Title:
Printability Analysis in Additive Manufacturing

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Introduction:
An important issue in Additive Manufacturing (AM), which to this date is still not well controlled, is predicting the build accuracy and its relationship to the total build time. For example, in Fusion Deposition Modeling, the accuracy of the as-manufactured geometry can be partially controlled in-process by choosing a convenient in-fill pattern [6] or modulating the printing head speed and nozzle temperature around small area features. Though, the effects of this process optimization are ultimately limited by the kinematic configuration of the AM machine as well as by the geometry and size of the extruder. These factors have the greatest potential for improving the build accuracy and, specifically, without negatively impacting the build time. The nozzle's shape has been traditionally constrained, unnecessarily, to a circular shape. On the other hand, the smallest printable feature, or print resolution [2], is fundamentally limited by both the shape and dimensions of this extruder. This resolution can be improved locally in-process by filament flow rate or under/over extrusion [11], but the extruder geometry remains the largest contributing factor to the build accuracy.

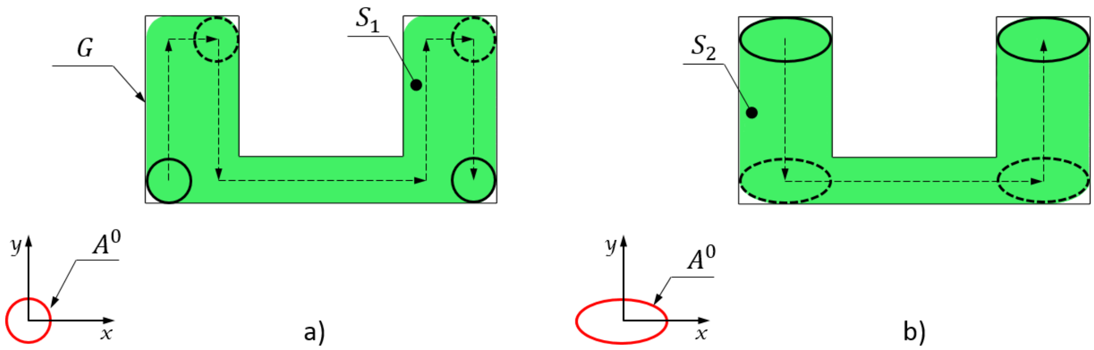

Fig. 1: Solid fill of a target contour G, with two extruders: (a) circular, (b) elliptical, in their initial configuration A^0. The motions allowed by the AM machine are two translations in the XY plane. Figures (a) and (b) show the simplest time-based motion of the nozzle (shown with dashed lines) which will fill the target contour without exceeding its boundary. The "as-manufactured" geometry is given by the solid sweeps S_1 and S_2. Although the elliptical extruder is larger in area than the circular one, it can still resolve small features (such as the narrow passage) due to its asymmetric shape.

We present a generic methodology for computing the as-manufactured geometry in additive manufacturing, which, in turn, provides a ranking of given nozzle geometries in terms of their corresponding build accuracy and subsequently build time. To illustrate the concept, Fig. 1 shows a conceptual example of a U-shaped contour *G*, which must be printed as close as possible to nominal dimensions, without exceeding the contour boundary. In Fig. 1(a) the nozzle has the usual circular opening, while in Fig. 1(b) the nozzle is elliptical. The elliptical nozzle in this case achieves a similar accuracy for the as-manufactured geometry and at the same time requires a smaller number of linear interpolations to cover the same target contour. In this research, we use the term "build accuracy" as the volumetric deviation of the build model when compared to the nominal, or target geometry. Even though the elliptical extruder has a larger surface area, it is still able to navigate through the narrower regions of the target contour due to its asymmetric design. In the examples section we show how an asymmetric extruder combined with a machine which allows rotations can vastly improve the accuracy by resolving smaller features in any orientation. For the example of Fig. 1, it is apparent that the elliptical extruder ranks higher than the circular one. However, when an arbitrary geometry to-be-printed is involved, build layers are non-planar, or the machine's kinematics includes rotations, the choice of extruders cannot be assessed simply by visual observation. Our method relies on the established properties of the inverse trajectory of a point [5] summarized in the next section. In Figure 1, the motion is defined by two parameters, the XY generalized coordinated of the printing head, thus we refer to the printing head as having a two-parameter motion. For clarity, we focused on only one layer thus the machine's layer advance on the Z axis is ignored.

