Title:
Image Analysis Using Recurrence Quantification Plots, Surface Modelling and Fused Deposition Modelling Tools to Design 3 Dimensional Models

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Introduction:
Many 'time-series' data analysis strategies are available to extract information from a data set. Various strategies can be applied to filter out noise to be able to estimate underlying trends; or transformations, such as analysing the data in the frequency domain or phase space [6], could be performed to provide insights into a dynamic system. Recurrence quantification analysis (RQA) is a non-linear data analysis tool and was developed to visualise recurrence behaviours [14, 15]. Recurrence plots (RPs) are N x N matrix based representations formulated by:

(1.1)

$$RP_{i,j=}\ \Theta(\varepsilon - \ \|\vec{x_i} - \ \vec{x_j}\|)$$

where i, j = 1 to N
$\vec{x_i}$ is the point in phase space at which the system is situated at time i,
ε is a predefined threshold, and
$\Theta(x)$ is the Heaviside function.

The input parameters for the analysis are the:
- Data series
- Embedding value
- Delay period value
- First point within the sequence to start the analysis
- Last point or N
- Threshold for filtering (or the Radius)

For a 'true condition', the grid element (i, j) is encoded black, else it remains white. The recurrence plot for a normalized sine wave with 8 periods within the data set is illustrated in Fig. 1. The sine wave was generated using the Grasshopper© Graph Mapper tool. Grasshopper© is a graphical algorithm editor [5] which has been design to integrate with Rhinoceros' computer aided design (CAD) modelling tools [7]. There are modules for manipulating curves, surfaces, vectors, data sets, and the resulting image's colour, which are employed in this research. The RQ plot is generated using Python script [4] for Grasshopper© along with geometry creation tools, and the data is displayed using Rhinoceros ©. After the points are created, boxes are generated around each grid point, and then bounded surfaces are created.

RQA has been performed in a diverse array of fields, including behaviour sciences [1, 13 – 15], advanced manufacturing [3], gas-solid fluidized beds [9], and climate change influences [16]. There are common characteristics for these research areas at the abstract level: these systems are dynamic, non-linear, noisy, and unpredictable.
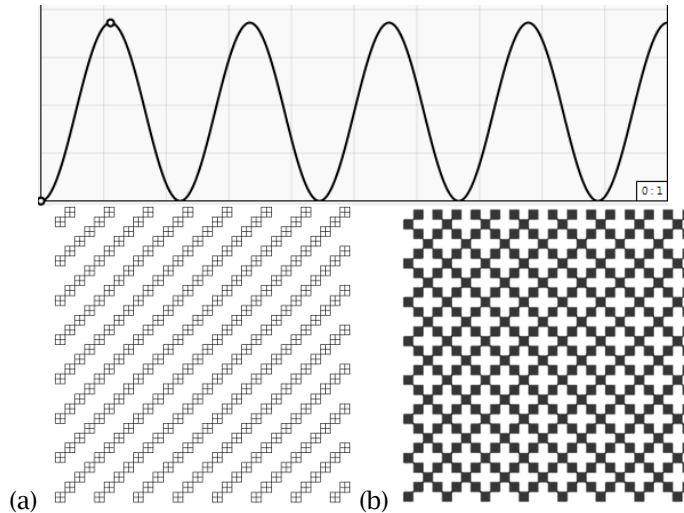
Fig. 1: Sine wave and the resulting RP showing the points, boundaries, and grids (a) embedding = 5, delay = 5, first point = 0, last point = 30, Radius = 5, (b) embedding = 5, delay = 8, first point = 0, last point = 30, Radius = 5.

Patterns within the observational data may not be readily evident. In addition to utilizing RQA to highlight recurrence patterns, the influence of noise within the observational data is studied [10] as the noise impacts information being plotted. However, as recognized by Webber and Zbilut [13], RQ plots are "intriguing graphical displays" and "tend more to evoke artistic than scientific appreciation". The resulting RQ plots from the various areas of research are visually intriguing; therefore, goal of this research is to explore recurrences in digital image colour or grey scale values, and create a three dimensional (3D) model of the results. Along with using the aforementioned computer aided design (CAD) modelling tools, Stratasys Insight© software [8], used for fused deposition modelling (FDM) process planning, is employed to create 'slice models' for dynamic visualization. Being able to dynamically change the RQA characteristics and visualize the results in 3D may provide more relevant and quantifiable pattern recognition insights.

