Title:
Virtual Simulation of Dynamic Interaction of Deformable Objects

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Introduction:
Collision response analysis is one of the most crucial parts in dynamic simulation of deformable objects. In the past, researchers proposed many solutions to this task [1-4]. Miller [5] presented a position-based dynamics framework that handles general constraints, and Shinar [6] developed a full two-way coupling of rigid and deformable bodies. Lenoir and Fonteneau [7] used Lagrange multipliers to model two-way interaction with a limitation to analytical deformable objects. The main purpose of this work is to propose a new algorithm for the collision response analysis, which is accurate in terms of description of collision response, general-purpose in terms of no extra assumptions on geometric constraint formula, nodal connection at contact surface, deformation zones, etc., and yet fast in terms of guaranteed O(n) time cost.

Main Idea:
The change in geometric configuration of objects can be decomposed into rotation and translation both of which can be further divided into local and global components. Global translation and rotation are called rigid-body movement, while local translation and rotation contribute to local deformation. Even though it is possible to include the local rotational degrees of freedom in the analysis, they are not used in this paper because of their high computation cost. For elastic continuum material, we can use different magnitude of translational displacements of nodes to approximate any moderate amount of large local rotation. In the case of huge local rotation, the object is partitioned into two sub-objects. Thus, in this paper the change in geometric configuration is partitioned into global rotation, global translation and local translation.

As a compromise between efficiency and functionality, the Lagrangian dynamics is used to describe global rotation because of its simplicity and the finite element method is used to describe translation because of its flexibility. On the basis of D'Alembert's principle and the principle of virtual displacement, the semi-discrete system governing equations that describe the equilibrium of a system is approximated by

$$\mathbf{M}\,{}^{n}\ddot{\mathbf{u}}_t + \mathbf{C}\,{}^{n}\dot{\mathbf{u}}_t + {}^{n-1}\mathbf{K}\,{}^{n}\mathbf{u}_t = {}^{n}\mathbf{F}, \tag{1}$$

$$\frac{d}{dt}\left({}^{n}\mathbf{M}_I\ {}^{n}\dot{\theta}\right) = {}^{n}\mathbf{Q}\ , \tag{2}$$

where the global mass matrix **M** and damping matrix **C** are assumed to be constant. ${}^{n}\mathbf{M}_I$ refers to the inertia tensor. ${}^{n-1}\mathbf{K}$ refers to the global stiffness matrix at step *n*-1. ${}^{n}\mathbf{F}$ is the external load vector and ${}^{n}\mathbf{Q}$ is the torque vectors caused by external forces. ${}^{n}\mathbf{u}_t$, ${}^{n}\dot{\mathbf{u}}_t$, ${}^{n}\ddot{\mathbf{u}}_t$ are the translational

displacement, velocity, acceleration vectors, respectively, and are determined by central difference as follows:

$$ {}^{n}\dot{\mathbf{u}}_t = \frac{1}{2h}\left\{ {}^{n+1}\mathbf{u}_t - {}^{n-1}\mathbf{u}_t \right\}, \tag{3a} $$

$$ {}^{n}\ddot{\mathbf{u}}_t = \frac{1}{h^2}\left\{ {}^{n+1}\mathbf{u}_t - 2\,{}^{n}\mathbf{u}_t + {}^{n-1}\mathbf{u}_t \right\}, \tag{3b} $$

where h is the interval in each time step.

There are two types of constraints which must be considered during a collision. The first one is the geometric constraint which imposes the requirement of geometric coherence to the displacement of two colliding objects, such as the prevention of inter-penetration between objects and the allowance of sliding between objects if the tangential force exceeds the frictional capacity at the interface. The global geometric constraint at time step n+1 can be expressed by

$$ {}^{n+1}\mathbf{G}\left\{ {}^{n+1}_{n}\Delta\mathbf{u} + {}^{n}\mathbf{X} \right\} = {}^{n+1}\mathbf{G}\;{}^{n+1}\mathbf{X} = 0, \tag{4} $$

where ${}^{n}\mathbf{X}$ and ${}^{n+1}\mathbf{X}$ are coordinate vectors at time step n and n+1, respectively. ${}^{n+1}_{n}\Delta\mathbf{u} = {}^{n+1}\mathbf{u} - {}^{n}\mathbf{u}$. ${}^{n+1}\mathbf{u}$ is the total displacement vector at time step n+1 with two components, ${}^{n+1}\mathbf{u} = {}^{n+1}\mathbf{u}_t + {}^{n+1}\mathbf{u}_r$, where ${}^{n+1}\mathbf{u}_t$ and ${}^{n+1}\mathbf{u}_r$ are the linear displacement vector contributed by translation and rotation, respectively.

