Title:
Linkage-FFD Algorithm for Dental Crown and Abutment Shape Design

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Introduction:
Dental CAD system [3], [6] has been widely used in clinical trials, greatly improving the success rate of surgery and shortening the operation cycle. For dental CAD, one of core functions is to model the crown shape. In current dental CAD packages, designing a crown includes two steps: 1) first take a corresponding standard tooth from the database and 2) repeatedly fine-edit it using some shape deformation algorithm to obtain the final ideal shape. So far, lots of research work has been done on the crown design, these studies only focus on the outer surface design of the crown, not synchronically considering the thickness between the designed crown and its abutment (the designed crown will further be glued to the abutment). In the crown design, meeting the standard thickness is a must [4]. As far as we know, all dental CAD systems complete the design of a crown without thickness consideration and the abutment-crown thickness requirement is guaranteed later by mutually adjusting the abutment shape with great care [5] or even repeatedly modifying the shape of already designed crown. This method requires more operating time and skilled experiences, and sometimes the trade-off between the crown thickness and crown shape should be made. If this compromise is not properly dealt with, cervical margin collapse may occur or the high quality occlusal morphology is difficult to obtain. The thickness between the abutment and corresponding crown should be in the range [1] shown in Fig.1.

Fig.1: The thickness distribution requirement for different parts of crowns.
The thickness requirement is varied at different parts of the crown: the thickness of the axial surface part is 1.0~1.5, but the thickness of the occlusal surface part is relatively thicker (1.5~2.0mm) due to bear the bite function. Too less thickness amount will result in tooth's strength not being enough, but too large thickness number may cause the sintering problem. Besides the thickness requirement, the abutment should hold the similar anatomical shape with the designed crown, which is conducive to further porcelain process and good mechanical properties. The designed crown's inside shape is nearly the same with the abutment shape and 0.05mm gap between them is made for adhesive to glue the crown and abutment together. In other words, once the shape of abutment is determined and the inner shape of the crown can be directly made from the abutment by surface-offset operating.

**Main Idea:**

The design of crown procedures in our system consists of the following steps: 1) vertex classification; 2) the crown shape modification using the algorithm DMFFD; 3) automatic adjustment on the corresponding abutment shape ensuring the thickness requirement. These steps are briefly introduced one by one in the following subsections.

**Vertex classification**

As mentioned before, different part of tooth needs various thickness numbers, so we first determine the part classification of all vertices of the crown mesh model. For each crown model, one local coordinate system is first constructed by splitting it into six anatomical sides: medial and distal sides, buccal and lingual sides, and gingival and occlusion sides. The result of local coordinate system and the six anatomical sides are shown in Fig.2 and Fig.3, respectively.

![Fig. 2: Build the local coordinate system for the input incisor and molar model: (a) the isometric view of incisor; (b) the top view of incisor; (c) the isometric view of molar; (d) the top view of molar.](image)

![Fig. 3: These bounding boxes for (a) an incisor and (b) a molar.](image)

Now, the classification of vertices can be simply determined in a simple way: connect each vertex and the origin of the local coordinate as one line segment, which will hit one plane of the bounding box; if the intersection point is on the top plane, this vertex will belong to the occlusal surface part, otherwise it belong to the axial surface.

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Direct manipulation of free-form deformation (DMFFD)

DMFFD algorithm [2] is directly used to design the crown shape in our system due to its excellent performance and operating easiness. DMFFD is evolved from the free-form deformation. In the FFD algorithm, a control lattice shown in Fig. 4 is used to control the shape deformation.

The lattice includes one frame denoted as $S$, $T$ and $U$ and the origin is denoted as $X_0$ which is coincident with the origin $O$ of the local frame in Fig. 4. For each model vertex $V$, a triple coordinate denoted by $(s, t, u)$ in the frame is represented as below:

$$V = X_0 + sS + tT + uU$$  \hspace{1cm} (1)

The coordinates $s$, $t$, $u$ are evaluated as Eq. (2):

$$s = \frac{T \times U \cdot (V - X_0)}{S \times U \cdot S}, \quad t = \frac{S \times U \cdot (V - X_0)}{S \times U \cdot T}, \quad u = \frac{S \times T \cdot (V - X_0)}{S \times T \cdot U}$$  \hspace{1cm} (2)