Methodology:
A six step methodology is utilized to develop a 3D model of RQ plots: (i) data acquisition, (ii) RQ analysis, (iii) plotting the results using CAD primitives (points, curves, and surfaces), (iv) creating a 3D model, (v) creating a mesh model that can be utilized by additive manufacturing process planning software, and (vi) using the additive manufacturing process planning software to create a 'slice' model to dynamically view all threshold results. The general graphical algorithm developed for this work (in Grasshopper®) is shown in Fig. 2.

The data series for this work is a decomposed digital image. This is utilized as the input 'rawSeries'. The RQA formulae are programmed using Python for Grasshopper. The input values can be adjusted dynamically by positioning the cursor over the '◇' symbol, and dragging the slider bar between the defined bounds. This enables rapid viewing of RQ plot variants. Once the RQ plots are generated for a parameter set, a 3D model is created, as discussed in detail in the research paper. The RQ plot results are converted into point geometry with position (i, j, 0) in Cartesian space (XY plane), and plotted. These points are the box centre points. Bounding box objects (0.9 x 0.9 units) are created. The box is deliberately smaller than the grid size to illustrate each element. The 3D model is created by extruding the rectangles by the Euclidean norm values for each bounding box in the + z direction. They joined to a base plate which has a bounding box slightly larger than the N x N grid, and extruded 0.25 units in the – z direction. The resulting extruded surfaces are 'capped' to generate watertight models. The resulting models have variable surface heights. The surface waviness versus a 'flat' result is evident in the 3D RQ plots (Fig. 3).
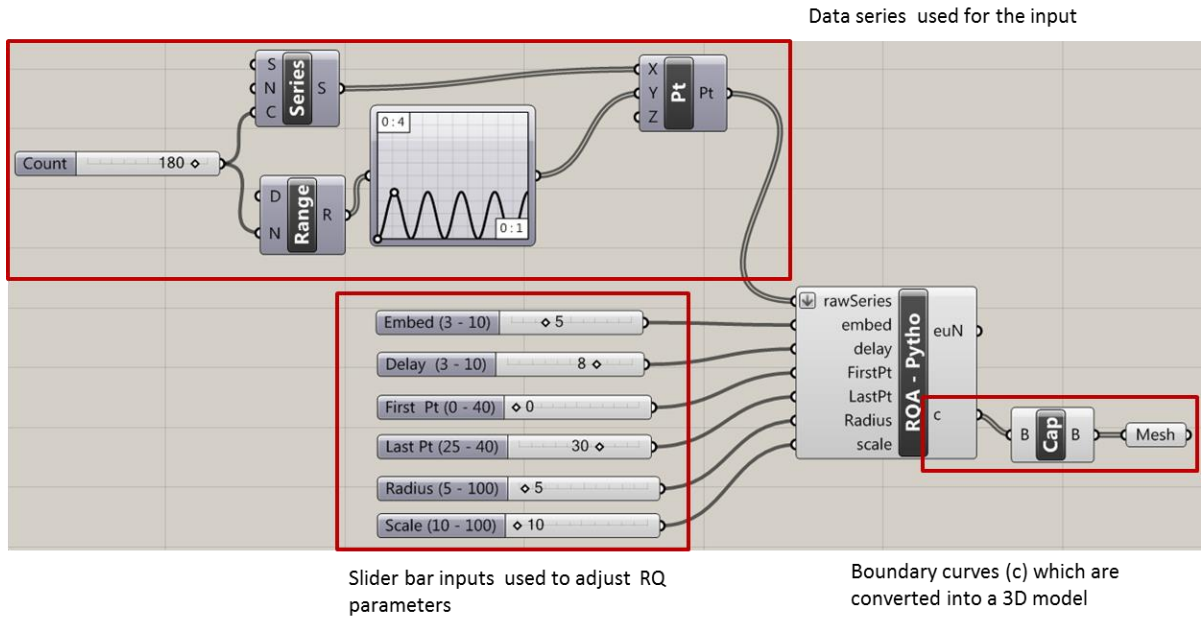
Fig. 2: RQ plot Grasshopper© program illustrating the input data series, and the RQ inputs which can be modified by the user for real time result updates.
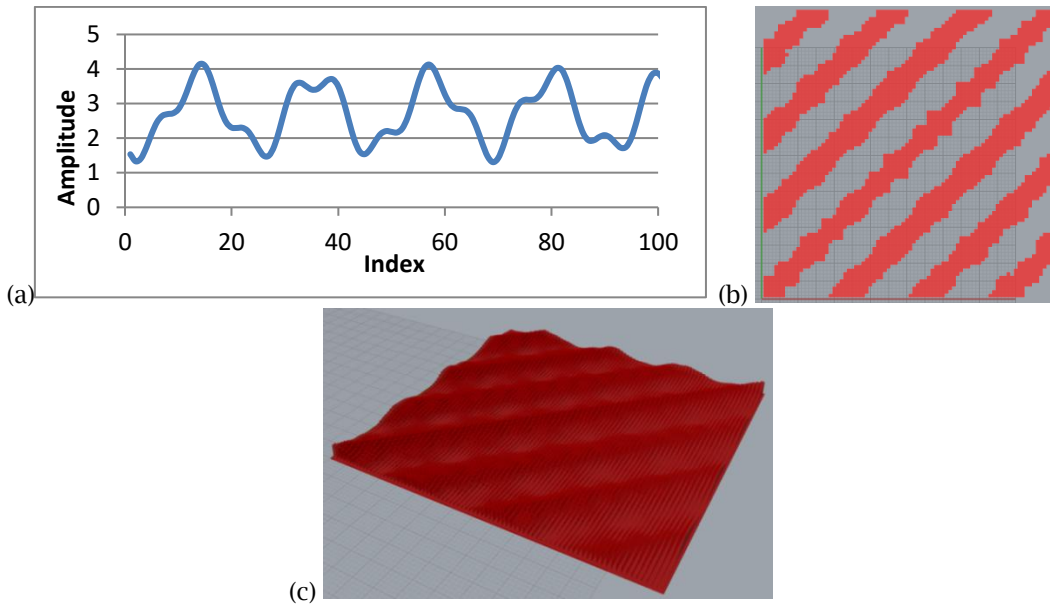

