

Title:

Two-step Mapping Parameterization of Scattered Points for Wrap-like Free-form Surfaces Reconstruction

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Introduction:

Reverse engineering (RE) is a process that the measured data from a physical part are transformed into a CAD model. It has been the focus of significant interest in the last three decades and used widely in aerospace, automobile industry to facilitate product design, analysis and manufacture. Especially when the nominal CAD model or original drawing is not available, it is crucial to reconstruct a CAD model from an existing part for the downstream geometric processing and manufacturing applications.

For free-form surface reconstruction in RE, the parameterization of scattered points is one of the most important step, there are many methods proposed for the data parameterization [6]. The commonly used ones are uniform parameterization, cumulative chord length parameterization and centripetal parameterization [4]. These methods work well when the data points are arranged in a chain for curves or a grid for surfaces, but they will fail when the measured data points are scattered unless the processes of slicing, segmentation and sorting are performed in advance. However, in practice, the slicing, sorting and segmentation often turn out to be very challenging when the data points are noisy or non-uniformly distributed. To deal with the parameterization of irregularly spaced and randomly distributed data points, several researchers proposed the base surface (BS) based method [3]. A basic requirement for BS is it has to be able to roughly reflect the global shape of the cloud of points. It might be a plane, a cylinder or a bilinear Coons patch. The parameters are then calculated by projecting the data points onto the BS. Earlier Piegler and Tiller tested various BSs and gave a thinning method to speed up the time consuming projection process, and at the same time they also pointed out, if the initial BS is poor, sometimes simple BS is difficult to approximate sufficiently the internal features of the data points, it may lead to an unaccepted reconstruction result [4]. To improve the BS based method, Azariadis proposed the dynamic base surface (DBS) [1]. In their method, DBS is gradually improved approximating more faithfully the geometry of the cloud of points by an iterative procedure, and then the parameterization are achieved by projecting the cloud of points onto the DBS. Although, theoretically either BS or DBS sounds easy to do, however, as pointed out by Weiss, they may fail if there are irregularly oriented highly curved sub-regions or the data points cannot be projected in an unambiguous way [6].

As an alternative to BS method, mesh parameterization (MP), which produces a topologically identical 2D triangulation, can be adopted for the scattered points, and the coordinates of the planar mesh vertices are used as the parameters of the data points [2]. Its running speed, in the past, is considered to be relatively slow when processing a large number of data points, so that it has not been used widely in free-form surface fitting, but with today's computer power, the main barriers such as memory and performance of CPU should be no longer perceived to be critical problems.

This paper proposes a practical method, which is particularly suitable to the wrap-like free-form surface fitting, to assign parameter values to scattered points. The basic ideas of the proposed method are discussed in the next section. In this paper, the experiments are performed to testify the effectiveness of the proposed method.

Basic Ideas:

The proposed method is realized using two-step mapping. Considering the wrap-like features of such surfaces, the cylindrical mapping is first constructed to map the scattered points onto a plane, which can avoid the wrong data overlap caused when projecting the scattered point simply along Z-direction and it can also simplify the following triangulation processing. The constructed triangular mesh is then mapped again into a topologically identical 2D triangulation bounded by a square. As a result, the coordinates of the mesh vertices in this square are used as the parameters of the scattered points and then the wrap-like free-form surface is nicely fitted.

In essence, the proposed method is a mesh-mapping (MP) based method. As mentioned above, before applying MP, scattered data points must be structured in a triangular mesh. Considering that the wrap-like surface is similar to the cylindrical surface, to simplify the following triangulation cylindrical coordinate mapping (CCM) is first constructed to map the data points onto a plane on which the 2D data point can be nicely triangulated into triangular mesh model, as shown in Fig. 1. Moreover, CCM can also avoid the wrong data overlap caused when projecting the scattered point simply along Z-direction. As we known, cylinder surface is a kind of developable surface and can be represented in cylinder coordinate system (CCS). The mapping of wrapped surface points to a cylindrical surface in CCS is actually to flatten the cylindrical surface onto its corresponding planar domain. As the wrap-like free-form surface has similar geometry feature of cylindrical surface, the mapping, namely CCM, can be extended to map the wrap-like data points onto a plane to simplify the following data processing.

