Title:
G1 Continuous Bifurcating and Multi-bifurcating Surface Generation with B-Splines

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Introduction:
Surface construction from cross sectional data, either scanned or manually determined, is a designing process used in various applications like automobile parts (manifold of engine's cylinder head), neuron tree construction [3], human vascular system, human bronchial tree [5], tree skeletal structure [8]; and many more. Most of the constructions cited above actually involve branching at either single level or multilevel. As in case of manifold design of an engine, bifurcation suffices the design requirements while bronchial tree construction involves bifurcation and sub branching. The methods used by other authors for the generation of bifurcating surface includes method of triangulation [7],[9], stitching of right circular cylindrical surface patches [4],[11], blending of half tubular Bezier patches [1], and few others. Each method had its own limitations which has been countered in the method proposed in this paper. With method of triangulation, continuity requirements could not be met while the stitching of surface patches require additional step of aligning tangential vectors of the corresponding stitched patches.

This paper subdue the complexities confronted in the modeling of dichotomous branching shapes by using disjoint open B-spline surface. As an extension to the works cited above, this paper proposes a new methodology to construct bifurcating surface by using only single open B-spline surface equation, which is contrary to conventional approach of stitching surface patches. The paper emphasizes on G1 continuity at the junction of two branches as well as overall G1 continuity of the generated surface, by exploiting the concept of disjoint surface through knot value repetition. The model proposed in this paper is neither limited to circular cross-section nor to uniform cross-section; and also B-spline surface offers better control than the Bezier surface. In this paper dichotomous branching surface and bifurcating surface are used interchangeably. Bhatt et al. [2] used a technique of disjoint surface, used in this paper, for generation of bifurcating surfaces using single equation but was limited to order 3 in one of the two defining directions of B-spline surface. The algorithm used in this paper uses a different arrangement of control points and a different technique to disjoin the surface than used in [2], which led to a model capable of handling any order of B-Spline in both of the defining directions; along with an additional benefit of using the algorithm in iteration to produce multiple bifurcating surface.

Overview of the Paper:
This paper illustrates a new methodology for constructing bifurcating surfaces from a set of data assumed to be already available. This set of data acts as control points arranged in a particular fashion for the open B-spline surface. These arranged control points form the control polyhedron for the surface generation using single open B-spline equation by exploiting disjoint surfaces. In real world

problems, where the surface has to be interpolated on real data set, the control polyhedron set can be formed by using methods described in [10].

The control point layout, forming control polyhedron, used for surface generation is showed in Fig. 1.The arrangement of control points along the z axis of the control polyhedron is referred as control polygon. The geometrical arrangement of the control polygons along the z axis are taken to be square shaped (either one as in stem or in a pair of two as in branches), one shaped like an open book structure and one shaped like an inverted T structure, as shown in Fig. 1. The horizontal levels having two squares in control polygons form the part of the surface termed as branches, while the horizontal levels with one square in control polygon forms the part of the surface termed as stem. Open book shaped control polygon along with inverted T shaped control polygon marks the junction of the two branches as well as the top of the stem. To have G1 continuity at the junction of two branches, open book polygon and inverted T polygon are made to have their corresponding control points in a straight horizontal line (as visible in Fig. 4(a)).

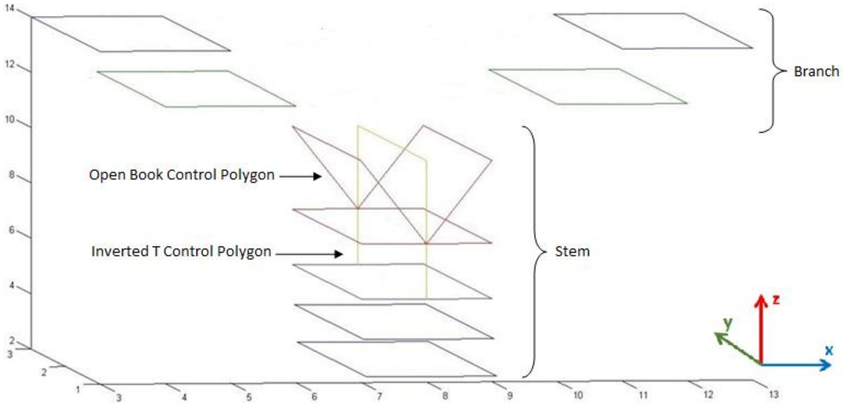

Fig. 1: General Layout of Control Polyhedron.