Fig. 4: RQ plot for the sine wave-based data series (embed = 5, delay = 8, Last point = 80, Radius = 56) and the 3D model.

To generate a dynamic model representation, process planning software is employed for the fused deposition modelling (FDM) additive manufacturing process. FDM is a bead deposition based additive manufacturing process. AM processes are capable of fabricating three dimensional parts with complex geometries by layering two dimensional (2D) cross sections of build material successively to create the

final part [11]. The FDM process planning software uses stereolithography files (*STL) as an input, and slices into the model into layers. Layer visualization tools are available for process validation; however, for an RQ plot model, the visualization of each slice is the equivalent of viewing the neighbouring points for a threshold value, and this inherently provides a dynamic 3D RQ plot. Illustrated in Fig. 4 is a crude 3D model for the Mobius strip image (Fig. 5) and two selected slices (Last point = 10, embed = 7, delay = 9), which was sliced in 0.254 mm increments. There are 100 slices for the RQ plot area. Note: the fill pattern varies 90° between the layers, which alters the appearance for the layers.
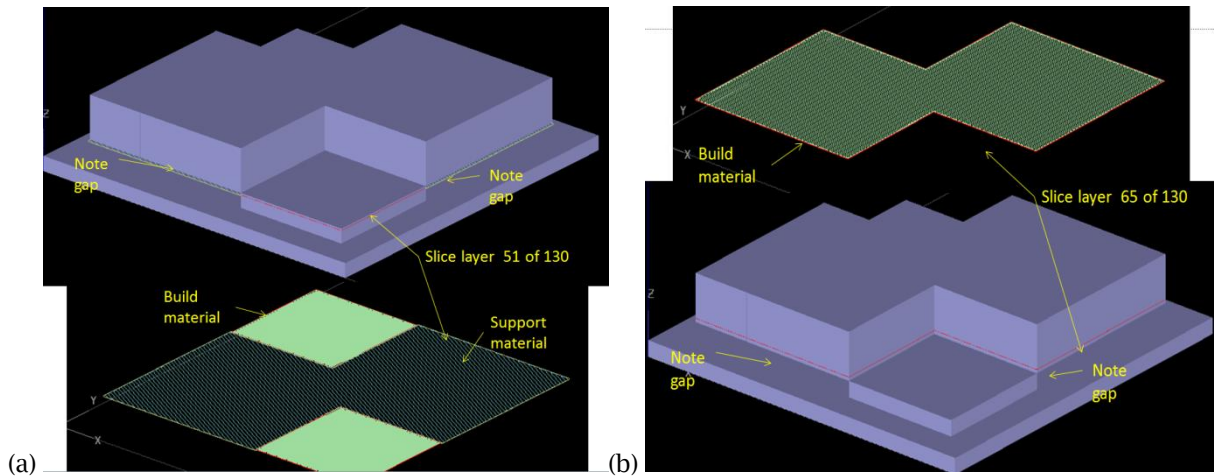

