Title:
Direct Simulation of Geometrical Models using the Finite Cell Method

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Introduction:
A typical task of an engineer – be it in civil, mechanical, aeronautical engineering and many other fields – is to design and develop new objects, which fulfill certain physical requirements, generally described by partial differential equations (PDE), e.g. for structural problems, heat transfer, fluid dynamics, etc. The realization of an optimal design naturally requires an iteration between geometric design, typically carried out by Computer Aided Design and numerical simulations, such as the Finite Element Method. Unfortunately, CAD models are not directly compatible with a numerical simulation, and thus need to be translated into a simulation suitable format, e.g. a mesh. This transition process is considered to be the bottleneck in the design process and has initiated the development of several new simulation techniques, which allow working directly on the CAD model.

The most prominent approach is Isogeometric Analysis (IGA) [1], which relies on the idea of using the same type of functions (e.g. B-Splines, or NURBS) for the description of the geometry and for the numerical approximation. Whereas IGA is best suited for dimensionally reduced models, such as shells, immersed boundary- or fictitious domain methods fit better to general solids, particularly those which are described by Boundary Representation (B-Rep) or by Constructive Solid Geometry (CSG). Very well suited in this context is the Finite Cell Method (FCM) [2], which has the advantageous property that the sole information needed from the CAD model is a reliable Point Membership Classification (PMC). FCM is a fictitious domain method employing high-order finite elements.

PMC for classical CSG models is trivial, as analytical solutions for the primitives are available. More elaborate are modern procedural modeling techniques. These are based on the CSG idea and use extended primitives such as sweeps or lofts. A PMC for swept volumes was proposed e.g. by Erdim et al. [3], based on inverted trajectories. However, the proposed PMC lacks generality concerning possible shapes provided by procedural CAD tools, e.g. changing the geometry during the sweeping process, as it is the case in lofts. PMC for B-Rep models is well-known for a long time and various methods are available, such as ray casting, swath method, signed distances, winding numbers, octree reconstruction, or point cloud methods. More recent approaches extend these methods to spline modeling techniques (see e.g. [4]). A problem concerning PMC in the context of B-Rep models is related to errors that typically occur during the modeling process, such as gaps, intersections, double entities, or wrongly oriented facets. These flaws can prohibit a reliable PMC and thus a subsequent simulation.

With the Finite Cell Method, we seek to circumvent a transition process from CSG and B-Rep to a simulation model to avoid healing and boundary conforming meshing, because both fo these processes represent a potential bottleneck in the analysis pipeline. In this paper, we present different PMC approaches for extended CSG models as well as for flawed B-Rep models.

Finite Cell Method:

The basic idea of the Finite Cell Method (FCM) is to embed a potentially complex geometry Ω_{phy} into a fictitious domain Ω_{fic} such that the resulting domain $\Omega_{\cup}$ is of simple shape which can be meshed easily. Consider as an example the problem of heat conduction described by:

$$-\mathbf{div}(\boldsymbol{C}\,\nabla\boldsymbol{T}) = \boldsymbol{Q} \qquad \forall \boldsymbol{x} \in \boldsymbol{\Omega} \tag{1}$$

$$\boldsymbol{T} = \widehat{\boldsymbol{T}} \qquad \forall \boldsymbol{x} \in \boldsymbol{\Gamma_a} \tag{2}$$

$$\boldsymbol{q_n} = \widehat{\boldsymbol{q}} \qquad \forall \boldsymbol{x} \in \boldsymbol{\Gamma_b} \quad , \tag{3}$$

where T is the unknown temperature, $\boldsymbol{Q}$ an internal heat source, $\boldsymbol{C}$ the thermal conductivity and $\mathbf{q}$ the heat flux. Following Galerkin's approach, (1-3) are reformulated to give the following weak form:

$$\mathcal{B}(\boldsymbol{u},\boldsymbol{v}) \coloneqq \int_{\Omega_{\mathrm{phy}}} \nabla\boldsymbol{v} : \boldsymbol{C} : \nabla\boldsymbol{T}\, \mathrm{d}\Omega_{\mathrm{phy}} = \int_{\Gamma_b} \boldsymbol{v}\widehat{\boldsymbol{q}}\ \mathrm{d}\Gamma_b - \int_{\Gamma_a} \boldsymbol{v}\boldsymbol{q_n}\ \mathrm{d}\Gamma_a + \int_{\Omega_{\mathrm{phy}}} \boldsymbol{v}\boldsymbol{Q}\ \mathrm{d}\Omega_{phy} = \mathrm{f}(\boldsymbol{v}) \quad , \tag{4}$$

which needs to be satisfied for all test functions $\boldsymbol{v}$ of a suitable test space (see e.g. [5]). While the Finite Element method discretizes Ω_{phy} directly in triangular, quadrilateral, tetrahedral or hexahedral elements, in the Finite Cell Method, the bilinear form of the problem is extended over the whole domain $\Omega_{\cup}$. Additionally, the constitutive (material) matrix $\boldsymbol{C}$ is multiplied by an indicator function $\alpha(\boldsymbol{x})$, changing the bilinear form $\mathcal{B}(\boldsymbol{u},\boldsymbol{v})$ to

$$\mathcal{B}_e(\boldsymbol{u},\boldsymbol{v}) \coloneqq \int_{\Omega_{\cup}} \nabla\boldsymbol{v} : \alpha C : \nabla\boldsymbol{u}\ \ \mathrm{d}\Omega_{\cup} = \mathrm{f}(\boldsymbol{v}) \quad , \tag{5}$$

with α defined as

$$\alpha = \begin{cases} 1 & \forall\, \boldsymbol{x} \in \Omega_{phy} \\ 10^{-q} & \forall\, \boldsymbol{x} \in \Omega_{fic} \end{cases} \quad . \tag{6}$$

When $q \to \infty$, (5) converges to the original weak form (4). In the context of FCM, the computational domain ($\Omega_{\cup}$) is discretized into simple rectangles or rectangular boxes whose boundaries do not necessarily conform with the boundaries of Ω_{phy}. The geometry of the original problem is recovered when evaluating the element stiffness matrices: each quadrature point is multiplied with the value of the indicator function associated to its location. This way, the only information that FCM needs from the geometric model is whether a given quadrature point lies inside or outside of Ω_{phy}. In order to avoid ill-conditioning of the discretized problem in practical computations the value of q is typically chosen between 6 and 12. For details on FCM see [2].

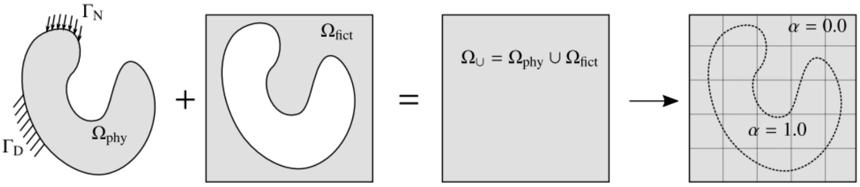

Fig. 1: Concept of the Finite Cell Method. Image from [6].

Point Membership Classification:

The Point in Membership (PMC) classification for flawless B-Rep [4] and classical CSG [7] models is straightforward and will not be revisited in this context. We present two approaches to obtain a fast and reliable PMC for extended CSG primitives and flawed B-Rep models respectively.

PMC of extended CSG primitives

A PMC on a CSG tree of simple primitives, such as cuboids, spheres, or cylinders can be carried out analytically and thus very fast [7]. Because this method is limited to a small set of geometric shapes, classical CSG is rarely used. More common are procedural modeling tools, which follow the basic CSG idea of combining primitives with the Boolean operations (difference, union, intersection). They offer an extended set of primitives (extrusions, sweeps, lofts and solids of revolution) and extended operations (fillet, chamfer, hole, etc.), allowing for a large variety of different shapes. While extended operations are in fact just a combination of primitives and the Boolean operations, extended primitives require special consideration.

Nevertheless, a fast and reliable PMC, which uses the inherent water-tightness of extended primitives, can be performed on these geometries. The basic idea is to reduce the dimension of the problem by taking advantage of the knowledge of how theses bodies are constructed, i.e. by moving 2D sketches along a curve $\boldsymbol{C}(\xi)$ (see Fig. 2).

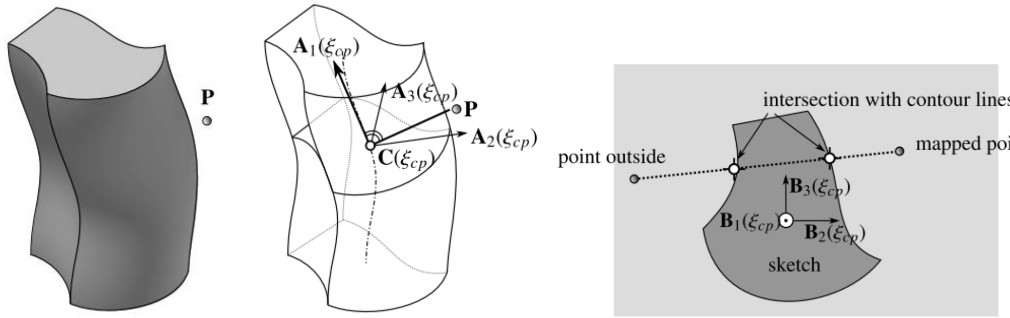

Fig. 2: Point Membership Classification on a simple sweep: (a) Sweep and point of interest. (b) Find local basis system $A(\xi_i)$ and (c) perform a PMC in 2D (here via ray-casting). Image from [6].

Let $\boldsymbol{P}$ denote the point of interest, which is in the case of the FCM an integration point. The general approach for a PMC of a simple sweep (i.e. the base coordinate system of the curve coincides with the sketch coordinate system at each point $(\boldsymbol{A}(\xi_i) = \boldsymbol{B}(\xi_i))$ reads as follows:

- Find the closest point $\boldsymbol{C}(\xi_{cp})$ on the curve $\boldsymbol{C}(\xi)$.
- Setup a local coordinate system $A(\xi_{cp})$ at the closest point following the construction instruction for the local basis system (e.g. as a Frenet basis).
- Map the point $\boldsymbol{P}$ onto the plane $\widetilde{\boldsymbol{P}}$.
- Perform a PMC in 2D on the initial sketch using e.g. ray casting or winding numbers

This idea can easily be extended to lofts, where the initial and final sketch is different. In this case, the 2D PMC test is carried out on both the initial and the final sketch. Additionally, the closest distance to each border is computed. The PMC emerges from an interpolation of theses distances according to the arc length of the closest point, w.r.t the overall length of the path.

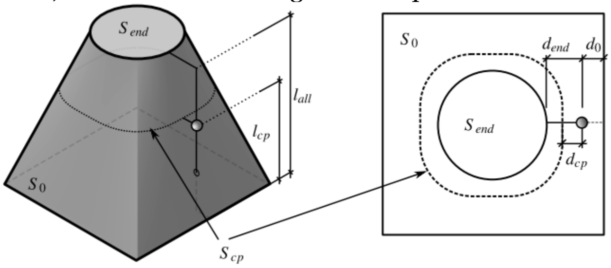

Fig. 3: Point Membership Classification on a simple loft: Interpolation of the distance to the intermediate border according to the arc length of the current curve point. Image from [6].

In the case of lofts, the property that the local basis systems of the curve $\boldsymbol{A}(\xi)$ and the sketch(es) $\boldsymbol{B}(\xi)$ coincide is typically not fulfilled. However, these cases can be evaluated similarly. To this end, the rotation of the sketch basis system $\boldsymbol{B}(\xi)$ is computed with respect to the curve basis system $\boldsymbol{A}(\xi)$ at the initial and final position. The PMC is adapted by not searching for the closest point, but for a curve position ξ where the z-coordinate of the mapped point $\widetilde{\boldsymbol{P}_z} = 0$ vanishes. The basis system into which the point $\boldsymbol{P}$ is mapped is the sketch basis system w.r.t the curve basis system $\boldsymbol{B}(\boldsymbol{A}(\xi))$. For a more detailed explanation we refer to [6].

Figure 4 depicts a procedural model containing extended primitives. Its construction history was extracted from a commercial modeling tool and converted into a neutral extended CSG format. The geometry is evaluated pointwise on the function $\boldsymbol{\alpha}(\boldsymbol{x})$ and requires no explicit boundary description. However, for visualizing results, the surface was reconstructed using the marching cubes algorithm, which needs as input, similar to the FCM, only a PMC.

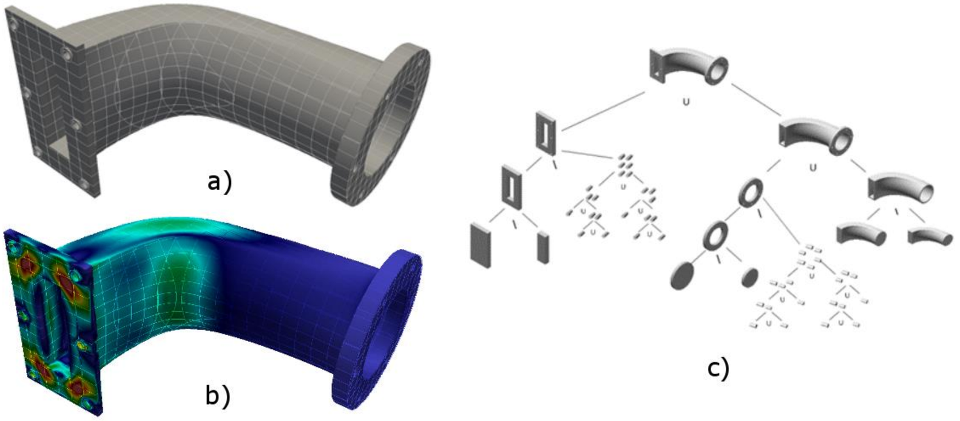

Fig. 4: (a) Surface reconstruction of a procedural model on the basis of the PMC presented herein. (b) von Mises stresses computed with the Finite Cell Method (c) Underlying CSG –tree containing extended primitives. Image from [6].

PMC of flawed B-Rep models

Due to their implicit volume representation, B-Rep models are prone to a broad variety of modeling errors, most commonly: gaps, intersections, wrongly oriented facets, or double entities. These flaws can lead to wrong results in the PMC. For example, the ray casting method is sensitive to gaps and double entities. Methods based on signed distances or point cloud techniques fail if facets are oriented wrongly, etc. In the following, we consider only 'small' flaws, which have no influence on the quality of the visualization. Nevertheless, they can render meshing or a numerical simulation impossible.

The basic idea behind the PMC on flawed geometries is an octree reconstruction based on intersections, with a subsequent membership classification using the flood fill method. Critical in this context are openings, such as gaps, which would mark the entire space to be inside (or outside). Consequently, the partitioning depth of the octree is bounded by the size of the openings, i.e. the dimension of the smallest cells must be larger than the faulty openings. An advantage of this method is the insensitivity towards other types of flaws, such as intersections, double entities, etc.

Figures 5 and 6 depict a four-sided pyramid, which possesses several flaws, including intersections, double entities, gaps and wrongly oriented facets. The maximum partitioning depth for a correct membership identification of inner, outer and cut cells is, in this case, limited to $n_{max} = 7$.

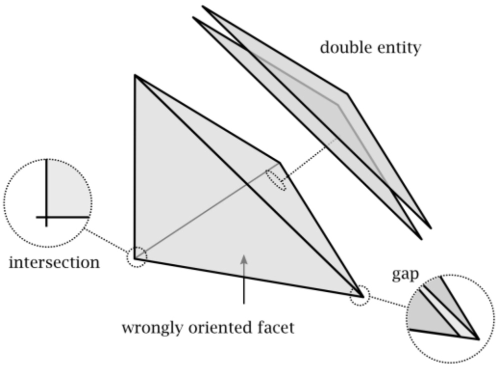

Fig. 5: Four sided pyramid with several flaws: gaps, intersections, wrongly oriented facets and double entities.

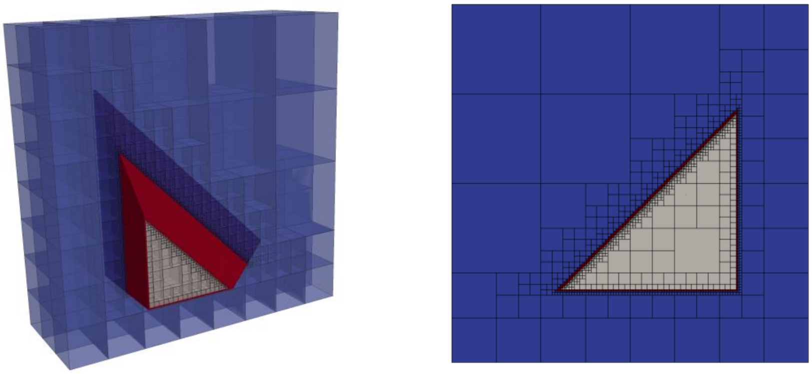

Fig. 6: Octree reconstructed model using the flood fill method of a four-sided pyramid. The outer domain (blue) is separated by the cut cells (red) from the inner domain (gray). The subdivision level is $n_{max} = 7$. (a) depicts the 3D reconstruction and (b) a 2D slice.

As a sole octree reconstruction leads to poor results, the PMC on cut cells is carried out by one or more inside-outside tests, e.g. by ray-casting, or using a point-cloud-based technique. This approach follows the observation that flaws typically appear only localized (e.g. at the joint of two patches). Even if this additional PMC test fails, the error is bounded by the size of the smallest cells on the lowest subdivision level and will (most likely) not spread along the entire boundary.

Conclusion:

Point membership classification is the most basic property, which each body must allow. Various tests are available for CSG models and B-Rep models. We presented additional PMC tests, which permit evaluating also flawed, three-dimensional B-Rep models and extended primitives, as provided by procedural modeling tools. We have shown that the combination of these PMC approaches and the Finite Cell Method allow for direct numerical analyses without the need of a pre-processing stage, even when the geometric model is potentially non-watertight.

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