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# A Generative System of Interlocking Structures with one Type of Octocube 

Authors:
Shen-Guan Shih, sgshih@mail.ntust.edu.tw, National Taiwan University of Science and Technology Yu-Ren Ho,d10113001@mail.ntust.edu.tw, National Taiwan University of Science and Technology

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Introduction:
A polycube is the configuration created by joining more than one cube of the same size with adjacent faces fully aligned. An octocube is a polycube composed of eight cubes. This paper investigates some properties of a specific type of octocube, instances of which can be systematically interlocked to form a large variety of 3D structures. In particular, we are most interested in three issues, first of which is the possibility for sophisticated propagation with just one type of polycubes. It is regarding what and how a type of simple polycube can be interlocked to form larger and ever more sophisticated structures. The second issue is regarding how the complex interlocking forms can be derived systematically. The third issue is to discover the essential geometric characteristics that enable the base polycube to acquire the above capacities. Interlocking is a very challenging geometric problem that closely related to CAD/CAM technology [2]. Interlocking of assembled structures can be analyzed by finding the degrees of translational freedom for individual part of the structure, and the network of relations for parts engagements [1]. Researches have been conducted to the development of algorithms for automatic generations of interlocking parts for desirable forms [1-3]. Among these researches, polycubes were often used as the basic structure of the assembled parts [2-3].

The finding unclosed in this paper may contribute to the discovery of methods to assemble large 3D forms by using just a few basic shapes as interlocking parts. The octocube as shown in figure 1 consists of an $S$-shape tetracube and an $L$-shape tetracube attaching to each other side by side. Eight instances of such type can be assembled into a 4-unit-size cube through the three steps shown in figure 2. The assembled set of eight elements is very well interlocked so that none of the element can be taken apart independently. The only way to dissemble the set of eight is through steps from right to left as shown in figure 2.


Fig. 1: An octocube with an $S$-shape part and an $L$-shape part.
The 4 -unit-size cube is not the only interlocking structure that can be assembled with the base octocube. It is possible to elongate the 4 -unit-size cube into any rectangular volumes with length and width as multiplications of 4 units (figure 3a). The interlocking structure can also be $X$-shaped (figure $3 b)$, $W$-shaped (figure 3c) or any other planar polycubes with 4 units in size. The interlocking structure can be further extended to including 4-unit-size planar polycubes with cavities inside (figure 4).


Fig. 2: Steps of eight identical octocubes to form a 4-unit-size cube.


Fig. 3: rectangular shape(a), X-shape (b), and W-shape (c).


Fig. 4: Shapes with cavity inside.
An interlocking structure can be defined based on a sequence of engagements between two adjacent octocubes. When one of such octocube with either its $S$ part or $L$ part attaching to the $S$ part or $L$ part of the other, it is called an engagement. Various interlocking structures shown above is derived through consecutively applying four types of engagements to form strings of octocubes. The four types are named as $H, h, A$ and $a$ engagements, as shown in figure 5a, 5b, 5c and 5d respectively.


Fig. 5: The $H$ engagement (a), The $h$ engagement (b), The $A$ engagement (c), The $a$ engement (d)

Figure 5a shows the $H$ engagement with two octocubes aligned along their longer axis but facing opposite directions, attaching to each other at the $L$ parts of both. Figure 5b shows the $h$ engagement with two octocubes aligned along their longer axis but facing opposite directions, attaching to each other at the $S$ parts of both. Figures 5c and 5d show that the $A$ and $a$ engagements both defined by two octocubes with their longer axis perpendicular to each other, as one octocube ride on the $L$ part of the other with its $S$ part. The two engagements of $A$ and $a$ result in the same structure if regardless of the order of placement. However, considering the different transformations that it takes to transform one octocube to the other with respect to each of the two, it is clearer to notate them as two different engagement types. Two pairs of octocubes formed by engagements $H$ and $h$ can be regarded as conjugates for they become an interlocked structure of four when they are joined together with tops facing against the other pair, as the model shown in the left-hand-side of figure 6. Two pairs of octocubes formed by engagements $a$ and $A$ would be conjugates for the same reason, except that the resulting structure is only semi-interlocked (figure 6 right), for parts of the structure allows not only one degree of freedom for translation, but also one degree of freedom for rotation.


Fig. 6: The Hh strand (left), and the aA strand (right).
More octocubes can be added through sequential applications of engagements. The left part of figure 7 shows a string of octocubes formed by sequential engagements of $h H h$, while the central part of the same figure shows the conjugates of the former with engagements of $H h H$. Since the two strings are conjugates to each other so that the eight octocubes would form an interlocked integrity when they are joined top against top as the structure shown in the right part of figure 7. The result can be called a strand of octocubes consisting of two conjugating strings of $h H h$ and $H h H$. With alternative engagements of $H$ 's and $h$ 's, the interlocked strand can be extended endlessly. The photo in figure 8 shows an interlocked $H h$ strand with 28 octocubes made with plastics supported at its two ends. The photo shows that the long and thin interlocking structure is strong enough to withstand the weight of a solid volume consisting of 96 octocubes loaded at the center without adhesive or other supporting materials.


Fig. 7: The strings of octocubes formed by $h H h$ (left), by $H h H$ (center), and their jointed strand.


Fig. 8: An Hh's strand made with 28 plastic octocubes supports a volume of 96 octocubes.

With $A$ and $a$ engagements, strings and their corresponding strands can make 90 degrees turns to left and right. Figure 9 shows one $H a$ string (figure 9 left) and one $h A$ string (figure 9 center) are jointed to form a strand that turns to the left (figure 9 right). Like a snake that eats itself from its own tail, a strand may follow an orthogonal path that eventually leads to join the end to its own start. The 4-unitsize cube shown in figure 2 is constructed as a cyclic strand consisting of an $A A A$ string and its conjugating aaa string.


Fig. 9: Ha string (left), $h A$ string (center) and the Ha\#hA strand (right).

The above examples show only a minute portion of interlocking structures that can be assembled with instances of the type of octocubes. More types of polycubes and engagements are to be unclosed for broader applications. The study leads towards applications in the fabrication and assembly of complicated structures. The generative system with just a few types of engagements may greatly reduce the difficulties of designing and fabricating interlocked structures.

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