(a)   (b)

Fig. 4: Mobius Strip 3D RQ plot (embed = 7, Delay = 9, Last Point = 10). Note: for illustration purposes, an *XY* reference plane is established at Radius = 50. Geometry that does not meet the threshold is extruded in the –z direction. Geometry which meets the threshold is extruded in the +z direction.

Results:
The 2D and 3D RQ plots are generated for a set of images for various parameter sets. N (grid size) is set to 40 and 120, and the other input parameters are varied to illustrate texture differences for the four images. The radius is set to 87, 56, 25, and 4 as noted. The Mobius strip image (Fig. 5), signal, and 3D model is illustrated here. The Mobius strip plots have much symmetry and a cubic structure being generated in both the 2D and 3D models, as shown in Fig. 5. The dynamic model can be viewed at: http://screencast.com/t/U3whpyjr.
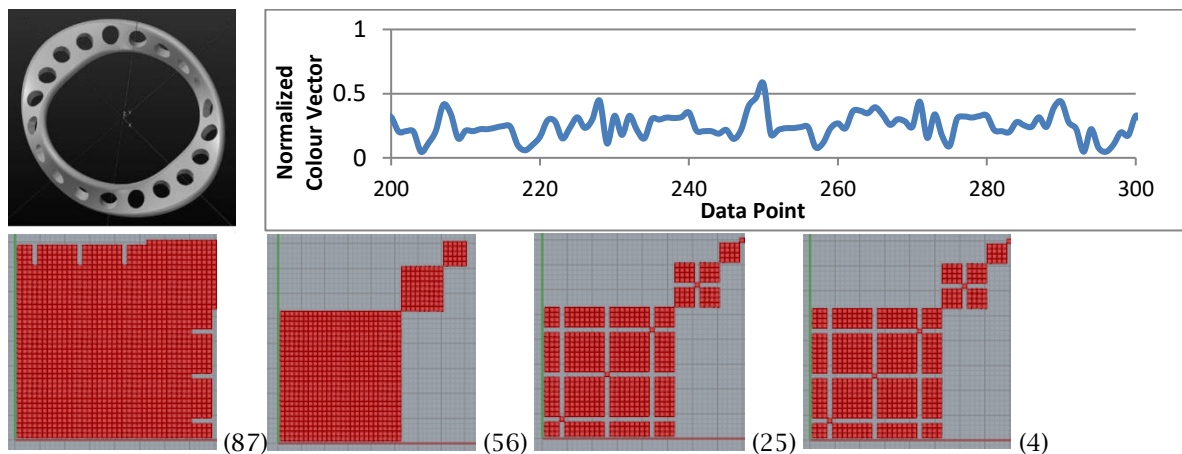

(87)   (56)   (25)   (4)

Fig. 5: Mobius strip image, (b) Mobius strip signal used as the raw data input, and Mobius strip RQ plots with parameter set 1: **embed = 7**, Delay = 9, **Last Point = 40**, Radius = (variable).

Summary:
RQ plots are generated for a set of digital images, which are chosen based on their mathematical and organic shape characteristics. Once the Euclidean norm values are determined from the extracted data set, they are normalized based on their maximum value, filtered based on the selected Radius value, and then passed through the Heaviside function. A set of points is generated, which are subsequently used as a centre point from which a 1.0 x 1.0 unit rectangular boundary is created. For the 2D plots, surfaces are created within the bounded rectangles. Parameters can be readily adjusted to visualize the results for various scenarios and images. By introducing a normalized height vector, and extruding the 0.9 x 0.9 unit rectangular bounding curves by the vector height provides a 3D model which contains all the radius values. The visual characteristics of the models can be modified by altering the normalizing and scaling values, as well as the parameters modified here. As a scripting language and CAD tools are utilized to implement this work, there are time lags when N is large. However, while working with these limitations, 3D RQ plots and dynamic models can be readily generated once the limitations of the tools are known.

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