Title:
CAD Models from Medical Images using LAR

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Introduction:
This paper points out the main design goals of a novel representation scheme of geometric-topological data, named Linear Algebraic Representation (LAR), characterized by a wide domain, encompassing 2D and 3D meshes, manifold and non-manifold geometric and solid models, and high-resolution 3D images [1]. To demonstrate its simplicity and effectiveness for dealing with huge amounts of geometric data, we apply LAR to the extraction of a clean solid model of the hepatic portal vein subsystem from micro-CT scans of a pig liver.

Technological advances made it possible to acquire large sets of biomedical data at a fast rate and affordable costs. In turn, the easiness of producing and collecting data in digital form has triggered a progressive paradigm shift from experiments on model organisms to simulation based on virtual prototypes and mathematical modeling [5].

The capability to extract geometrical models from medical images fosters the development of quantitative, evidence-based medicine, where laboratory and clinical observations are cumulated and made accessible to integrative research. In the near future, the collected knowledge of multifarious physiological subsystems on a hierarchy of dimensional scales and of a variety of biological functions will be formalized, catalogued, organized, shared and combined in many ways, providing integration across subsystems, temporal and spatial scales, biomedical and bioengineering disciplines, to give rise to personalized healthcare.

Consistently with the availability of quantitative data, the interest in physically-based simulations, customary in engineering CAD, is now growing also in medicine, with the clinical aim of getting a better understanding of physiology and pathologies on a single-patient basis, using personalized models extracted from patient's body scans. A meaningful example of this trend, akin to the application we focus on in this paper (the extraction of the liver portal vein system), is provided by the current developments in techniques aiming at providing surgeons with accurate, patient-specific guidelines when designing partial hepatic resections for the treatment of liver tumors [2].

The rising applications of 3D medical modeling [3], computer-based training of medical doctors, computer-assisted surgery, etc. call for the convergence of methods and know-hows from computer imaging, computer graphics, geometric/solid modeling, and physical modeling and simulation. Similar challenges are posed also by more established endeavors, such as materials science—think of soft matter, engineered surfaces, nano-materials and meta-materials—and biophysics, where modeling issues range from the molecular/protein level to multi-scale modeling of subcellular organelles, cellular structures, tissues and organs. Serious progress in these directions demand major innovations, from cooperative collaboration to multi-physics support, where different field equations imply different geometric structures at the level of basic descriptive data, to enhanced robustness toward scale mismatch in coupled problems, complexity of the simulation environment, terascale number of elementary entities or agents.

In this paper we show that most common geometric and imaging operations reduce in LAR to simple compositions of linear operators, implemented by sparse matrix multiplication and transposition, possibly supported by advanced graphics hardware. We expect this approach to be beneficial for producing the CAD tools of the next generation, capable to face the challenge posed by the treatment of big geometric data, when solid models are to be derived from 3D and 4D high-resolution images. A sample application of this sort is presented and discussed.

Linear Algebraic Representation:

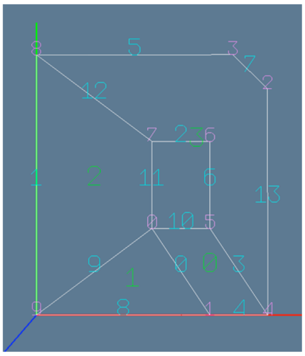
A representation scheme is a mapping between the mathematical spaces to be represented by a computer system and their symbolic representation in computer memory [4]. The Linear Algebraic Representation (LAR) scheme [1], uses Combinatorial Cellular Complexes (CCC) as its mathematical domain, and various compressed representations of sparse matrices as its codomain.

Since LAR provides a complete representation of the topology of the represented space, the matrix $[\partial_d]$ of the boundary operator may be used to compute the coordinate representation $[\partial_d\ c]$ of the boundary chain of any collection c of cells, through a single operation of SpMV multiplication between the CSR (Compressed Sparse Row) representation of $[\partial_d]$ and the CSC (Compressed Sparse Column) representation of the $[c]$ chain.

Importantly, the matrices of coboundary operators $[\delta_0]$, $[\delta_1]$, and $[\delta_2]$, computable in the LAR scheme by means of multiplications between sparse matrices, provide respectively the discrete *gradi- ent*, *curl*, and *divergence* on the given space decomposition. The *Laplacian* operator Δ is computed as a combination of *boundary* and *coboundary* operators. Last but not least, the standard operators of *mathematical morphology* on images (*dilation*, *erosion*, *opening* and *closing*) are obtained by product of sparse matrices of topological incidences times sparse matrices of boundary and/or coboundary.
The first important concept introduced by LAR is the definition of the *model* of a cell complex, as composed of a list of vertices, each of which is given as a list of coordinates, and by one or two topological relations.