The Open B-spline surface is generated by

$$S(u,v) = \sum_{i=0}^{n} N_{i,k}(u) \sum_{j=0}^{m} N_{j,l}(v)\ P_{ij} \tag{5.1}$$

where,

S(u,v)	= Point on B-spline surface
n+1	= No. of control points in *u* direction
$N_{i,k}(u)$	= Basis function in *u* direction.
m+1	= No. of control points in *v* direction
$N_{j,l}(v)$	= Basis function in *v* direction
k	= Order of the curve in *u* direction (k -1 = Degree of the curve in *u* direction)
l	= Order of the curve in *v* direction (l -1 = Degree of the curve in *v* direction)

P_{ij} contains the control points in defined order. In this paper *u* direction is in x-y plane and *v* direction is along z axis. The information about the control points at which the surface will be disjoined both in *u* and *v* directions is also made available as input data in matrices Mu and Mv respectively.

Basis function is defined as:

$$N_{i,k}(u) = (u-u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1}-u_i} + (u_{i+k}-u)\frac{N_{i+1,k-1}(u)}{u_{i+k}-u_{i+1}} \tag{5.2}$$

where

$$N_{i,1} = \begin{cases} 1, & u_i \le u \le u_{i+1} \\ 0, & otherwise \end{cases}$$

u_i are called the parametric knots or knot values. These values form a sequence of non-decreasing integers called knot vectors. To have a disjoint B-spline curve, the parameter u was experimented upon, and the results were then extended to B-spline surfaces.

<u>Disjoint B-Spline Curve:</u>
As stated, the aim is to make a bifurcation shaped structure using only single surface equation i.e. not using surface patches, hence the concept of disjoint surface has been explored vividly by knot value repetition. How the multiplicity of a knot value affects the nature of the curve is demonstrated below.

For a curve of degree 3 and number of control points = 12,
n = 11
k = 4
u = [0 0 0 0 1 2 3 4 5 6 7 8 9 9 9 9]

