



Title:

Moving Sequence Generation Based on Hungarian Method for Pick-and-Place Process

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Introduction:

Just like bricks are used in building construction, in rapid prototyping identical components can be aggregated to form 3D objects by a process called pick and place (PAP). Recently, an aerial construction PAP platform was developed [1-3], in which unmanned aerial vehicles (UAV) were programmed to carry blocks from an original to a designated position. Over time, the UAVs could build complex structures, such as a curved wall and a tower [1], as shown in Fig. 1a. The moving sequence was preset in order to avoid aerodynamic interference between the UAVs. In the long term, this new technology would benefit from an efficient PAP algorithm.



Fig. 1: (a) UAV assembled architecture installation built a tower from a pile of foam bricks [1], (b) A pavilion constructed in the campus of the University of Applied Sciences in Detmold. It consists of more than 2000 beer boxes, (c) A model of Arc de Triomphe made of around 10,000 food cans.

Recently, designers created fascinating structures by reusing discarded materials such as boxes (Fig. 1b) and cans (Fig. 1c). In these projects, the PAP process was also pre-determined and was based on the position of material stock and destination structure. The transformation from one structure to another poses an interesting research question in rapid prototyping: how can a system recognize the overlapping regions of two structures and minimize the number of moves in a PAP process.

To address the question, we propose an efficient algorithm to generate 3D moving sequence for a PAP process, in which 3D models represented by identical components, such as cubes, bricks and cans, can be constructed and transformed from one to another. This algorithm minimizes the cost in PAP rapid prototyping while maintaining high computational efficiency compared with alternative algorithms.

Principle:

The generation of the moving sequence is based on Hungarian method [4, 5], which is originally used to solve assignment problems in graph theory. For example, n workers (X_1, \dots, X_n) each can do any of the n jobs (Y_1, \dots, Y_n) at a specific cost. The cost can be any reasonable application-dependent efficiency metric, such as time consumption. It is often expressed in a matrix form:

$$C(i, j) = \begin{matrix} & Y_1 & Y_2 & \cdots & Y_n \\ \begin{matrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \end{matrix} \quad (2.1)$$

where a_{ij} denotes the cost for the i -th worker to complete the j -th job. If one job is assigned to each worker, the total cost is sum of the cost of all workers. For instance, if X_i is assigned to Y_i , the total cost would be $\sum_{i=1}^n a_{ii}$; if X_i is assigned to Y_{n-i} , the total cost would be $\sum_{i=1}^n a_{i, n-i+1}$. It can be seen that different assignment strategies end up in different total costs. A brute-force search for the minimum cost requires comparison of $n!$ different sequences. The Hungarian method [4, 5] is a much more efficient search algorithm which has time complexity $O(n^3)$.

Given two models in a global coordinate, components in the first model but not in the second are named mover (M) components; those in the second model but not in the first are named unfilled (U) components; those present in both models are named fixed (F) components. Fig. 2 illustrates a 2D example of the M , U and F which represented by squares.

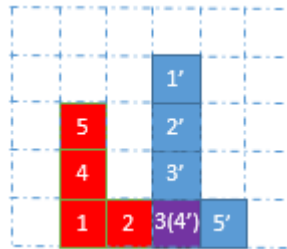


Fig. 2: A 2D example of square representation. The red and cyan squares form the first and second model respectively. No. 1, 2, 4 and 5 red squares are M ; No. 1', 2', 3', and 5' cyan squares are U ; No. 3 red and No. 4' cyan squares are F .

The problem of model transformation can be stated as finding an optimal PAP sequence to transport all M components to U components. This problem definition assumes that the number of M and U components is equal. If this is not the case, boundary conditions can be incorporated such that additional components may be added or removed. To apply the Hungarian method to find an optimal sequence, M components are treated as workers, U components are treated as jobs, and the moving cost such as distance from an M to a U component is treated as the cost. F components stay where they are; so they do not participate in calculation.

The cost matrix shown in Eq. (2.1) can be obtained by calculating the moving cost from any M component to any U component. The cost can be defined in various ways according to different circumstances. For simplicity, one example is to calculate the moving distance from M_i to U_j in Cartesian coordinate, which is shown as the following equation.

$$C_{ij} = |z_{max} - z_i| + |x_j - x_i| + |y_j - y_i| + |z_j - z_{max}| \tag{2.2}$$

where (x_i, y_i, z_i) denotes the position of M_i , and (x_j, y_j, z_j) denotes the position of U_j , and z_{max} is the highest position of the global coordinate. A mechanical device such as a Cartesian robot would pick an M component vertically to z_{max} (Fig. 3 path A), move it in z_{max} plane to (x_j, y_j, z_{max}) (Fig. 3 path B), and place it vertically to the destination position U (Fig. 3 path C). This strategy is a simple approach to avoid the mechanical device being in contact with the two models during the PAP process. The resultant cost matrix can be solved by the Hungarian method and an optimal solution indicates the minimum-cost pairing of M and U components. A moving sequence can then be generated based on the minimum-cost pairs, in considering physical constraints.