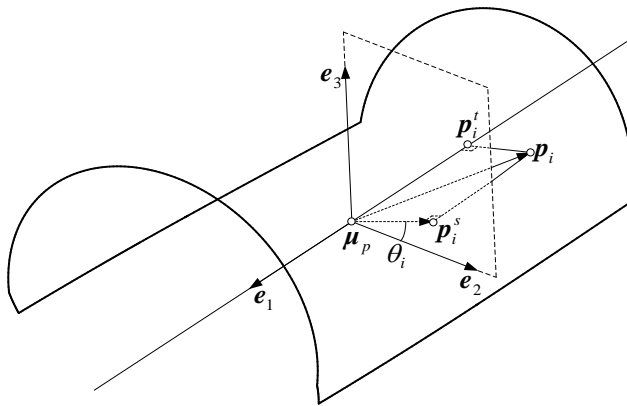


Fig. 1: Cylindrical coordinate mapping (CCM)

In the engineering applications, aircraft body surface, pipe-like surface and other wrap-like surfaces all have the wrap-like feature of cylindrical surface such that CCM can be applied to data points of these surfaces and generate 2D data points that can maintain nicely the neighbor relationships in 3D data points. Fig. 2 gives an example of mapping wrap-like data points to 2D data points.

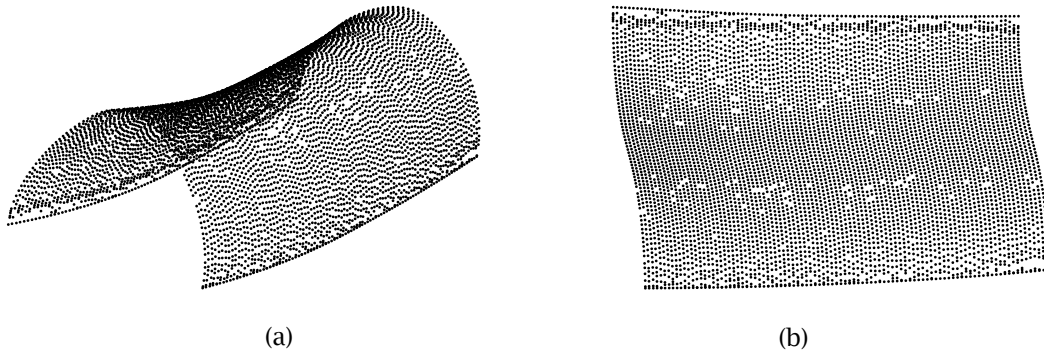


Fig. 2: An example of mapping wrap-like data points to 2D data points: (a) Scattered data points; (b) 2D data points obtained using CCM.

Next task is how to construct the CCM on the wrap-like cloud of points. Once 2D data scattered points have been acquired, the scattered point data needs to be triangulated into organized triangular mesh. Although the triangulation of 3D scattered data points is very challenging, its 2D data points are relatively easy to be handled. In this paper, an energy optimization method is developed to handle the 2D data scattered points triangulation and the mesh mapping processing. As shown in Fig. 3, the parameterization of scattered data points is processed by executing Sloan's Delaunay triangulation procedure [5]. The process turns to be robust and efficient in handling the scattered point data. The basic steps of triangulation of scatter point data based on the energy optimization are briefly summarized as follows:

- 1th triangulation
- 2th triangulation
- 3th triangulation
- 4th triangulation

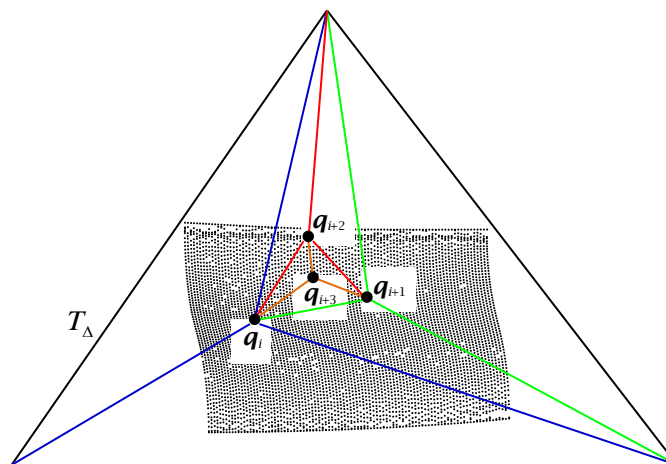


Fig. 3: Triangulation of scattered points by the Sloan Delaunay triangulation

- (1) Select three points arbitrarily to form a big triangle T_{Δ} which completely contains all 2D data points to be triangulated, as shown in Fig. 3;
- (2) Select randomly a new point q_i from the 2D data point set $Q = \{q_i, i = 0, 1, \dots, m\}$, find an existing triangle that encloses q_i and form three new triangles by connecting q_i to each of its vertices;
- (3) Use the swapping algorithm to update the existing triangulation to a Delaunay triangulation after q_i is inserted;

- (4) Repeat steps 2 and 3 until all points in Q have been added to the triangular mesh;
- (5) Delete all triangles associated to the big triangle T_{Δ} to form the Delaunay triangulation.