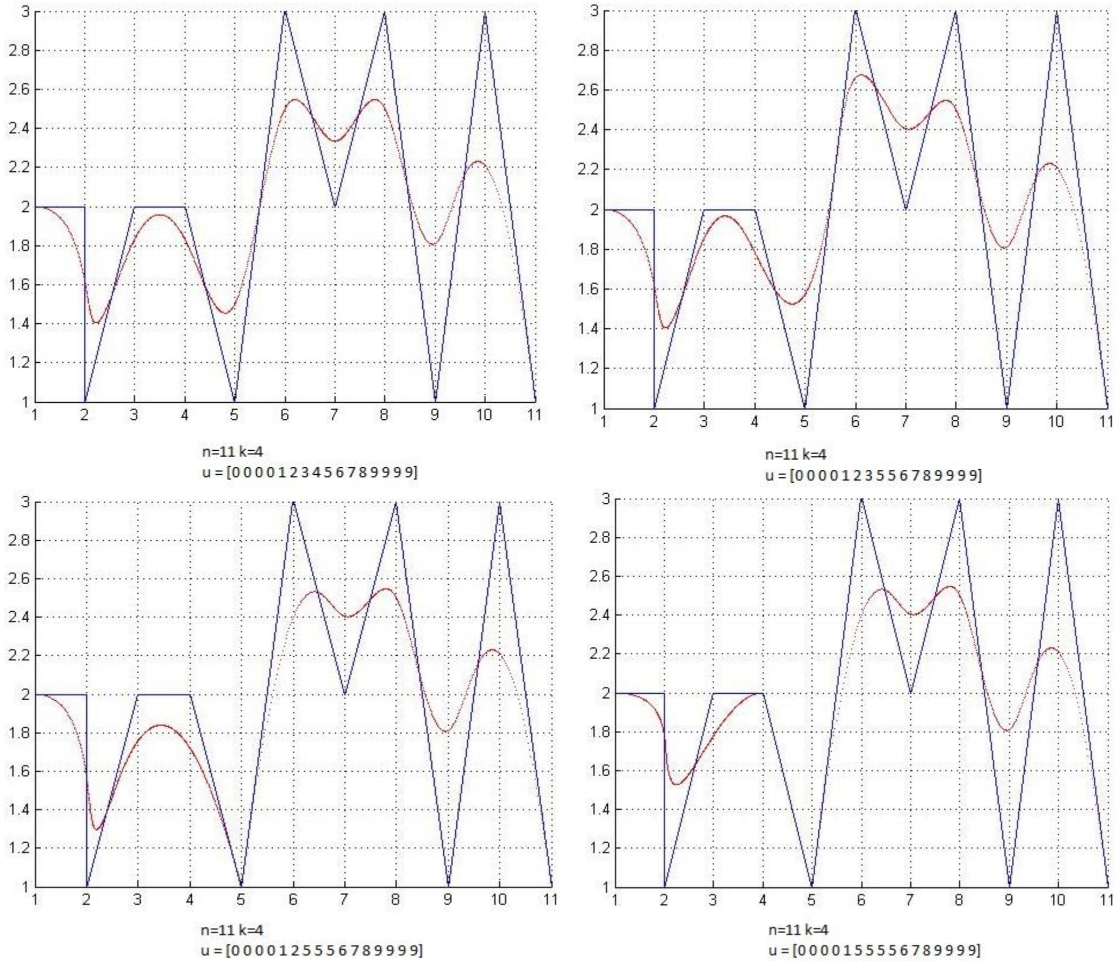

Fig. 2: B-Spline Curve with: (a) Standard knot values: (b) Multiplicity of 2 of knot value 5 (c) Multiplicity of 3 of knot value 5 (d) Multiplicity of 4 of knot value 5.

It can be seen in Fig. 2(c) that when a knot value is repeated for (k-1) times, i.e. 3 times in this example, the original single curve of Fig. 2(a) ceases at an intermediate control point and a new curve from the same control point originates, with the two curves satisfying C0 continuity. If the knot value is repeated for k times, i.e. 4 times in this example as in Fig. 2(d), the original single curve divides into two curves by skipping an intermediate control point. In this manner, we can make two or more disjoint curves without even having C0 continuity from a single B-Spline equation. One of the noteworthy outcome of this disjoining of curve is visible in Fig 2(c) and Fig 2(d), where the second curve for both the figures are same but the first curve of both the figures are different.

When this technique was extended for surface generation, certain open spaces were observed as shown in Fig. 3. The reason for these open spaces in the surface is due to regular stepwise increment of knot value in the code used to generate the surface. For example, in the case when u is changed from 1 to 2 in 10 steps, i.e. incrementing u each time by 0.1, open space will be more than when u is incremented in 20 steps. Thus even if u is incremented in 100 steps some open space will still be present which may not be visible to naked eyes. To counter this, the spline was made to terminate at a control point before disjoining it (similar to Fig. 2(c)). The disjoining was then incorporated in the model by increasing the multiplicity of this particular control point.

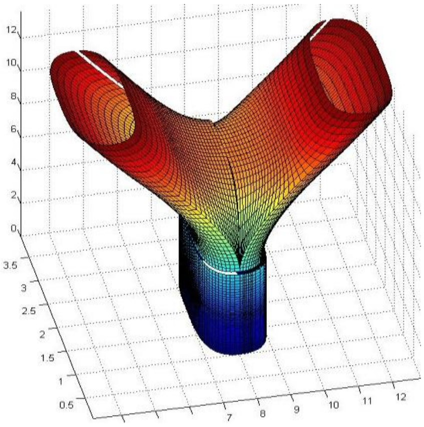

Fig. 3: Open space in generated surface.

Methodology:

In generating the surface, sections of control polygons in stem region are traversed twice. This repeated traversing is caused due to the property of defining rectangular control polyhedron matrix of B-spline i.e. the number of control points for each layer has to be same. Thus to have same number of control points in the branch and stem region, control points in the stem are repeated and overlapped (except for open book polygon and inverted T polygon).

B-splines produce same curve (or surface) irrespective of its direction of traversing. But as shown in Fig. 2(c) and Fig. 2(d), after disjoining the first part of the curve changes its shape while the second part remains same. In order to have same contour upon repeated traversing of the control points, the direction of traversing is made to be same. The spline is made to terminate before disjoining so as to have no open spaces and better control. The termination of spline and subsequent disjoining is done by knot value repetition. This is shown in Fig 3 where arrow head defines the direction of traversing and number of arrows define the number of times a particular section is being traversed in each control polygon. The direction of traversing as well as the disjoints in the surface can also be understood by following the sequential numbering.

The layer of stem in inverted T shape is overlapped by a layer same as one below it, though it's not visible in the figure. The inverted T layer is used to disjoint the spline in v direction and begin the second part of the curve from the overlapped layer. The first four layers of stem region (seen from top to bottom, 3 layers which are visible in Fig 4(a) and one overlapping layer with inverted T layer) should be in a straight line to have G1 continuity in v direction. The generated surface using this algorithm is shown in Fig. 4(b).