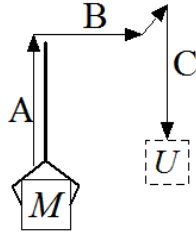


Fig. 3: An illustration of cost in PAP process.

The physical constraint considered in this study is called top-surface-access constraint, meaning PAP is assumed to be feasible only on the top surface of a model, i.e. at any step, an M component and a U component involved in a feasible move must be both on the top surface of the respective model. Bottom surfaces are assumed to be supported so that gravity is not considered.

Based on the example shown in Fig. 2, we have the cost matrix:

$$C(i, j) = \begin{matrix} & 1' & 2' & 3' & 5' \\ \begin{matrix} 1 \\ 2 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 7 & 8 & 9 & 11 \\ 6 & 7 & 8 & 10 \\ 6 & 7 & 8 & 10 \\ 5 & 6 & 7 & 9 \end{bmatrix} \end{matrix} \tag{2.3}$$

where each element of the matrix is calculated by Eq. (2.2). The Hungarian method produces the following minimum-cost pairs: 1->1', 2->2', 4->3', 5->5'. The total cost is 7+7+8+9 = 31. A feasible moving sequence is 5->5', 4->3', 2->2', 1->1', as shown in Fig. 4.

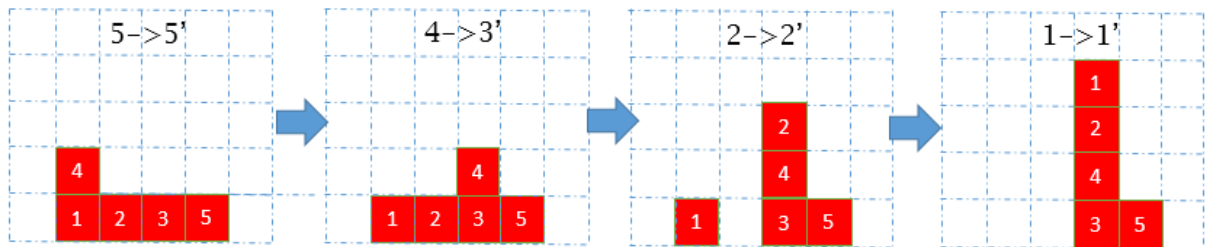


Fig. 4: A feasible moving sequence based on the minimum-cost pairs found by the Hungarian method.

In order to automatically obtain a moving sequence, the following strategy is applied. Instead of applying the Hungarian method to the entire M and U components, we extract M and U components that are on the top surface of the respective model, and apply the Hungarian method to these components only. The generated minimum-cost pairs by default would satisfy the physical constraint; hence, feasible for PAP. After the subset of M and U components are transported, we extract top-surface M and U components again, and apply the Hungarian method to the second subset of M and U components. This process is repeated until all M components are moved to U positions.

Based on Fig. 2 and the modified strategy, in the first round, M squares are $\{2, 5\}$ and U squares are $\{3', 5'\}$. The cost matrix is

$$C(i, j) = \begin{matrix} & \begin{matrix} 3' & 5' \end{matrix} \\ \begin{matrix} 2 \\ 5 \end{matrix} & \begin{bmatrix} 8 & 10 \\ 7 & 9 \end{bmatrix} \end{matrix} \quad (2.4)$$

The Hungarian method produces $2 \rightarrow 5'$, $5 \rightarrow 3'$ minimum-cost pairs. PAP can be applied in any order to these pairs. Fig. 5(a) and Fig. 5(b) show a particular PAP sequence. In the second round, 4 is the only M square and $2'$ is the only U square [Fig. 5(c)]. In the last round, 1 is the only M square and $1'$ is the only U square [Fig. 5(d)]. The total cost is $10+7+7+7=31$, which is the same as applying the Hungarian method to all M and U components in one go.