By examining the intersection set between the nozzle geometry and the inverse trajectory of points sampled from the target volume we determine the total set of motion parameters for which the nozzle is permanently contained within the target contour. This allows us to compute the volume of the as-manufactured geometry as a sweep, defined by the nozzle's "smallest printable feature" [2] and the restricted *p*-parameter motion (assuming a constant extrusion flow and constant velocity of the printing head). We can then compute the volumetric deviation between the built and the nominal volume, as a Lebesgue measure defined over the set difference between the two respective volumes. Given this restriction, the problem of motion synthesis, which will be covered in a subsequent paper, recasts in our framework as a space filling curve problem [1], under certain build quality requirements, such as a controlled overlap [8]. A paper examining the properties of as-manufactured parts for AM can be found in [9].

<u>The property of the inverse trajectory:</u>

In general, the trajectory of a point X is defined as $T = \bigcup_{m \in M} X^m$ where $m \in SE(3)$ are the instantaneous transformations of an arbitrary rigid-body motion *M*. If the motion is time dependent, all generalized coordinates that control the motion (joint angles, prismatic translations, etc.) are coupled together by *time*. Hence a *time*-dependent motion is in fact a *one*-parameter variety. By eliminating time, and consider all generalized coordinates as independent of each other, we obtain a non-physical motion, called multi-parametric [3,7]. In the most general case, the trajectory of a point moving according to a one-parameter motion is a spatial curve, for a two-parameter motion is an *n*-dimensional surface, or respectively a volume if the number of parameters is three or more. It is worth noting that the multi-parameter trajectory of a point is the superset of all one-parameter trajectories that can possibly be synthesized in a time-based reference system, which is why in the following paper we explore the problem of time-based motion synthesis using this generic formulation of multi-parameter motions.

By applying the inverse of these transforms $\hat{m} \oplus m = id_3$ to a sample point $Y \in E^d$, where id_3 is the Identity transformation, we obtain the so-called inverse trajectory of the sample point *Y*, similarly defined as $\hat{T}_Y = \bigcup_{\hat{m}=m^{-1}} Y^{\hat{m}}$. In this formulation, $\oplus$ is the group additive operator; for example, if the transformations are represented as homogeneous transformation matrices, then the group operator is the usual multiplication of matrices. The property of the inverse trajectory states that only the points $X \in \hat{T}_Y$ will pass through the sample point *Y* during the motion *M* [4,5]. This property can be readily verified from the definition of the inverse transformation $\hat{m} \oplus m = id_3$. This property applies without restrictions to the number of motion parameters and the dimensionality of the Euclidean space where the points are embedded since it has a fundamental set theory formulation.

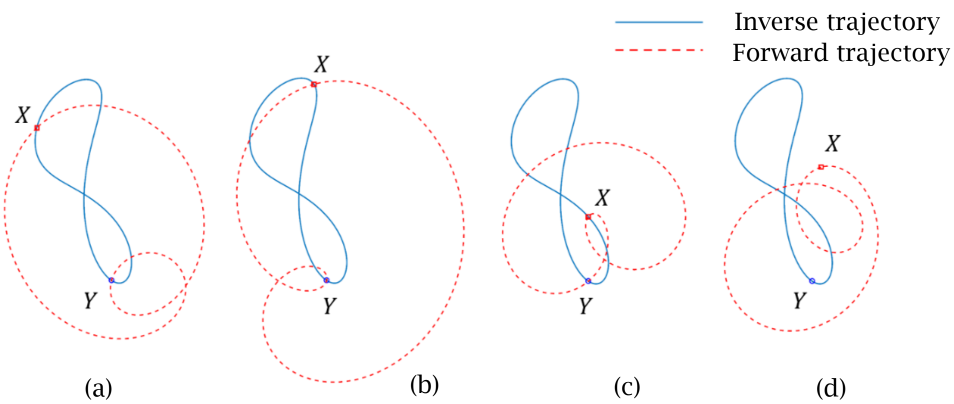

Fig. 2: The inverse trajectory of a random point $Y \in E^2$ shown here in solid blue color. Fig. 2(a) through 2(c): the forward trajectory of any point $X \in \hat{T}_Y$ intersects point Y. Fig. 2(d): in the Euclidean space there are no other points $X \notin \hat{T}_Y$ which will pass through Y during the forward motion.

To provide a visual aid, Fig. 2 shows in solid color the one-parameter inverse trajectory of an arbitrary sample point Y, embedded in a two-dimensional Euclidean space. The point Y is contained in the forward motion of any arbitrary point $X \in \hat{T}_Y$. Moreover, it has already been shown that the particular configuration m_i where forward trajectory of point X intersects the sample point Y, is given by $X = Y^{\widehat{m_i}}$.