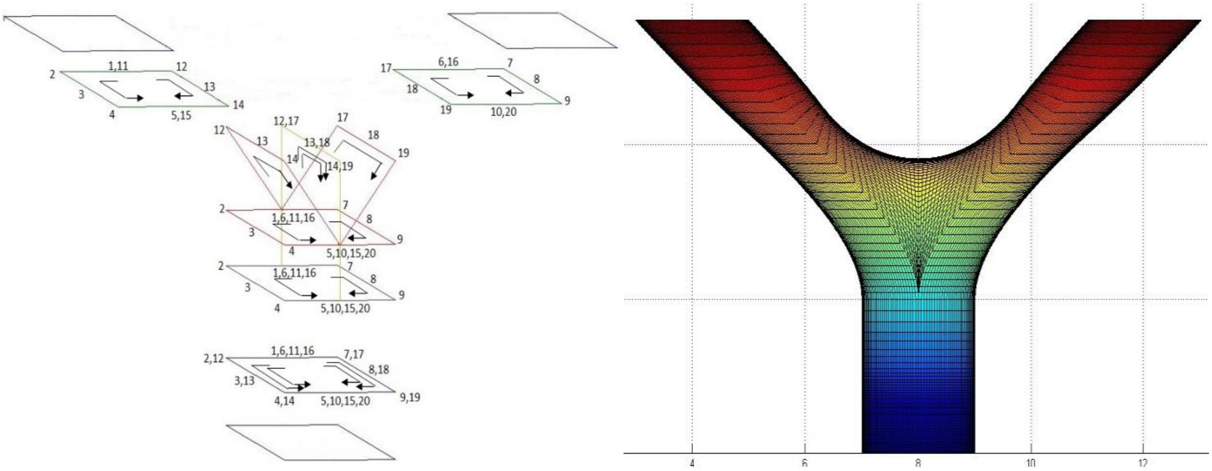

Fig. 4: (a) Control polyhedron for surface generation, and (b) Generated Surface.

The next stage is to incorporate G1 continuity at all the critical points of the surface as well as at the sides of the junction so as to have a better surface, not just a surface with C0 continuity. T*hese critical points are those points on the surface where the spline is terminated; and started again.* Also, since in this model we have terminated the spline before it is made to disjoin, hence the points of disjoining are also included in the critical points when points of termination are considered. G1 continuity of B-spline is achieved by aligning the control polygon segments on either side of the critical point in a straight line. This is done by adding one control point very close to the critical point on either side of it, with all three points forming a straight line. This results in a smoother G1 continuous surface.

The model presented above offers a wide range of advantages in the designing or reconstruction of surface in terms of shape and also in terms of parameters used. All the experiments conducted above used the degree of 3 in u direction and degree of 2 in v direction (i.e. k=4, l=3); but it is not necessary, the model can handle any degree both in u and v directions. With this model we can have asymmetric branches, irregular cross sectional shapes along the length and if algorithm is used in iteration, we can have multi-bifurcations as shown in Fig. 5.

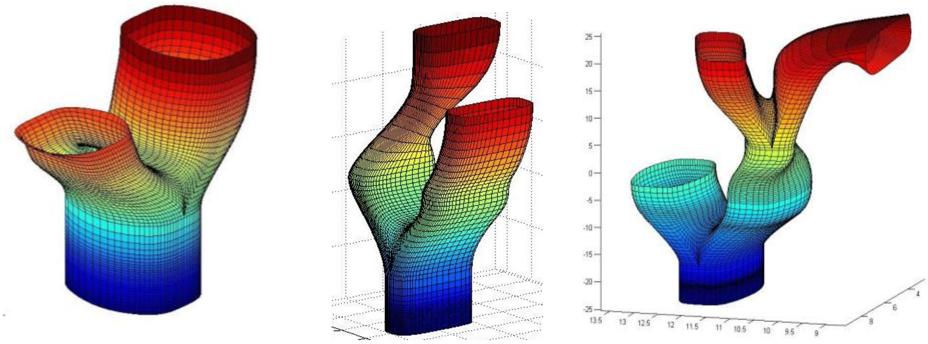

Fig. 5: Generated surfaces G1 continuous with (a) Asymmetric branching, (b) irregular cross section, and (c) multi-bifurcation.

Conclusion:

The method discussed in this paper allows an easy and effective way of generating dichotomous branched surface using single open B-spline surface equation. It allows the generated surface to have at least G1 continuity both in parametric space and in visibility. This method differs from the conventional approach of stitching various surface patches together, which sometimes can be a very complex step both computationally and logically for achieving continuity, to achieve the final surface. Moreover, the flexibility to have irregular cross sections at different sections of the surface and asymmetric branching allows the generated surface to be more realistic. Thus it is a tool which can handle varying topological and geometrical complexities while constructing the surface. Some applications of this can be in reconstruction of human bronchial tree, engine's manifold design, and Y & T frames for automobiles.

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