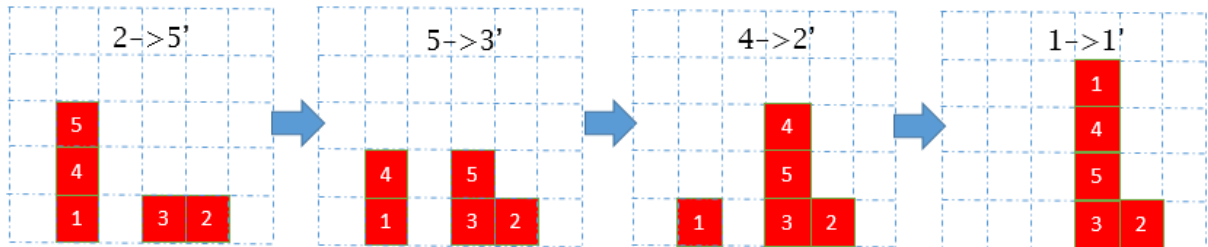


Fig. 5: An automatically generated moving sequence based on applying the Hungarian method iteratively to M and U squares on the top surface.

Experiments and results:

The above modified strategy is named as local moving sequence (LMS) approach. It can be easily transplanted to parallel system such as the UAV PAP system [5]. The components for construction can be arbitrary stackable shape, such as cubes, bricks or cans (cylinders). Examples of transformations among several architectural models (Fig. 6) in cylinder form are given in Fig. 7.

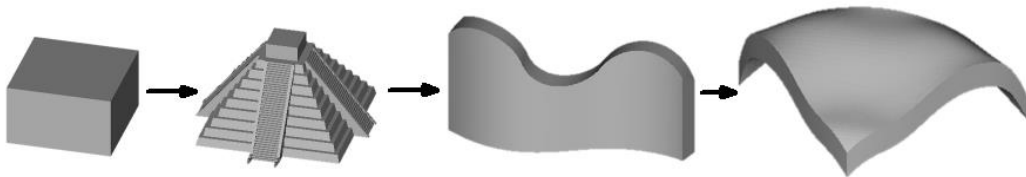


Fig. 6: Original architectural models for transformation (from left to right): Block, Pyramid, Curved Wall and the Boxel Pavilion (Fig. 1b).

