Title:
An Extended Solid Modelling Kernel for Combined Analytic and Mesh B-Rep Faces

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Introduction:
In several application fields, ranging from computer graphics to industrial design, 3D modeling and production systems, users face the design and editing of 3D virtual geometric models. 3D models can be classified in two families: polygonal models, that represent real shapes at different levels of approximation, and analytical models, which are representations of nominal mathematical shapes, usually formulated as geometrical primitives and Non-Uniform Rational B-Splines (NURBS) [7, 15]

Each family has its own modeling pipeline. Polygonal models could be the result of a conversion of an analytical model, an acquisition process through 3D scanners, the extraction of the external boundary of voxel-based representations, such as tomography applications, virtual sculpturing systems and topological optimization software. Oppositely, analytical models are typically created by conventional computer aided modeling systems, which are nowadays diffused and consolidated on the market to transpose shape concepts into 3D models.

Up to now, only limited research efforts have been devoted to put together these two families of models [2], while the common practice is the conversion of one type into the other. However, the conversion of geometric primitives often implies expensive computations and possible loss of information. Given the usual complexity of the shapes represented by polygons, their conversion to analytical models is onerous and often unfeasible [3, 11]. On the other hand, converting continuous surfaces into polygonal meshes through tessellation prevents the representation from easy editing process in the following design stages and loss of explicit geometrical information.

The paper reports a research aiming at integrating designed models, represented by continuous surfaces, and digital models, represented by polygonal meshes, in a unique 3D model in which entities such as NURBS analytical surfaces and meshes coexist. This would close the gap existing between designed and digital models and it would simplify many processes that nowadays require the conversion of one representation in the other.

To support this new modeling paradigm an Extended Solid Modeling System (ESMS) is proposed. It relies on Boundary Representation (B-Rep) of solid models where faces are described by different kinds of representations, both continuous and discrete, i.e. NURBS surfaces and meshes. The representations coexist, interact and, since they do not have to be converted into a common form, they always keep their shape features and properties. The regions of the model represented by meshes maintain their faithful compliance to the real data, while those represented by continuous models are easily editable.

The possible applications of such ESMS spread through cultural heritage, medical science, passing through industrial design and engineering applications. In the cultural heritage applications, analytical models representing regular architectonic elements are often combined with decorative patterns and artistic shapes, acquired by 3D scanning systems. In the biomechanical field, for instance in the implant design, plastic surgery or maxillofacial surgery, the ESMS could be used to integrate the digitalized parts of the human patient with parts modeled by a biomedical designer, such as in a prosthesis or dental implant. This would make the design and prototyping processes quicker, cheaper and more efficient (Fig 1a). Finally, in the emerging field of the Additive Manufacturing (AM), topological optimization is an effective way to design parts minimizing material consumption and then part weight. However, such algorithms produce freeform polygonal shapes from the intrinsic voxel-based representation which must be combined with analytic shapes at the interfaces with other mechanical components (Fig 1b).

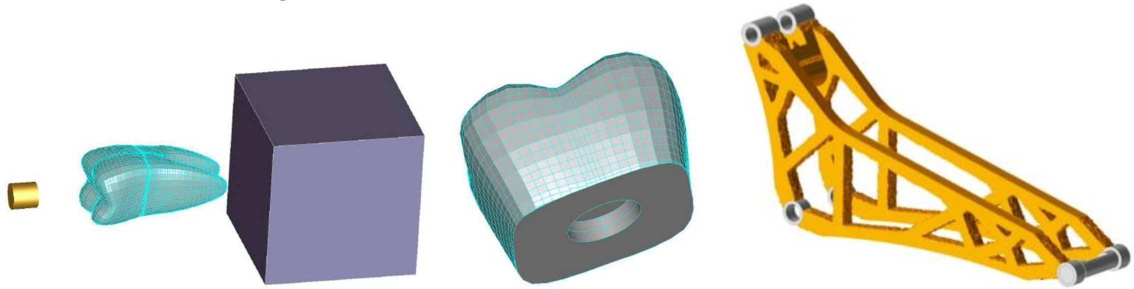

Fig. 1: Examples of possible uses of extended solid models: (a) Dental implant definition by Boolean operations among analytic and mesh solids, (b) Model of a stirrup obtained by topological optimization combined with connections modelled as analytic solids.

The research has been focused on the following goals:

- The study and design of a theory for the new paradigm of the ESMS;
- The development of tools to extend a traditional solid modeling kernel, aimed at integrating the new primitives and the new paradigm.

Since digital models are represented by meshes, the first goal aims at an extension of the representation scheme to include a new entity, i.e. the Mesh-Face. Then, an extended Boolean Operation has been introduced investigating the mesh/NURBS intersection algorithm, in order to realize new mixed models in which NURBS and meshes coexist.

To control the quality of the model, especially in joining operations between faces of different type, it has been necessary to include a notion of continuity for Extended models, called Approximate Geometric continuity (AG), and a set of conditions to guarantee that extended models be manipulated keeping prescribed continuity constraints between their constituting patches.

