} and K_{LAC_v} are scaled to $r^{2\alpha-1}$ times. Furthermore, microelements du and dv also are scaled to rdu and rdv. Therefore, we obtain the following equation.

$$J_{LAS}' = r^{2\alpha-1} \int K_{LAC\_u} r dv + r^{2\beta-1} \int K_{LAC\_v} r du = r^{2\alpha} \int K_{LAC\_u} dv + r^{2\beta} \int K_{LAC\_v} du \tag{3.10}$$

Finally, we obtain the scale-invariant objective function of Eqn. (2.5) by comparing Eqn. (3.7) with Eqn. (3.10) as follows

$$J_{LAS-SI} = \frac{\int K_{LAC\_u} dv}{A^{\alpha}} + \frac{\int K_{LAC\_v} du}{A^{\beta}} \tag{3.11}$$

Here, $A = r^2$ are the area of the surface.

<u>Parametarization independence:</u>
The objective function of minimum variation log-aesthetic surface Eqn. (2.5) is defined so that the isoparametric curves of minimized surface become log-aesthetic curve. However, this formulation includes surface parameter u, v and depend on parametarization. We archive parametarization-independence by using principal curvature of radius $\rho^{\max}$ and $\rho^{\min}$.

$$J_{LAS-PI} = \int \left\{ \frac{1}{|e_{\max}|} \alpha^2 (\rho^{\max})^{2\alpha-2} (\rho_{\max}^{\max})^2 + \frac{1}{|e_{\min}|} \beta^2 (\rho^{\min})^{2\beta-2} (\rho_{\min}^{\min})^2 \right\} dA \tag{4.1}$$

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Here, we note that $|e_{\max}|$ and $|e_{\min}|$ are derivatives of surface with principal direction (i.e. $|e_{\max}|=|e_{\min}|=1$ is not always consist).

Especially, when α and β = -1, from ρ=1/κ and ρ_t=d/dt(1/κ)=-κ_t/κ_2, (4.1) becomes:

$$J_{LAS-PI}\Big|_{\alpha=\beta=-1} = \int \left\{ \frac{(\kappa_{\max}^{\max})^2}{|e_{\max}|} + \frac{(\kappa_{\min}^{\min})^2}{|e_{\min}|} \right\} dA \tag{4.2}$$

If we assumed $|e_{\max}|=|e_{\min}|=1$, Eqn. (4.2) is locally equivalent to the objection function of the minimum variation surface [5].

Additionally, by the same discussion in previous chapter, we obtain the following scale-invariance objective function:

$$J_{LAS-PI-SI} = A^{-\alpha} \int \frac{1}{|e_{\max}|} \alpha^2 (\rho^{\max})^{2\alpha-2} (\rho_{\max}^{\max})^2 dA + A^{-\beta} \int \frac{1}{|e_{\min}|} \beta^2 (\rho^{\min})^{2\beta-2} (\rho_{\min}^{\min})^2 dA \tag{4.3}$$

Especially, when α and β = -1, we obtain following:

$$J_{LAS-PI-SI}\Big|_{\alpha=\beta=-1} = A \int \left\{ \frac{(\kappa_{\max}^{\max})^2}{|e_{\max}|} + \frac{(\kappa_{\min}^{\min})^2}{|e_{\min}|} \right\} dA \tag{4.4}$$

Eqn. (4.4) is locally equivalent to the scale-invariant objection function of the minimum variation surface Eqn. (3.1).

Results:

In this section, we adopted the objective function given in Eqn. (4.4) for B-spline surfaces and optimize the control points of the surface by minimizing the objective function. At that time, we impose constraints on coordinates of and tangent vectors across the boundary curves as boundary conditions. Hence, we fix two control points from the boundary to fix the shape of the boundary curves and tangent vectors across them and input these control points. We used the downhill simplex method [8] for optimization.

We applied our method to complete log-aesthetic surfaces [6] to which noise had been added. One of the formulations of a complete log-aesthetic surface is given by the following equation.

$$\begin{pmatrix} x(\theta,\phi) \\ y(\theta,\phi) \\ z(\theta,\phi) \end{pmatrix} = R_z(\phi) Sc(e^{b_g \phi}) \begin{pmatrix} t_r + e^{b_p \theta} \cos\theta \\ 0 \\ t_z + e^{b_p \theta} \sin\theta \end{pmatrix} \tag{5.1}$$

where θ and ϕ are surface parameters, b_p, and b_g are shape parameters, t_r t_z are offset parameters, $R_z(\phi)$ is a rotation function around the z axis, and $Sc(e^{k\phi})$ is the scaling function. First, we generate a complete log-aesthetic surface with b_p=0.2, b_g=0.2, t_r=5, and t_z=3. Next, we cut part of the surface and approximate this surface with bicubic B-spline, which has 10×10 control points. Finally, noise is added to the surface and our objective function is applied (i.e., we optimize the inner 6×6 control points). We used a PC with a Core i7-7700 3.60 GHz CPU.

Fig.3 shows generated surfaces. In the figure, the original surface is shown on the left, the surface with noise is shown in the middle, and the surface optimized by our method is shown on the right. The processing time for optimizing surfaces is 140 [s]. Fig.4 and Fig.5 show the mean curvature distribution and zebra map of these surfaces. These results showed that the surface with added noise is markedly disturbed. In contrast, after optimization, the surface is not disturbed and has almost the same quality as the original surface.