Where $0 < s < 1$, $0 < t < 1$, $0 < u < 1$, ‘$\cdot$’ and ‘$\times$’ stand by the vector dot product and cross product, respectively. The lattice control point $P_{ijk}$ is calculated by Eq. (3):

$$P_{ijk} = X_0 + \frac{i}{l}S + \frac{j}{m}T + \frac{k}{n}U$$  \hspace{1cm} (3)

Where $l$, $m$ and $n$ are the numbers of the control lattice grids in the $S$, $T$ and $U$ direction, respectively. In Fig. 4, $l = 1$, $m = 2$, $n = 3$. The model vertices $V(s, t, u)$ can be updated by the lattice control points using Eq. (4)

$$V = \sum_{i=0}^{l-1} \sum_{j=0}^{m-1} \sum_{k=0}^{n-1} [[(1-s)^i] / \binom{l}{i} \cdot [(1-t)^j] / \binom{m}{j} \cdot [(1-u)^k] / \binom{n}{k}] \cdot [V_{ijk}]]$$  \hspace{1cm} (4)

It can be seen from the above formula, the modification of these control points in lattice can change the shape of the model, but this scheme is not intuitive and one hopes to change the shape in an easier manner, in which one specific vertex of the model is selected and moved to the expected position. In order to meet this requirement, the DMFFD algorithm is proposed in [2]. The Eq. (4) can be rewritten in a matrix form, i.e., $V = BP$, where $P$ stands for the matrix recording all control points, $B$ represents the Bernstein basis matrix and the symbol $\Delta V$ is the modification of $V$. In DMFFD, Eq. (5) is used to evaluate the movement amount of the control points of the lattice.

$$\Delta P = B^* \Delta V$$  \hspace{1cm} (5)

Where $B^*$ represents the pseudo-inverse matrix of $B$ and $P$ will be replaced by $P = P + \Delta P$. After updating $P$, all the model vertices will be re-calculated using the Eq. (4) with their corresponding $(s, t, u)$ coordinate one by one. In our package, the above algorithm is used to design the shape of the crown by editing the initial standard tooth from the database.
**Linkage Free-form Deformation Algorithm (L-FFD)**

For our specific application, linkage free form deformation (L-FFD) algorithm is proposed to guarantee the crown-abutment thickness requirement. In the proposed algorithm, the shape modification of the crown is proceeded using the above DMFFD algorithm and its deformation information together with the thickness constraint is considered when automatically deforming the corresponding abutment. The process flow of L-FFD algorithm is shown in Fig. 5.

![Flowchart](image)

**Fig. 5:** The flow of L-FFD.

As shown in Fig.5, this paper introduces a monitoring mechanism, which can real-time monitor the crown-abutment thickness during the deformation procedure. Once the thickness is less than the minimum number, the shape of abutment will be automatically adjusted within a preset region to meet the thickness requirement, which will ensure the validity of the crown design. The abutment shape will be deformed using direct manipulation of free-form deformation (DMFFD). After adjusting all unsatisfied points on the abutment, the surface of abutment may be not smooth, so the laplacian smoothing step will be performed. These three steps including thickness detection, modification by DMFFD and smoothing are alternately and iteratively done until the thickness requirement is met. In practice, after repeating 3 to 5 times, the ideal thickness can be achieved and the surface of abutment is smooth enough. Fig.6 shows the final results by the proposed algorithm and Tab.1 gives the statistics of the running time showing that it can get an instant response, which is important for the crown design.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Vertex number/triangle number</th>
<th>DMFFD for crown (ms)</th>
<th>DMFFD for abutment (ms)</th>
<th>Thickness detection (ms)</th>
<th>Smoothing (ms)</th>
<th>Number of iteration</th>
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<td>9084/18088</td>
<td>78</td>
<td>78</td>
<td>5</td>
<td>20</td>
<td>3</td>
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<tr>
<td>Fig.9 (d)</td>
<td>12798/25843</td>
<td>98</td>
<td>98</td>
<td>5</td>
<td>21</td>
<td>5</td>
</tr>
</tbody>
</table>

Tab. 1: Running time for the L-FFD algorithm.
Fig. 6: The thickness distribution after the L-FFD algorithm: (a)&(d) the initial abutments and their corresponding standard tooth model; (b) & (e) the results after L-FFD are smooth enough; (c)&(f) the distance distribution map of (b)&(e), showing that the thickness is within the required ranges.

References: