Title:
Topology Optimization for Manufacturability Based on Visibility Map

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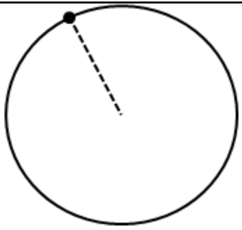
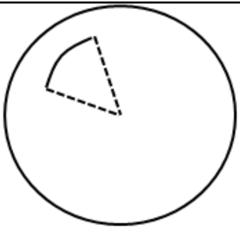
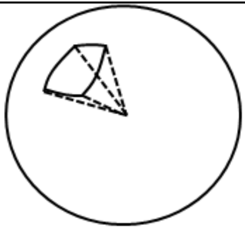
Introduction:
As a popular and effective tool for structural design, topology optimization has been increasingly used for mechanical part design in recent years. However, the effectiveness of topology optimization in mechanical design has been seriously affected by the poor manufacturability of parts generated. In this study, manufacturability in the topology optimization process is described by using the concept of a visibility map. Apart from additive manufacturing, almost all manufacturing processes can be associated with a visibility map. A part generated by topology optimization must conform to the visibility map of a manufacturing process thus generating optimized design that is manufacturable by the proposed manufacturing process. Since the visibility map concept can be used to describe most manufacturing processes, the proposed approach can be used as a general method for all mechanical part design when topology optimization is needed.

Literature Review:
The wide spread use of topology optimization in structural design has caught the attention of researchers in mechanical engineering design. Over the years, there are some reports about practical applications of topology optimization in automotive [6] and prosthetics design [4], etc. However, current topology optimization methods have the tendency to generate hollow and framework-like features in the optimized design. To convert the optimized structure into a sensible mechanical design, manual intervention must be done. This process will defeat the purpose of optimization as the manually modified structure may not be the optimal any more.

To generate sensible mechanical parts directly from topology optimization, effort had been made to integrate manufacturing constraints into the structural optimization processes. Zuo et al had considered the minimum feature size and geometric symmetry as manufacturing constraints to generate parts that can be machined [8]. A hybrid of moving asymptotes and wavelets had been used to solve the topology optimization problem. Chang et al considered cost of manufacturing in their optimization process [1]. Niclas [5] had considered draw direction or draft angle as sample manufacturing constraints. Harzheim et al have reviewed optimization methods for cast parts [3]. All previous studies have considered only one or two constraints for a specific manufacturing process. No one has reported a general topology optimization approach that is applicable to most commonly used manufacturing processes.

In most manufacturing processes such as machining, casting/molding, or forging, etc., there are some primary directions. For instance, the tool approach directions for machining, parting directions for casting/molding, and punching direction for forging. Geometric features of a part design should be properly aligned with respect to these directions in order to be manufacturable. Inspired by this observation, this study proposes and implements a visibility map constrained topology optimization approach for mechanical part designs. The visibility map of a 3D object is generated on a unit sphere that encloses the object that is to be optimized [2,7]. Using visibility map, the complex problem of visibility can be addressed by simple spherical algorithms that invoke the intersection between the visibility map and a point, a great circle or s spherical rectangle.

<u>Visibility Map and The discretization of a Unit Sphere:</u>
Visual capacity is a term used to describe a manufacturing process. It is determined by its visual style and visual field. Different manufacturing processes have different visual styles, and different visual style could do different unit work in manufacturing. The visual capacity of manufacturing processes could also be expressed on the unit sphere. An illustration diagram about visual capacity (defined by visual style and visual field) on a unit sphere is shown in Tab. 1. Take “1 DOF” visual field and “Surface” visual style as an example, its visual capacity is just a point on the unit sphere. That is, the manufacturing process has just one manufacturing direction. The practical examples of this kind of processes include casting, molding and so on. Now, take a 3-axis CNC machining process as an example. It has a “Point” visual style and “3 DOF” visual field. Its visual capacity is also a point on the unit sphere. That is, the tool can only access the object from one direction. For the case of 5-axis CNC machining, it has a “Point” visual style and a “5 DOF” visual field. Its visual capacity is represented as a spherical polygon region on the unit sphere.

Visual Styles	*Visual Fields*		
Surface	1DOF	2DOF	3DOF
Line	2DOF	3DOF	4DOF
Point	3DOF	4DOF	5DOF
Spherical expressions of visual capacity			

Tab. 1: Unit sphere expressions of visual capacity.

Since the Vmap (Fig. 1(a)) of an object and the visual capacity (Fig. 1(b)) of a manufacturing process are both expressed on a unit sphere, the visible region can be identified by moving the two-unit spheres to concentric unit spheres as shown in Fig. 1. Now, if the visual capacity (represented as the thick lines) intersects with all Vmap (represented as thinner lines) in Fig. 1(c), then the part is manufacturable. Otherwise, the non-intersected Vmap will not be manufacturable.

Now, the problem is how to define the manufacturing directions. Refer to Tab. 1, the visual capacity of a manufacturing process is defined on a unit sphere. In this paper, the unit sphere is discretized by dividing it along longitude and latitude as shown in Fig. 2. Each intersection point between a longitude and a latitude represents a potential manufacturing direction. Of course, the more longitude and latitude lines we use, the higher accuracy the sphere discretization will be.

Suppose we have m longitude lines and n latitude lines, the total number of intersection point can be represented as a two dimensional array S[i,j] where i=[1,m] and j=[1,n]. Because both the south pole and the north pole converges to a single point, they are represented independently as P_s and P_n.