When a node of one object penetrates a triangular surface patch of another object, the penetration intersect point within each triangular patch can be expressed by barycentric coordinates t_1 and t_2. Let the sub-matrix ${}^{n+1}\mathbf{G}^{(j)}$ of ${}^{n+1}\mathbf{G}$ represent part of the geometric constraint matrix corresponding to the contribution of the penetrating node j whose starting and ending position is e and f, respectively, in time step n+1. The local geometric constraint with respect to penetrating node j and triangular patch (a, b, c) can be expressed as in the following generic way (here, 'generic' means that the constraint matrix is independent upon the geometric format of object surface):

$$ {}^{n+1}\mathbf{G}^{(j)}\;{}^{n+1}\mathbf{X}^{(j)} = 0\ , \tag{5} $$

where

$$ {}^{n+1}\mathbf{G}^{(j)} = \begin{bmatrix} 1-t_1-t_2 & 0 & 0 & t_2 & 0 & 0 & t_1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1-t_1-t_2 & 0 & 0 & t_2 & 0 & 0 & t_1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1-t_1-t_2 & 0 & 0 & t_2 & 0 & 0 & t_1 & 0 & 0 & -1 \end{bmatrix}, $$

$$ {}^{n+1}\mathbf{X}^{(j)} = \begin{bmatrix} x_a & y_a & z_a & x_b & y_b & z_b & x_c & y_c & z_c & x_j & y_j & z_j \end{bmatrix}^T, \tag{6} $$

where x_a, y_a and z_a are the Cartesian coordinates of node a, etc.

The global geometric constraint ${}^{n+1}\mathbf{G}$ is formed by assembling the local geometric constraint ${}^{n+1}\mathbf{G}^{(j)}$ over all penetrating nodes similar to assembling the element stiffness matrices into the global stiffness matrix. However, in the proposed local finite element method, such assembly process will be unnecessary.

The second type of constraint is the loading constraint. It is assumed that the contact force at each penetrating node is transferred to the three nodes of the penetrated triangular patch through a linear interpolation which is the same as the interpolation of coordinates in equation (6). The contact forces at both the penetrating node and the nodes of the penetrated triangular patches are considered as external loading to the system. Such extra loading contributed by the penetrating node j is calculated

by $({}^{n+1}\mathbf{G}^{(j)})^T \; {}^{n}\mathbf{\Lambda}^{(j)}$, where ${}^{n}\mathbf{\Lambda}^{(j)} = \left[{}^{n}\lambda_x^{(j)} \quad {}^{n}\lambda_y^{(j)} \quad {}^{n}\lambda_z^{(j)} \right]^T$ contains the components of the contact force at node *j* in directions *x*, *y*, and *z*. It should be noted that only the contact forces at the penetrating nodes are independent unknowns, while those at the nodes of the penetrated triangular patches are dependent through the Newtonian action-reaction law and linear interpolation. Such a linear interpolation of the penetrating force among three nodes of the penetrated triangular patch guarantees the equilibrium of translational forces but not of rotational forces in the system. In order to conserve angular momentum for the contact, algorithmic moment arms (numerically-corrected moment arms) should be used. In computer animation, this numerical correction may not be needed depending upon the desired accuracy imposed by researchers. The assembly of the extra loading caused by all penetrating nodes leads to the global contact force vector $({}^{n+1}\mathbf{G})^T \; {}^{n}\mathbf{\Lambda}$, which will be used in a set of modified system governing equations.