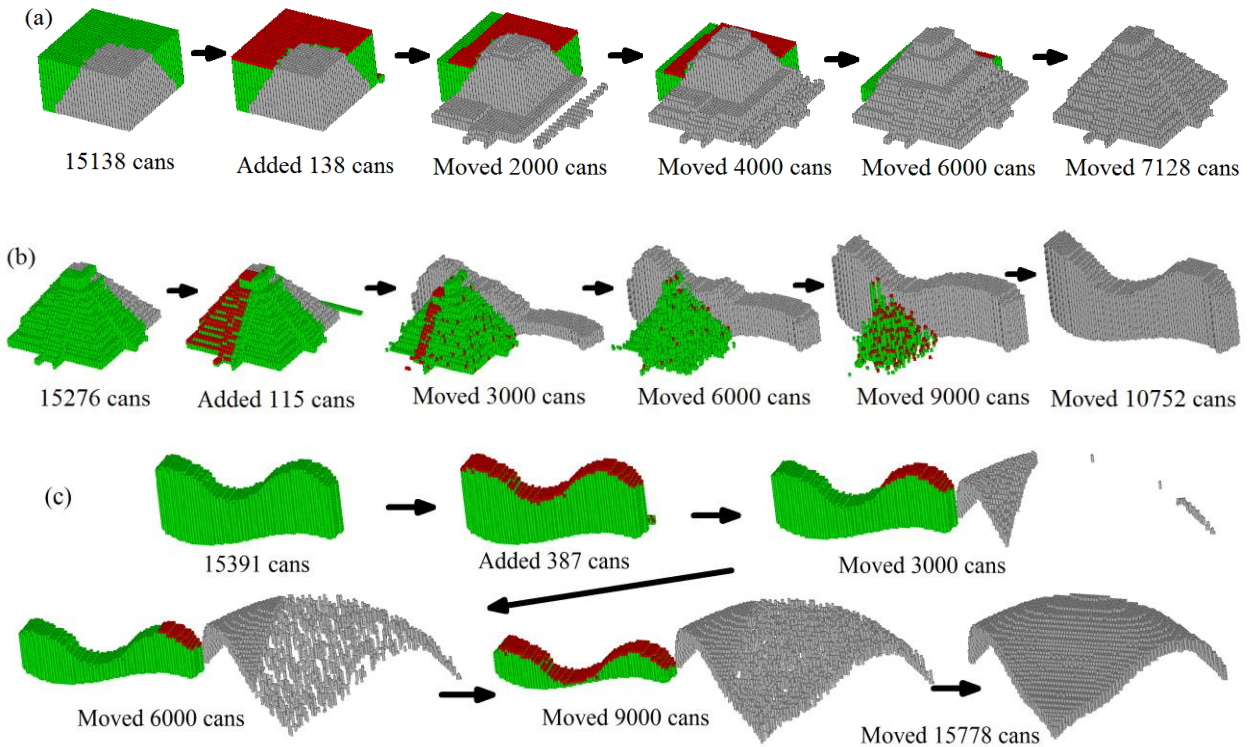


Fig. 7: Specific Models transformations by LMS approach. The red, green and gray cans are M cans in current round (being moved), M cans and F cans. (a) A Pyramid structure (15276 cans) was transformed from the block (15138 cans), (b) The Curved Wall (15391 cans) was transformed from the Pyramid in the previous process, and (c) The Boxel Pavilion (15778 cans) was transformed from the curved wall.

In order to evaluate our modified strategy, non-optimized random sequence (NORS) approach is also compared. In this approach, M and U components on the top surface are extracted and without optimization, an M is randomly paired and moved to a U until all M components are moved to U components.

Based on many experiments, we find that the LMS approach, can produce comparable low-cost sequences as the global moving sequence (GMS) approach (Fig. 7a) while maintaining a much higher computational efficiency, which is comparable as NORS due to the small number of components involved in each round. Fig. 7b shows a time comparison of these approaches based on several experimental datasets.

References:

- [1] Augugliaro, F.; Lupashin, S.; Hamer, M.; Male, C.; Hehn, M.; Mueller, M.; D'Andrea, R.: The Flight Assembled Architecture Installation: Cooperative Construction with Flying Machines, Control Systems, IEEE, 34(4), (2014), 46-64. <http://dx.doi.org/10.1109/MCS.2014.2320359>
- [2] Lupashin, S.; Hehn, M.; Mueller, M.-W.; Schoellig, A.-P.; Sherback, M.; D'Andrea, R.: A platform for aerial robotics research and demonstration, The Flying Machine Arena. Mechatronics, 24(1), 2014, 41-54. <http://dx.doi.org/10.1016/j.mechatronics.2013.11.006>
- [3] Mirjan, A.; Gramazio, F.; Kohler, M.; Augugliaro, F.; D'Andrea, R.: Architectural fabrication of tensile structures with flying machines, Green Design, Materials and Manufacturing Processes, 2013. <http://dx.doi.org/10.1201/b15002-99>

- [4] Kuhn, H-W.: The Hungarian method for the assignment problem, Naval research logistics quarterly, 2(1 - 2), 1955, 83-97. <http://dx.doi.org/10.1002/nav.3800020109>
- [5] Munkres, J.: Algorithms for the assignment and transportation problems, Journal of the Society for Industrial & Applied Mathematics, 5(1), 1957, 32-38. <http://dx.doi.org/10.1137/0105003>

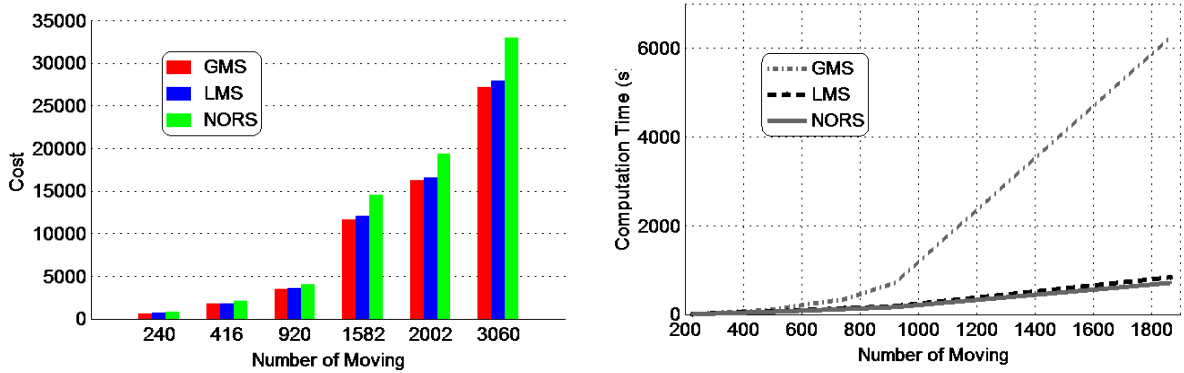


Fig. 7: Difference of computational cost (a) and time (b) of GMS, LMS and NORS.