The Extended Solid Modelling System

The structure of a solid modeling system can be subdivided in three parts: the representation data structure, the mathematical foundations and the algorithms necessary for the applications. The representation data structure is the scheme used to represent a virtual 3D model. The mathematical foundations are all those abstract concepts that allow a physical object to be idealized and approximated. Such concepts consider the shape as a perfect and homogeneous 3D point set, ignoring internal structures and boundary imperfections. Furthermore, continuity between geometric representations is considered in order to join solid objects. These are the basis for the algorithms necessary to model the object, i.e. tools to represent, modify and interrogate solid objects.

The Mesh-Face and the Extended B-Rep

In the B-Rep model, a face is the topological element connected to a geometric entity in the form of a plane, an analytic surface or a NURBS surface [12, 14]. The proposed Extended B-Rep (EB-Rep) exploits all these kinds of primitives, adding a new type, i.e. the Mesh-Face. A Mesh-Face consists in a mesh of polygonal facets with boundaries associated to other faces of the EB-Rep representing a solid object.

The proposed scheme includes both the trivial cases, where the solid is described by only one Mesh-Face, and the more general cases, where the Mesh-Face, delimited by a closed polyline, represents one face of the EB-Rep. The Mesh-Face is handled exactly as a standard face in a B-Rep data structure. The loop of the face is defined by the polygonal boundary of the Mesh-Face.

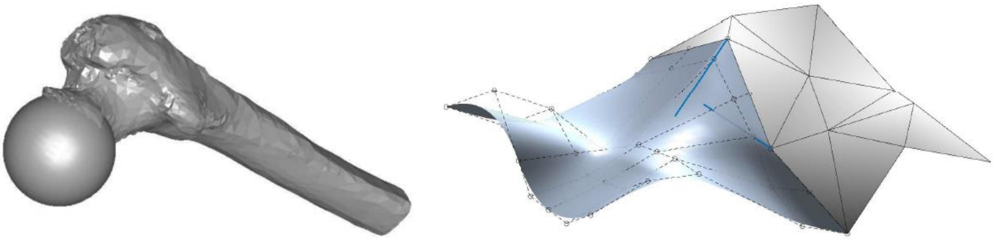

Fig. 2: Example of EB-Rep representing a femur (a) and of G1-AE Continuity (b).

Therefore, the following definition can be given:

Definition 1 (Extended B-Rep). An Extended B-Rep is a representation scheme Be = (Ge; T) where the geometry is described by Ge = (V; E; Fe) and the set of the faces Fe admits also Mesh-Faces.

An example of EB-Rep representing a femur composed with a NURBS surface and a Mesh-Face is shown in Fig.2(a).

This new data structure has to maintain the same properties of the standard B-Rep, in particular the same topology T, and to provide the same tools while holding the new potential for Mesh-Face primitives.

EB-Rep form of a Valence Semi-Regular Mesh

An Extended B-Rep paradigm can be realized as a new data structure in an ESMS. However, a most typical scenario could require the integration of the Mesh-Face primitive into an existing solid modeling system based on a classical B-Rep paradigm. In this case the data structure cannot be modified and thus finding an alternative way to represent a mesh in a standard B-Rep data structure becomes necessary. The most intuitive way to represent a mesh surface is to associate a plane to every face. A new approach, suitable for quadrilateral valence semi-regular meshes, is here proposed. If a triangular mesh is considered, a conversion in a quadrilateral valence semi-regular mesh is performed [16]. The original mesh is subdivided into submeshes with rectangular topology. Then the E-BRep structure is created associating bilinear NURBS surfaces to each submesh.

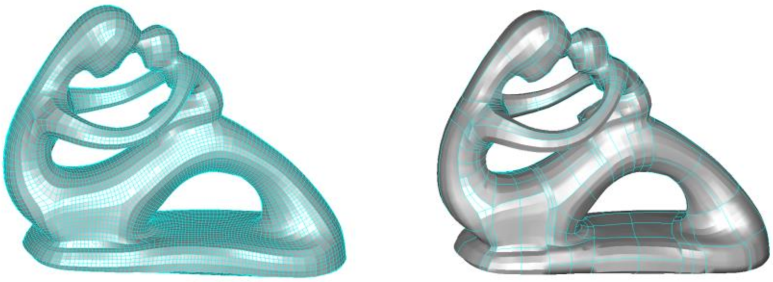

Fig. 3: Example of polygonal mesh (a) and the resulting EB-Rep by the QMTP algorithm (b).

Two variants, called Quad Mesh Patching (QMP) and Quad Mesh T-Patching (QMTP), have been defined. The first method creates rectangular patches without T-Junctions, while the second one allows T-Junctions. In both methods, the subdivision is created considering extraordinary vertices (EV) of the original mesh and tracing "straight" polylines from an EV to another. In QMP every polyline ends when an EV is reaches, while in QMTP a polyline ends when another polyline is intersected, creating a T-Junction. Finally, the boundary edges of every rectangular patch are determined and a bilinear NURBS surface is associated to them. Control points of every NURBS surface are the vertices of the original mesh inside the region delimited by the boundary edges of the associated patch.