Another aspect of the loading constraint is that the contact forces should obey the basic friction law. In this paper, the basic Coulomb law is adopted and the static friction coefficient is assumed to be the same as the dynamic one. Let us consider the penetrating node *j* with a contact force vector (FC_x, FC_y, FC_z), i.e., $FC_x = \lambda_x^{(j)}$, $FC_y = \lambda_y^{(j)}$, $FC_z = \lambda_z^{(j)}$. If the normal vector of the penetrated triangular patch is expressed by (X_1, Y_1, Z_1), the normal force $\mathbf{FC}_n$ and the norm of the tangential force $|\mathbf{FC}_t|$ in Figure 2 are expressed as

$$\mathbf{FC}_n = (X_1 FC_x + Y_1 FC_y + Z_1 FC_z) \frac{\mathbf{n}_{abc}}{|\mathbf{n}_{abc}|} \tag{7a}$$

and

$$|\mathbf{FC}_t| = \sqrt{FC_x^2 + FC_y^2 + FC_z^2 - |\mathbf{FC}_n|^2}\,, \tag{7b}$$

respectively. If $\mathbf{FC}_n$ is along the direction $-\mathbf{n}_{abc}$ (i.e., pointing to the outside of the object), then a tension mode is reached. In such a case, both the normal and tangential contact forces at the penetrating node are set to be zero. Let ϕ be the friction angle between two objects. The Coulomb friction law, $|\mathbf{FC}_t| = |\mathbf{FC}_n| \cdot \tan\phi$, is enforced by using a Gauss-Seidel iterative algorithm in which the tangential contact force is updated at each iteration.

The basic concept of the forward Lagrange multiplier method is that displacement constraints at time step ${}^{n+1}t$ are correlated with Lagrange multipliers at time step ${}^{n}t$. It is originally proposed in numerical analysis in a simple 2-D format [8]. In this paper, we extends it to 3-D format

We propose a new variation of the conventional finite element method, the local finite element method. Here, 'local' means that no factorization, inversion and assembly of any global matrix are required, i.e., all calculations are performed at the local element level. Actually, there is even no need for storing any global matrix in this approach.

The proposed approach is implemented using MS Visual C++ on a Window workstation. Figure 1 demonstrates the impact process of a hammer dropping onto a fictitious flexible table. Figure 1 (a) is the initial state, while (b) and (c) are different views of the moment at which impact occurs. Both the hammer and the table deform in a free style as expected.

The second example is an elastic cup collides with a rigid wall obliquely as illustrated in Figure 2 which demonstrates the unsymmetrical local deformation of the cup. This delicate detail is difficult, if not impossible, to be produced by using the global deformation approach.

The accuracy of the proposed formulation is verified by using the example shown in Figure 3. For this problem, there is an analytical solution for the elastic collision between two blocks. The analytical

solution in solid mechanics is compared with the numerical solution shown in Figure 4. The relative error between the two solutions w.r.t. the impulse during the collision process is within 3.3%.

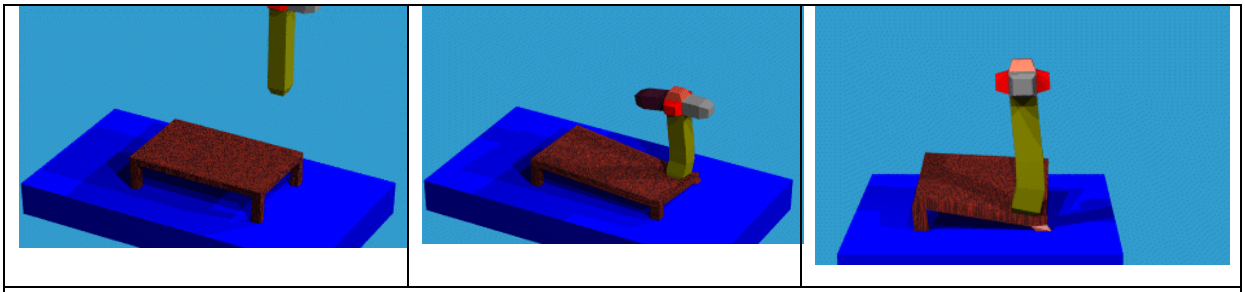

Fig. 1: Frames of image indicating the collision among a flexible hammer and a flexible table.