Fig. 3 show first four inserting and triangulating of points, other points can be addressed in a similar manner. The obtained triangular mesh models using Sloan's method of 2D data points and 3D data points are shown in Fig. 4.

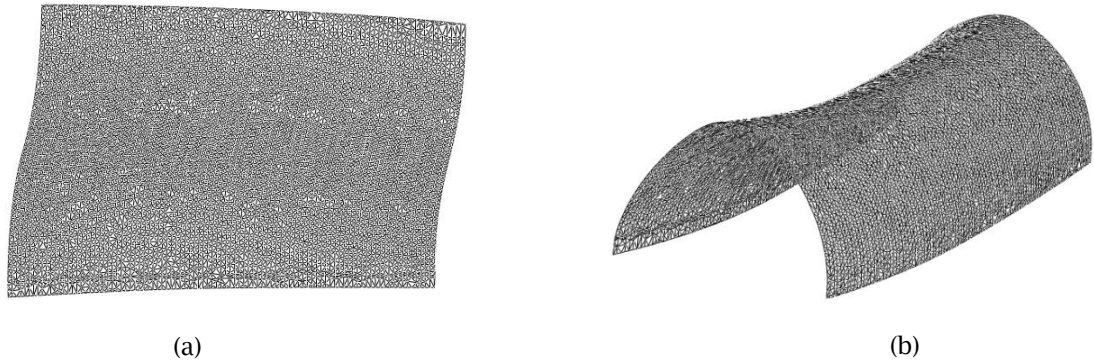


Fig. 4: Triangulation results: (a) Triangulation of 2D data points. (b) Triangulation of 3D data points.

The process of mesh mapping involves mainly two steps: the first step is to specify the boundary of planar region, in this paper which is a unit square; and the second step is to arrange the interior vertices of M_{3D} in S_{2D} to minimize $E(M_{3D} \rightarrow S_{2D})$. For the first step, the mesh boundary needs to be divided into four segments by specifying four boundary corner points according to the design intent so that these four segments are consistent to the four sides of the square. Then, the boundary points of each segment can be mapped onto the corresponding planar boundary according to the chord length between the adjacent points on the 3D boundary segment. Fig. 5 shows the mesh mapping of the triangulation shown earlier in Fig. 4. For each scattered data point p_i , a pair of parameter values (u_i, v_i) is associated by the mesh mapping. The surface degree and knot vectors in the u - and v - directions can be specified by the users or calculated.

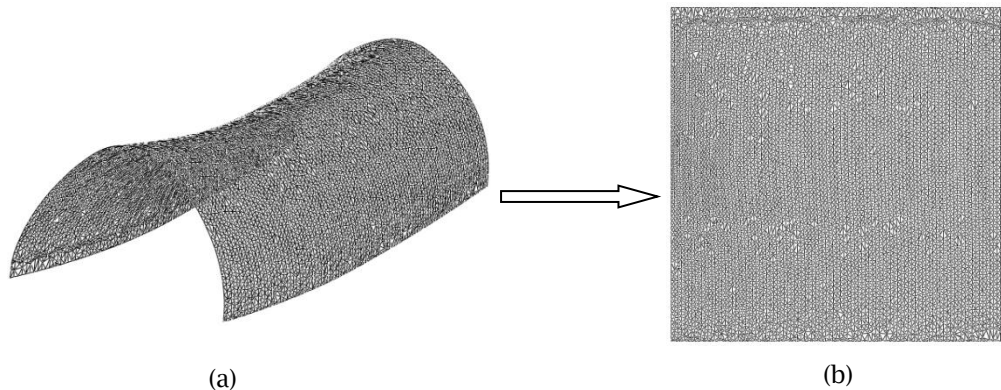


Fig. 5: Mesh parameterization: (a) 3D mesh model. (b) Parameterization in the unit square.

Results and Discussion:

The presented techniques have been implemented and tested for validation. The algorithm proposed in this paper have been coded in C++ language and implemented on a PC with an Intel 3.4 GHZ and 8.0G physical memory. In the following, the examples of typical wrap-like surface reconstruction are to be given to test the validity of the proposed method. Fig. 6 shows an example of a bicycle seat surface scattered point data of 6,776 points. A 15×15 control net is reconstructed for the scatted data points.

The fitted surface model is shown in Fig. 6 (a) with an average error of 0.041 mm and a maximum error of 0.305mm, which is then imported into CAM module of UG 7.5 and is machined using a 3-axis CNC machine. The example was also been machined to generate the reverse engineered part from the original scattered point data. The tool paths generated based on the fitted surface is shown in Fig. 6 (b), Fig. 6(c) shows the machining simulation and the final machined part is shown Fig. 6(d).

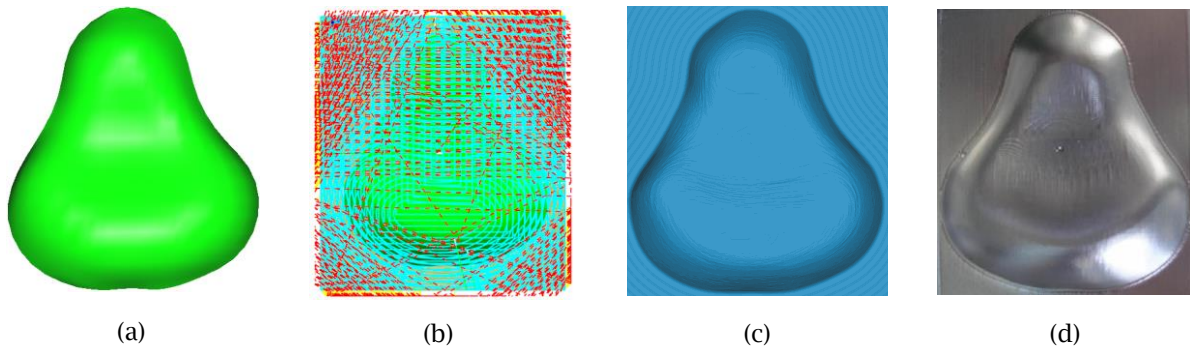


Fig. 6: Example of wrap-like free-form surface reconstruction and machining: (a) CAD model obtained by the proposed method. (b) Tool paths generated in CAM module of UG 7.5. (c) Machining simulation. (d) The machined part.

Conclusions:

This paper presents a new method of surface re-parameterization from scattered points processing for industry and manufacturing applications. A two-step mapping algorithm was presented by first map the scatter data onto a 2-dimensional space and then a smoothing step is carried out by considering the third-dimension continuity and smoothness for surface reconstruction. In the presented method, an error-correcting process was developed to detect and eliminate overlapping errors in the mapping procedure of the scattered data. The re-constructed free-form surfaces are generated by a surface triangulation process that generates a topologically identical triangulation bounded surface facets. The presented method has been implemented and used for industry and manufacturing applications with wrap-like surfaces. Practical examples using the presented techniques are presented for industry and aerospace manufacturing. The presented techniques can be used for computer-aided design and reverse engineering manufacturing of complex surfaces products development.

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References:

- [1] Azariadis, P. N.: Parameterization of clouds of unorganized points using dynamic base surfaces, *Computer-Aided Design*, 36(7), 2004, 607-623. [http://dx.doi.org/10.1016/S0010-4485\(03\)00138-6](http://dx.doi.org/10.1016/S0010-4485(03)00138-6).
- [2] Floater, M. S.: Parameterization and smooth approximation of surface triangulations, *Computer aided geometric design*, 14(3), 1997, 231-250. [http://dx.doi.org/10.1016/S0167-8396\(96\)00031-3](http://dx.doi.org/10.1016/S0167-8396(96)00031-3).
- [3] Ma, W. Y.; Kruth J. P.: Parameterization of randomly measured points for least squares fitting of B-spline curves and surfaces, *Computer-Aided Design*, 27(9), 1995, 663-75. [http://dx.doi.org/10.1016/0010-4485\(94\)00018-9](http://dx.doi.org/10.1016/0010-4485(94)00018-9).
- [4] Piegel, L. A.; Tiller W.: Parameterization for surface fitting in reverse engineering. *Computer-Aided Design*, 33(8), 2001, 593-603. [http://dx.doi.org/10.1016/S0010-4485\(00\)00103-2](http://dx.doi.org/10.1016/S0010-4485(00)00103-2).

- [5] Sloan S. W.: A fast algorithm for constructing Delaunay triangulations in the plane, *Advances in Engineering Software*, 9(1), 1987, 34-55. [http://dx.doi.org/10.1016/0141-1195\(87\)90043-X](http://dx.doi.org/10.1016/0141-1195(87)90043-X).
- [6] Weiss, V.; Andor, L.; Renner, G.; Várady, T.: Advanced surface fitting techniques, *Computer Aided Geometric Design*, 19(1), 2002, 19-42. [http://dx.doi.org/10.1016/S0167-8396\(01\)00086-3](http://dx.doi.org/10.1016/S0167-8396(01)00086-3).