Title:
Truss-like Structure Design with Local Geometry Control

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1. Introduction:

Truss and truss-like structure design has been actively investigated in the past three decades for its broad application scope, and the most popular approach is the ground structure method which can be tracked back to [7]. With this method, a dense grid is initially generated and the nodes are mutually connected to form the input ground structure. Cross-section area of each beam is applied as the design variable, and it could approach to zero value which means a topological change. A review about early developments of the ground structure method can be found in [3], and an educational Matlab program can be found in [10]. Even though intensively investigated, there are still drawbacks about the ground structure method that: the optimized solutions tend to employ complex topology; the result optimality is dependent on the initial guess and therefore extremely dense grid is generally required for the input ground structure; in addition, the optimized solution may be impractical because only selected sizes of the beams are available in reality.

In order to reduce the topology complexity, one possible approach is to remove the beams with sizes below a threshold value, and add the removed material back to the remaining beams. Given the negative impact, global optimality is lost and the optimal solution is strongly dependent on the threshold value [2]. Another approach is to penalize the intermediate beam sizes into either the lower or the upper bounds through the SIMP (Solid Isotropic Material with Penalization) method [4]. In this approach, the lower bound is assigned a small positive value to avoid singularity phenomenon, and the upper bound is the full beam size. The penalization factor should be big enough to derive the pure binary solution.

To eliminate the dependence on the initial guess, exploration about adding nodes and beams during optimization [6] is conducted which has demonstrated good potential. However, a more popular solution is to allow additional design freedoms of moving nodes. As summarized by [1], two approaches: the alternative optimization and the simultaneous optimization, have been widely applied to involve the additional nodal freedoms. For the alternative optimization, the simple implementation [8,9] is to alternately optimize the topological variables (beam cross-section size) with fixed shape variables (nodal positions) and shape variables with fixed topological variables. With this implementation, the optimality criteria cannot be applied, but a local minimum is likely obtained even though it is not guaranteed [1]. Another implementation is to transform the original problem into a two-level nested problem [1,5], which is solved through the so called “implicit programming approach”. However, only limited scale of design variables can be handled because of the non-smooth high-level problem. A more direct approach is about the simultaneous optimization. This approach generally leads to a drastic increase of design variables, which may make the optimization algorithm quite complicated [8]. However, as commented by [1], the standard theory of nonlinear optimization applies and at least a local minimum can be found given certain scenarios [6].

2. Main idea:

In this work, the authors adopt the simultaneous shape and topology optimization. And more importantly, the main contribution is to realize the local geometry control. Specifically, the ground structure is optimized about the shape and topology while the local grid size or the local grid incircle radius is constrained with an upper bound. Practically, it is meaningful of realizing the local geometry control in fulfillment of some functioning requirements, e.g. the sand protection of wire-wrapped screens and the permeability control of lattice structures. To the authors' knowledge, there is no similar work based on the literature survey.

2.1 Optimization problem:

The poplar compliance-minimization problem is employed to demonstrate the simultaneous optimization with local geometry control. The optimization problem is formulated as:

$$\begin{gathered} min.\ \ \boldsymbol{F}^T\boldsymbol{U} \\ s.t.\ \ \boldsymbol{KU}=\boldsymbol{F} \\ \sum_{e=1}^{n}\rho_e v_e < v_{max}\ \ or\ \ \sum_{e=1}^{n}\rho_e < v_{max} \end{gathered} \tag{2.1}$$

where $\boldsymbol{K}$ is the assembled global stiffness tensor, $\boldsymbol{U}$ is the global displacement vector, and $\boldsymbol{F}$ is the global force vector. v_e is the volume of truss element e and v_{max} is the maximally allowed volume of the truss structure or the maximally allowed number of truss elements. The density based method [4] is applied, so a topological variable ρ_e is added to each truss element which varies within (0,1].

It is noticed that:

$$\boldsymbol{K}_e = \boldsymbol{T}_e{}^T \rho_e{}^p \overline{\boldsymbol{K}}_e \boldsymbol{T}_e \tag{2.2}$$

where $\boldsymbol{T}_e$ is the coordinate transformation tensor; $\boldsymbol{K}_e$ and $\overline{\boldsymbol{K}}_e$ are the stiffness tensors of truss element e in global coordinate system and local coordinate system, respectively; and p (≥3) is the penalization factor to prevent intermediate densities.

To solve this shape optimization problem, the Lagrangian function is constructed as:

$$L = \boldsymbol{F}^T\boldsymbol{U} - \widetilde{\boldsymbol{U}}(\boldsymbol{KU}-\boldsymbol{F}) \tag{2.3}$$

in which $\widetilde{\boldsymbol{U}}$ is the adjoint displacement field.

Correspondingly, the sensitivity analysis result on the nodal coordinate is:

$$\begin{aligned} \frac{\partial L}{\partial x_i} &= -\sum_{e=1}^{n}\boldsymbol{U}_e{}^T\frac{\partial \boldsymbol{K}_e}{\partial x_i}\boldsymbol{U}_e \\ &= -\sum_{e=1}^{n}\boldsymbol{U}_e{}^T\left(\frac{\partial \boldsymbol{T}_e{}^T}{\partial x_i}\overline{\boldsymbol{K}}_e\boldsymbol{T}_e + \boldsymbol{T}_e{}^T\frac{\partial \overline{\boldsymbol{K}}_e}{\partial x_i}\boldsymbol{T}_e + \boldsymbol{T}_e{}^T\overline{\boldsymbol{K}}_e\frac{\partial \boldsymbol{T}_e}{\partial x_i}\right)\boldsymbol{U}_e \end{aligned} \tag{2.4}$$

where $\boldsymbol{U}_e$ is the displacement vector of truss element e and x_i is the ith nodal coordinate.

Then, the sensitivity analysis result on the truss element density ρ_e is:

$$\frac{\partial L}{\partial \rho_e} = -\sum_{e=1}^{n}\boldsymbol{U}_e{}^T\frac{\partial \boldsymbol{K}_e}{\partial \rho_e}\boldsymbol{U}_e \tag{2.5}$$

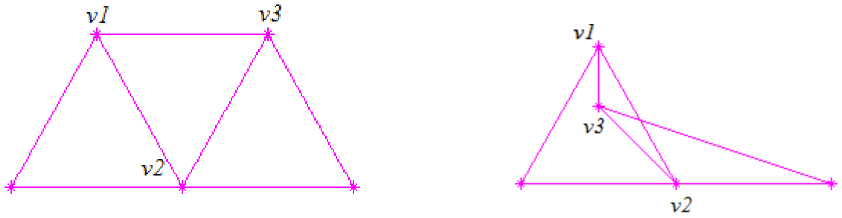

Fig. 1: Example of truss element intersection: (a) structure before truss element intersection, (b) structure after truss element intersection.

2.2 Local geometry control:

In [9], geometry control was realized to prevent truss elements from intersection. As shown in Fig. 2.1, if the vertice *v3* flips over the edge *v1-v2*, the truss elements intersect which is unreasonable.

A non-intersection constraint was developed, as:

$$S_j \geq \underline{S} \quad j = 1, \dots, m$$
$$S_j = 0.5 * \det \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \tag{2.6}$$

where S_j is the jth triangle grid area, and it is guaranteed positive by counting the vertices in the contour-clockwise order. $\underline{S}$ is the lower bound of the triangle grid area, and m is the number of triangle grids involved.

Additionally, areas of the triangle grids are constrained with an upper bound, as well, for the functioning purpose. It is:

$$S_j \leq \overline{S} \quad j = 1, \dots, m \tag{2.7}$$

in which $\overline{S}$ is the upper bound of the triangle grid area.

Another method of realizing local geometry control is to constrain the incircle radius (see Fig. 2), with which the maximum incircle radius constraint can be applied as:

$$R_j \leq \overline{R} \quad j = 1, \dots, m$$
$$R_j = \frac{2S_j}{P_j} \tag{2.8}$$

where P_j is the perimeter of the jth triangle grid.

As well, the minimum incircle radius constraint should be simultaneously applied to prevent the edge flipping, which is:

$$R_j \geq \underline{R} \quad j = 1, \dots, m \tag{2.9}$$

The benefit of controlling the incircle radius is that, the aspect ratio can be kept small which means close-to-isotropic properties.

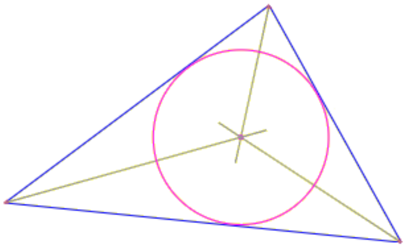

Fig. 2: Incircle of the triangle grid.

2.3 A case study:

One numerical example is studied in this subsection to prove the effectiveness of the local geometry control.

Fig. 3 presents the input truss structure and the attached boundary conditions. To be specific, all truss elements employ the section area of 0.04 and the material Young's modulus of 1.3.Two point forces are applied with the magnitude of 0.1/each.

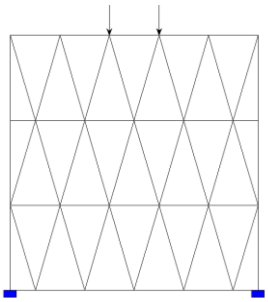

Fig. 3: The initial truss structure (5*5.19) and the boundary conditions.

A few shape optimization processes are gone through with different $\overline{S}$ values and fixed $\underline{S}$=0.3, and the other a few are gone through with different $\overline{R}$ values and fixed $\underline{R}$=0.2. Correspondingly, the optimization results are demonstrated in Fig. 4 and Fig.5.

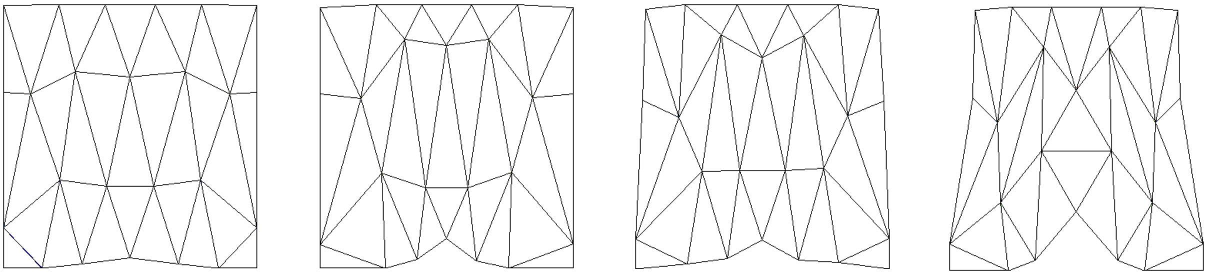

Fig. 4: Shape optimization results with different $\overline{S}$ values: (a) $\overline{S} = +\infty$, (b) $\overline{S} = 1.2$, (c) $\overline{S} = 1.0$, (d) $\overline{S} = 0.8$.

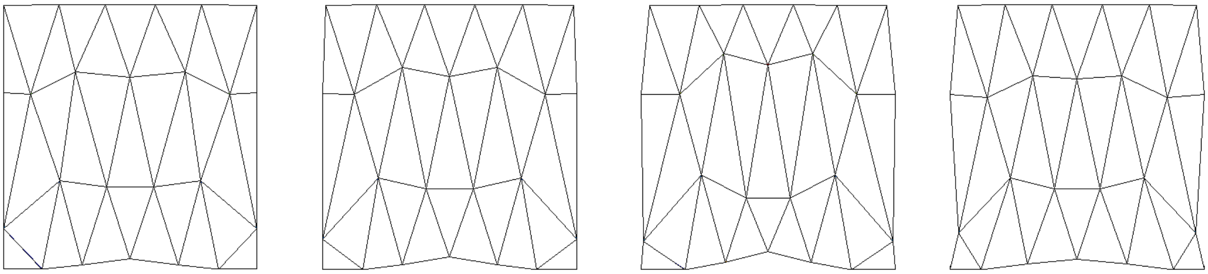

Fig. 5: Shape optimization results with different $\overline{R}$ values: (a) $\overline{R} = +\infty$, (b) $\overline{R} = 0.38$, (c) $\overline{R} = 0.37$, (d) $\overline{R} = 0.36$.

Through data analysis, there can draw some interesting conclusions that:

- The shape optimization result without maximum grid area constraint is only a local optimum, and a similar conclusion can be found in [9];
- Restricting the maximum grid area can help converge the result at a better local optimum. The overall trend is that the result optimality increases with the decreasing $\overline{S}$ values (see Fig. 6a) or $\overline{R}$ values (see Fig. 6b), even though there are fluctuations. It is noted that, compliance of the initial truss structure is 2.1189.

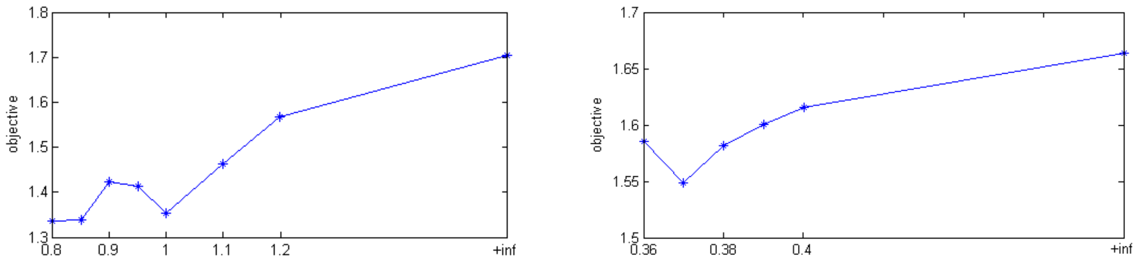

Fig. 6: Evolvement of the structural compliance (objective) against the (a) $\overline{S}$ value, and (b) $\overline{R}$ value

3. Conclusion:

This work realizes a novel idea of local geometry control while implementing the simultaneous shape and topology optimization of truss-like structures. For the control purpose, two different methods have been developed through either controlling the local grid area or the local grid incircle radius. Both control methods have been proven effective through a shape optimization case study. Some case studies under the topology optimization framework will be supplemented in the full paper. The authors believe that the contributed techniques in this work employs great engineering significance and will improve the practice of truss-like structure design.

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