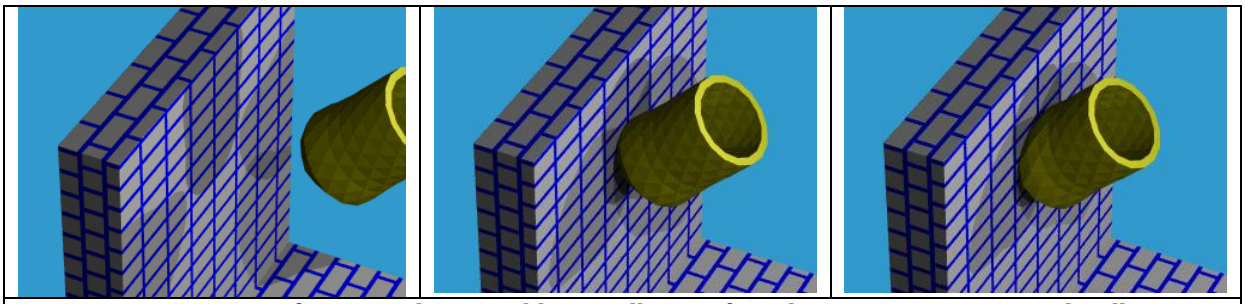

Fig. 2: Frames of image indicating oblique collision of an elastic cup against a rigid wall.

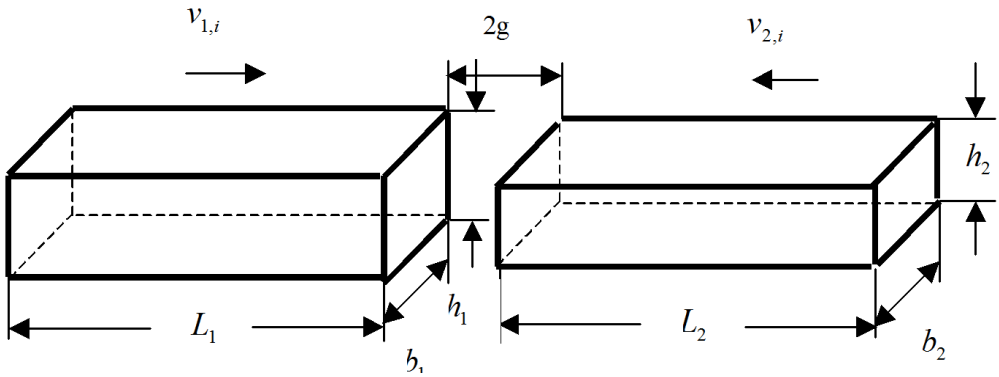

Fig. 3: Impact problem between two prismatic rods. ($L_1 = L_2 = 0.254$ m, $b_1 = b_2 = 0.0254$m, $h_1 = 0.0254$ m, $h_2 = 0.0224$ m, g = 2.54×10^{-4} m, $v_{1,i} = 5.1359$ m/s, $v_{2,i} = -5.1359$ m/s, $\rho = 7844$ kg/m^3, $E = 206.82 \times 10^9$ N/m^2)

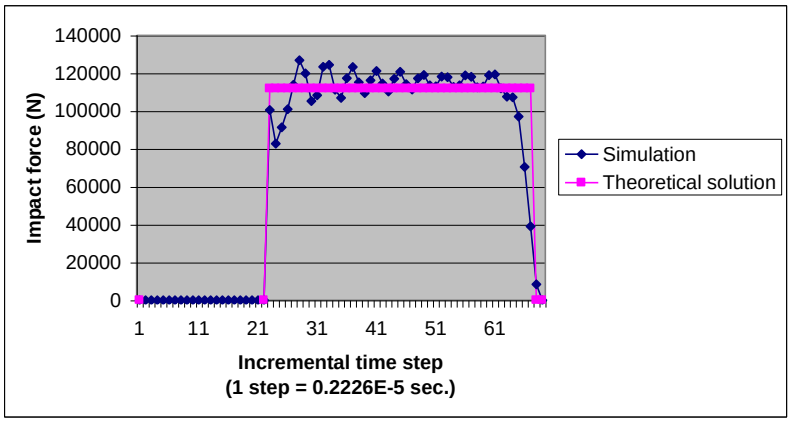

Fig. 4: Impact-force relationship for the problem in Figure 3.

Conclusions:

Overall, the main feature of the proposed solution is accurate, general-purpose and yet fast in a collision response analysis, which represents a new compromise between efficiency and functionality. It overcomes the shortcoming of the conventional finite element method for being computationally expensive and the limitations associated with the boundary element method. It also provides more realistic contact deformation than the global deformation approaches, avoids the undesired properties associated with the penalty method, and avoids additional assumptions on deformation zones made in the global-local approach.

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