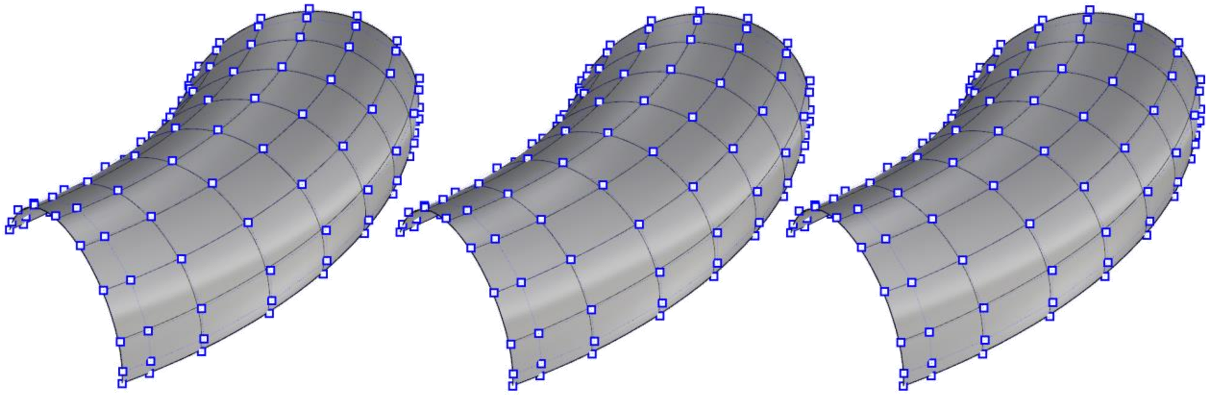

Fig. 3: Generated surfaces. Left: before optimization. Middle: surface with added noise. Right: after optimization.

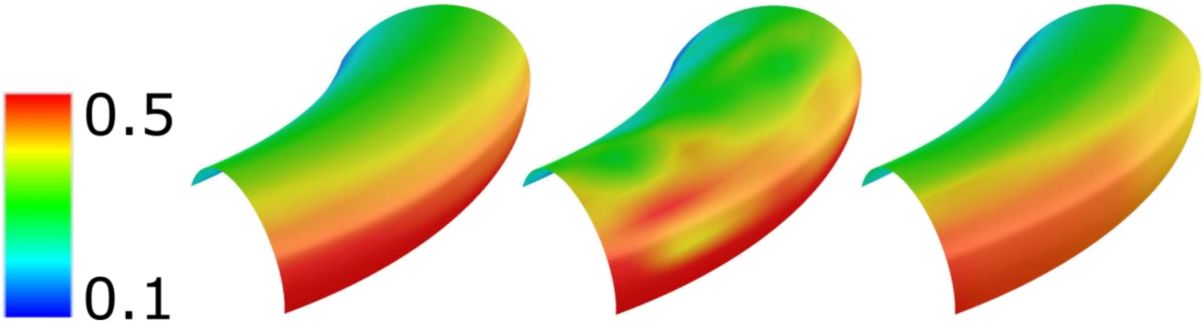

Fig. 4: Mean curvature distribution. Left: before optimization. Middle: surface with added noise. Right: after optimization.

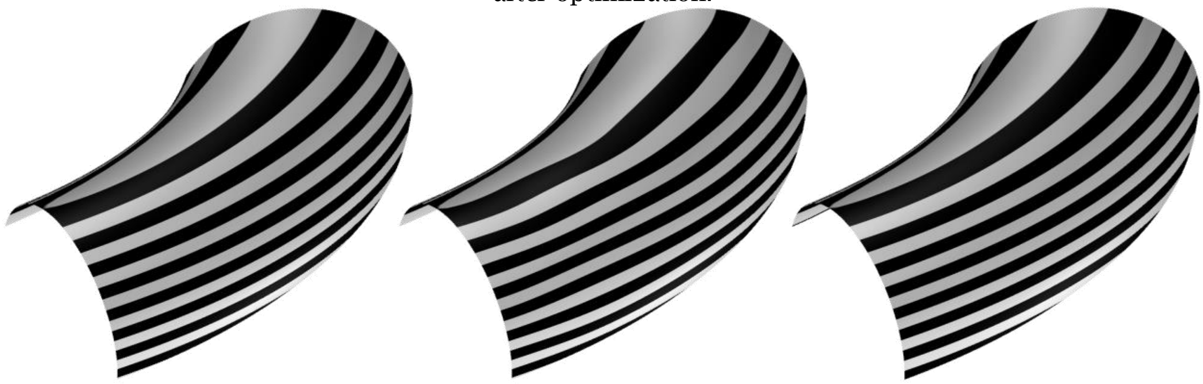

Fig. 5: Zebra map. Left: before optimization. Middle: surface with added noise. Right: after optimization.

Conclusion:

In this research, we derive a new formulation of the minimum variation log-aesthetic surface for Scale-invariance and Parameterization-independence. Furthermore, we generate surfaces by minimizing the objective function. The results indicated that we can obtain free-form surfaces of high quality. However, the processing times required for relatively large surfaces are expected to be very long.

Therefore, in future, we will use GPU processors to reduce the processing time and hope to achieve a real time.

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