An example is illustrated in Fig.3 in which a mesh representing the Fertility statue (3357 faces and 3351 vertices) is represented with an E-BRep made of NURBS surfaces (132 faces and 191 vertices) applying QMTP method.

Continuity for an EB-Rep model

The notion of continuity between Mesh-Face and NURBS entities has to be investigated, because it is necessary to define a new concept of continuity between smooth and discrete entities [8]. In fact, in EB-Rep it is impossible to create an exact smooth join between a Mesh-Face and a NURBS surface [4, 6]. Meshes are piecewise linear approximations, under a given tolerance, of analytic surfaces, thus it is only possible to give some less restrictive conditions in order to obtain a join that is smooth under a given tolerance.

A definition of continuity between discrete and continuous entities, the Approximated Geometric (AG^0 and AG^1) continuity is given in [2] and formalized in [9]. Moreover, the following definition has been considered:

Definition 2 (G1-AE continuity). Given a surface $s(u; v)$ with a boundary curve $c(t)$ and a mesh M bounded by a polyline p with vertices $p_1, \ldots, p_n$. Let M and s be joined C^0 along p and c. M and s join with G^1-Almost Everywhere Continuity along c and p if and only if they are G^1 connected along all points of c except at $p_1, \ldots, p_n$, where c can not be differentiable.

An example of G^1-AE Continuity is illustrated in Fig.2(b) where a NURBS surface, on the left, and a mesh on the right, join with G^1-AE continuity. The surfaces have C^0 continuity along the boundary curve and G^1 continuity everywhere except at the vertices of the mesh on the boundary polyline.

The definitions of AG^0 and AG^1 coincide with the main idea of numerical approximation. Instead, the introduced G^1-AE Continuity definition allows joining operators to be smooth except for a finite number of points and in these points the two tangent vectors have a distance angle that is related with the dihedral angle between the adjacent polygons.

Operators for the Extended B-Rep

In this section, Boolean Operations, Cutting and Join Operators are introduced for the EB-Rep, allowing for basic solid modelling functionalities.

The concept of Regularized Boolean Operation is maintained from the traditional approach [8], in which the result of the operations are the closure of the operation between the interior of the two processed solids in order to eliminate the remaining lower-dimensional structures. Similarly, the cutting operation between a solid and a surface, which results in two distinct solids, is considered.

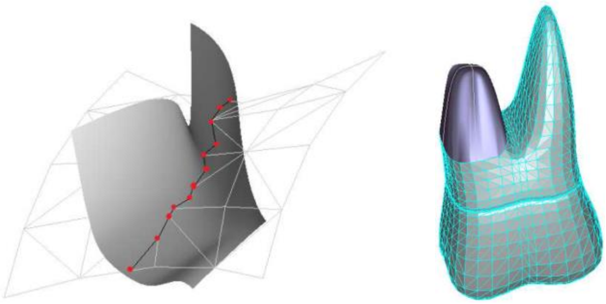

Fig. 4: Intersection between a NURBS and a mesh (a); 1-n Face-Join (b).

The main issue regards the necessity to handle more intersection cases, i.e. NURBS-NURBS, mesh-mesh and NURBS-Mesh. Traditional algorithms are employed for the first two cases [1, 5, 10]. The NURBS-

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Mesh intersection is the novel case to be considered. In particular, the intersection curves between a Mesh-Face M and a NURBS surface s are respectively a polyline p that bounds the Mesh-Face and a NURBS curve c that bounds the NURBS surface. The result surfaces are a trimmed NURBS and trimmed Mesh-Face (Fig. 4)(a). As it can be noticed, the polyline is C^0 with the Mesh-Face, while is AG^0 with the NURBS surface. The precision of the intersection polyline respects the tolerance used in the surface-surface intersection algorithm and depends from the tolerance associated to the AG^0 continuity.

Join operator

The Join Operation attaches two entities changing one or both entities according to AG^0/AG^1 or G1-AE continuity. The possibility of modifying the entities makes this tool differ from Boolean Operation, in which both entities are fixed. We distinguish between *1-1Face-Join* operation, *1-n Face-Join* and *n-m Face-Join* of open solids. *1-1 Face-Join* matches two surfaces along an edge, creating a connected surface. *1-n Face-Join* closes the hole of an open solid with a surface (see Fig.4(b)), while *n-m Face-Join* matches two open solids closing a hole on both entities.

Conclusions:

The paper presents the Extended Solid Modeling System to manage both Mesh and NURBS entities and represents solids obtained by modeling these entities using an Extended B-Rep scheme. A prototype of the geometric solid modeling kernel has been implemented in the OpenCascade [13] environment. The proposed algorithms has been then tested on some models demonstrating the feasibility of the approach.

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