Formulation:

As stated in the introduction, in this paper we seek to derive a generic method for ranking known extruder geometries, of arbitrary design, based on their corresponding build accuracy, given a certain target geometry we wish to reproduce. We formulate our ranking criteria as a Maximum Material Condition (MMC). This tolerance condition, standardized in ISO 2692 and ISO/TC 213 [10], enforces that no material can be deposited *outside* the target boundary. In effect, the highest-ranking extruder will have the smallest MMC tolerance, or in other words, the largest volume of deposited material without exceeding the target boundary of the nominal geometry.

The key to our formulation is to compute a restriction of the printing head's motion, represented as a set of instantaneous configurations, for which the nozzle is fully contained in the target domain. We represent all motion configurations as points in a p–dimensional linear space where p is the number of generalized coordinates. This "parametric space" is denoted as Ω, thus a restriction of the motion is nothing else than a set $\mathcal{S} \subset \Omega$. To eliminate any confusion, we denote sets of the usual Euclidean space with an uppercase letter (i.e. A, X, S, etc.) and sets of the parametric space with a stylized font (i.e. $\mathcal{B}, \mathcal{C}, \mathcal{P}$, etc.). The trajectory T_X of a point X is simply the union set of the instantaneously transformed points $Y = X^{M(\boldsymbol{u})}$, where $\boldsymbol{u} \in \Omega$ is an instantaneous motion parameter. Therefore, the inverse trajectory $\hat{T}_Y$ of an arbitrary point $Y \in E^d$ contains all points $X \in E^d$ such that $T_X \equiv Y$, for some parameter $\boldsymbol{u} \in \Omega$. In our case, if $Q \in G$ is a point of the target geometry $G \subset E^d$, and $A^0 \subset E^d$ is the nozzle geometry in its initial configuration, the parameter values $\boldsymbol{u}$ at which the inverse trajectory $\hat{T}_Q$ intersects the nozzle geometry A^0 are precisely the configurations at which the nozzle A intersects the target G during the given (forward) motion. This inversion allows us to build the set of configurations $\mathcal{C} \subset \Omega$, , where $\mathcal{C} = \bigcup \boldsymbol{u}_Q$ for all $Q \in G$, for which the motion generator (the extruder in our case) sweeps over the target geometry. In other words, the parametric range $\mathcal{C}$ contains all configurations where the nozzle can potentially deposit material over the target G. The complete set $\mathcal{C}$ also contains the configurations for which the extruder only partially intersects the target geometry. This is unwanted since we want to enforce an MMC tolerance. Therefore, these undesirable configurations, which we will denote with $\mathcal{B} \subset \Omega$, must be subtracted from $\mathcal{C}$. The configurations $\boldsymbol{v} \in \mathcal{B}$ are those parametric values for which the extruder intersects the boundary of the target geometry. This observation follows naturally from the partitioning of space. Hence, the set of configurations $\mathcal{B}$ is given by $\mathcal{B} = \bigcup \boldsymbol{v}_P$, where $P \in \partial G$. The parametric set which will enforce the MMC tolerance is given by $\mathcal{P} = \bigcup_{q \in G} \boldsymbol{u}_q \setminus \bigcup_{p \in \partial G} \boldsymbol{v}_p$, or in short $\mathcal{P} = \mathcal{C} \setminus \mathcal{B}$. This restriction allows us to determine the as-manufactured volume corresponding to the MMC tolerance as a solid

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sweep $S_{MMC} = \bigcup_{t\in\mathcal{P}} A^t$. Finally, the given nozzle geometries are ranked automatically based on their corresponding build accuracies formulated as the smallest Lebesgue measure $\min[\lambda(G \setminus S_{MMC})]$, taken over the set difference between the nominal and the manufactured geometry.

Examples:

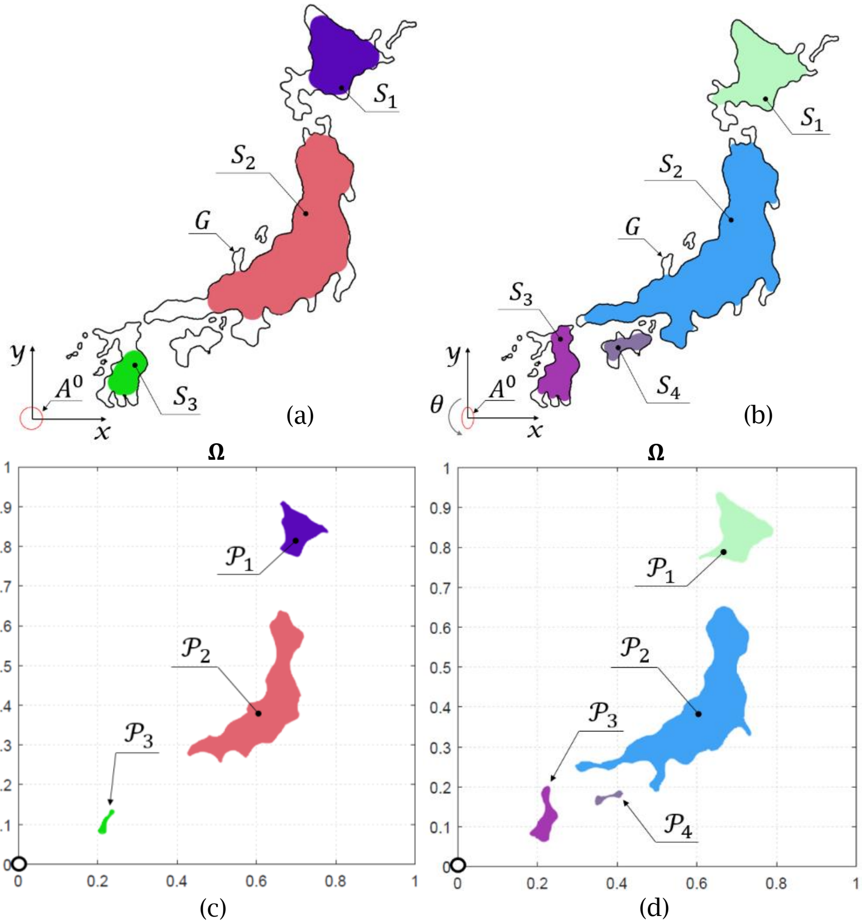

Fig. 3: Planar as-manufactured models and their motion restrictions: (a) the circular extruder A^0 executes a planar *XY* motion, (b) the elliptical extruder is attached to a T-T-R machine. The target contour contains many small features that are comparable in size with the dimensions of the extruders. The parametric domain Ω in Fig. (c) corresponds to the example in Fig. (a), and Fig. (d) corresponds to Fig. (b).

In Fig. 3 we show a planar example where we observe an increase in resolving power when we attach an elliptical extruder to a T-T-R machine. The target geometry in this case is a sketch of Japan's coastline. This contains many small features which would otherwise be difficult to resolve with the circular extruder from Fig. 5(a). The extra DOF in Fig. 5(b) allows the print heat to reorient the extruder about its *Z*-axis and thus eliminating any variability in the build resolution for the entire printer workspace. The restrictions $\mathcal{P}_{1-4}$ are those configurations for which the circular extruder is permanently contained within the target domain *G*. The example also shows that our formulation has an inherited capability of processing *p*-parameter motions and is also able to detect build volumes with disconnected components. This advantage can be used for building the model simultaneously with two or more extruders fitted to the printing head. With the setup in Fig. 5(b) we achieved a 17.62% increase in combined build volume, when compared to the setup in Fig. 5(a).

Conclusions:

In this paper we present a generic method for computing the as-manufactured geometry in additive manufacturing, which, in turn, provides a ranking of given nozzle geometries in term of their corresponding build accuracy and subsequently build time. While the details are presented in an accompanying archival paper, this method is applicable to printing on planar and non-planar surfaces, with extruders of arbitrary geometry and with machine kinematics that have multiple degrees of freedom, including rotations. We enforce the Maximum Material Condition as a dimensional tolerance to have a meaningful comparison of build volumes. In effect, this tolerance acts as a maximizer of the built volume while ensuring that the nominal boundary is not exceeded. We exemplify our method with an elliptical extruder as an alternative to nozzles with a circular opening, although our generic method accepts any arbitrary geometry. This formulation is derived in set theoretical terms and uses the concept of the inverse trajectory, which allows it to be implemented in virtually any geometric representation which supports distance computations. Our method does not include time in our derivations, instead we treat the motion as a set of configurations parametrized by the mechanism's generalized coordinates. The ability to calculate the motion restriction which will prevent the nozzle to exceed the nominal boundary can be viewed as a precursor to motion synthesis. Any one-parameter coupling of all generalized coordinated is represented in this parametric space as a curve. Therefore, a re-parametrization of this motion under time (i.e. motion planning) recasts itself as a space spanning curve in a p-dimensional space, given certain quality requirements. This formulation does not enforce the planarity of the build layer, which makes our approach suitable for layer-less AM technologies such as 6-axis Fusion Deposition Modelling and laser jet cladding.

Acknowledgements:

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