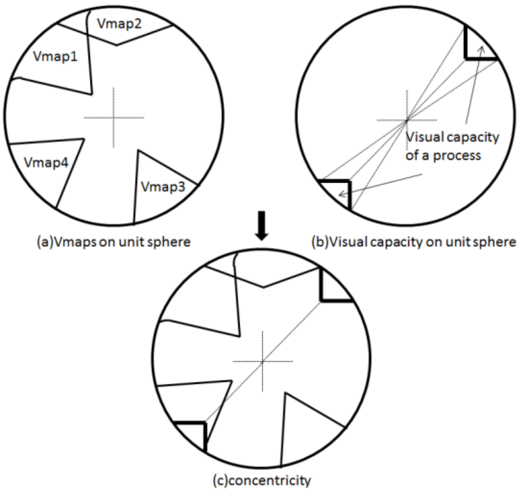


Fig. 1: Concentric spheres of Vmaps and visual capacity.

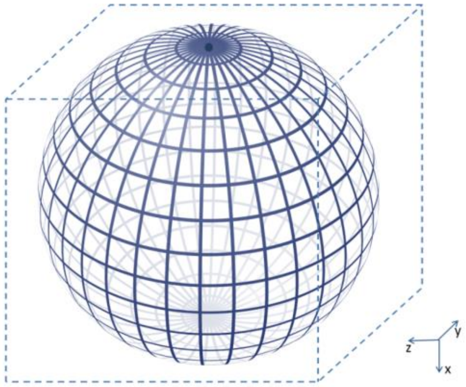


Fig. 2: Discretizing a unit sphere.

Constrained Topology Optimization:

Generally, a topology optimization problem for minimum compliance could be expressed as Eqn(1):

$$\begin{aligned} \min_{\mathbf{u},\rho_e} &: c_{(\rho)} = \mathbf{f}^T\mathbf{u} \\ s.t. &: (\sum_{e=1}^{N} \rho_e^p \mathbf{K})\mathbf{u} = \mathbf{f}, \\ &: \sum_{e=1}^{N} v_e \rho_e \le V, \\ &: 0 < \rho_{\min} \le \rho_e \le 1, \quad e = 1, \dots, N. \end{aligned} \tag{1}$$

Where $c_{(\rho)}$ is the compliance of the structure. **f**, **u** and **K** are the global load, global displacement and stiffness matrix respectively. ρ_e indicates the relative density of each element; ρ_{min} presents the minimum relative density. p is the penalization index which is normally assigned to 3. N is the number of elements. ve is the element's volume and V is the total volume of the design domain.

The SIMP scheme uses an Optimality Criteria method to update relative densities. The formulation of this process is expressed in Eqn (2):

$$\rho_{K+1} = \begin{cases} \max\{(1-\zeta)\rho_K, \rho_{\min}\} \\ \quad if \ \ \rho_K B_K^{\eta} \le \max\{(1-\zeta)\rho_K, \rho_{\min}\}, \\ \min\{(1+\zeta)\rho_K, 1\} \\ \quad if \ \ \min\{(1+\zeta)\rho_K, 1\} \le \rho_K B_K^{\eta}, \\ \rho_K B_K^{\eta} \\ \quad otherwise. \end{cases} \tag{2}$$

Where η is the damping coefficient with a value assigned to 0.5; ζ is the move limit with value assigned to 0.2. These values are determined based on experiments of Sigmund and Bendsoe. In Eqn(2), the variable BK could be obtained by Eqn(3) as follow:

$$B_K = \frac{-\dfrac{\partial c}{\partial \rho_e}}{\lambda \dfrac{\partial V}{\partial \rho_e}} \tag{3}$$

In Eqn(3), λ is a Lagrangian multiplier which is decided by a bi-sectioning method. The sensitivity of objective function ∂c/∂ρe could be computed by Eqn(4) as:

$$\frac{\partial c}{\partial \rho_e} = -p(\rho_e)^{p-1}\mathbf{u}_e^T \mathbf{k}_e \mathbf{u}_e \tag{4}$$

When the design space is converted to a finite element environment, the visibility could be described by using Fig. 2. In a given direction d, the boundary element is visible if the solid elements are lined as shown in Fig. 2(a). On the contrary, Fig. 2(b) shows a structure with invisible boundary element in direction d. Therefore, given a manufacturing direction d, if the boundary element in this direction is visible, the following equation about element densities must be satisfied:

$$\rho_i \ge \rho_{i+1} \ge \dots \ge \rho_n \tag{5}$$

Using Eqn (5) as constraint in the above described topology optimization algorithm, the resulting structure will be visible from the given direction.

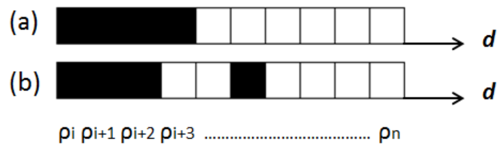


Fig. 2: An element's visibility in a given direction d.

<u>Conclusions:</u>

This study presented a general approach for topology optimization in mechanical part design so that a part generated could be manufactured by the intended manufacturing processes. Visibility is thought to be well associated with manufacturing capabilities of most manufacturing processes. It is thus used to constrain the topology optimization such that the resulting part could be manufactured. Two examples will be used to show the effectiveness of the proposed approach in which results generated from the proposed approach have very simple geometry and good manufacturability.

In this paper, only point visual capacity manufacturing processes are presented. In future works, the line and surface visual style for manufacturing processes such as 5-axis CNC machining will be explored. Meanwhile, in order to fully consider manufacturability, accessibility (with consideration of tool geometry and size) must be taken into consideration together with visibility because even if a surface is visible by the effector, the interferences between the effector and the part surface could still make the surface inaccessible.

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