Definition **1** (LAR model). *A* LAR model *is either a pair V,FV, or a triple V,FV,EV, where:*

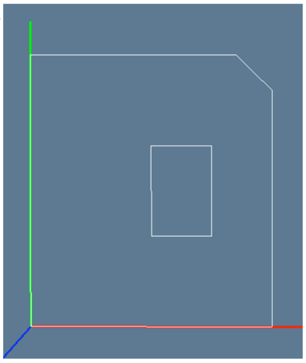
1. *V is the list of* vertices, *given as lists of coordinates;*
2. *FV is a* cell-vertex *relation, given as a list of cells, where each cell is given as a list of vertex indices;*
3. *EV is a* facet-vertex *relation, given as a list of cells, where each cell is given as a list of d vertex indices and* facet *stands for* $(d-1)$*-face of a* d*-cell.*



```
V = [[0.395,0.296],[0.593,0.0],[0.79,
 0.773],[0.671,0.889],[0.79,0.0],
 [0.593,0.296],[0.593,0.593],
 [0.395,0.593],[0.0,0.889],[0.0,0.0]]

FV = [[0,1,4,5],[0,1,9],[0,7,8,9],
  [2,3,4,5,6,7,8]]

EV = [[0,1],[8,9],[6,7],[4,5],[1,4],
  [3,8],[5,6],[2,3],[1,9],[0,9],[0,5],
  [0,7],[7,8],[2,4]]
```



(a) (b) (c)

Fig. 1: (a) LAR model with 0−, 1−, and 2−cells; (b) the triple V,FV,EV of *vertices*, *faces* and *edges* (indexed on vertices); (c) the extracted boundary. Note that 2-cells have different numbers of vertices, and may be *non-convex*.

Several basic representations of topology are used in the LARCC library, including some common sparse matrix representation: CSR (Compressed Sparse Row), CSC (Compressed Sparse Column), COO (Coordinate Representation), and BRC (Binary Row Compressed).

A BRC representation is an array of arrays of integers, with no requirement of equal length for the component arrays. The BRC format is used to represent a (typically sparse) binary matrix. Each component array corresponds to a matrix row, and contains the indices of columns that store a 1 value. Zero values are not stored.

```
FV = [[0,1,4,5],
      [0,1,9],
      [0,7,8,9],
      [2,3,4,5,6,7,8]]
```

$$\mapsto M_2 = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0] \end{pmatrix} \mapsto$$

```
<4x10 sparse
matrix of type
'numpy.uint8'
with 18 stored
elements in CSR
format>
```

Fig. 2: The binary characteristic matrix M_2 (centre) of the cellular complex in Figure 1 and its BRC (left) and CSR (right) representations.

Solid boundary representations: The representation scheme of topology most frequently used by solid modelers is a decompositive representation of the boundary, to be coupled with a meshing of the interior just in case of need. The boundary is usually decomposed into faces, with face boundaries represented in turn by a decomposition into edges, given as pairs of vertices. In the case of manifold representations, storing only a subset of the binary incidence between such boundary elements is sufficient. Usual non-manifold representation relations include by necessity some set of pointers between incident pairs of boundary elements, usually circularly ordered to discriminate locally between interior and exterior, so doubling (at least) the storage size of the representation. Contrariwise, LAR includes only lists of cells as unordered lists of vertex indices, and manages equally well both manifold and non manifold models.

Comparison: The common reference term for comparing the memory requirement of solid boundary representations in 3D is the Winged-Edge scheme by Baumgart, which makes use of relation tables with a storage occupancy $8|E| + |V| + |F|$, where F, E, V stand for the sets of boundary faces, edges and vertices, respectively. An equivalent LAR representation of topology of the boundary of a 3D solid (B-rep) needs only the storage of the CSR(M_2) sparse matrix, corresponding to the *FV* incidence relation, and the computation of the CSR(M_1) sparse matrix, to obtain the *EV* relation, for a total memory size of $2|E| + 2|E|$, according to [6].

<u>LAR of Images:</u>

In this section we mainly discuss how to map a d-image, with normally d $\{2, 3\}$, to the coordinate representation of *chains* (collections of voxels) within the linear space C_d generated by the *cellular complex* corresponding in (generalized) row-major order to the image voxels, using LAR.

From d-images to chains and cochains: In order to generate the coordinate representation of a chain in a multidimensional image (or d-image), we choose a basis of image elements—i.e., of d-cells—and a total ordering of image voxels, then map the multi-index identifying each d-cell to a single integer, so labelling the cell with its ordinal position within the chosen basis ordering. Assuming that vertices are located on a 3D lattice of points with integer coordinates, it is easily seen that an explicit storage of coordinates is not required, because of an explicit bijective mapping μ between the ordinal index of cells and the tuples of coordinates of their vertices.

Our model of a *d*-image is a cuboidal grid with integer coordinates. Every *d*-cell is identified by a *d*-tuple of integer coordinates, mapped to a single integer, in order to compute the basis vector corresponding to the cell in the linear space C_3 of chains of 3-cells (voxels).

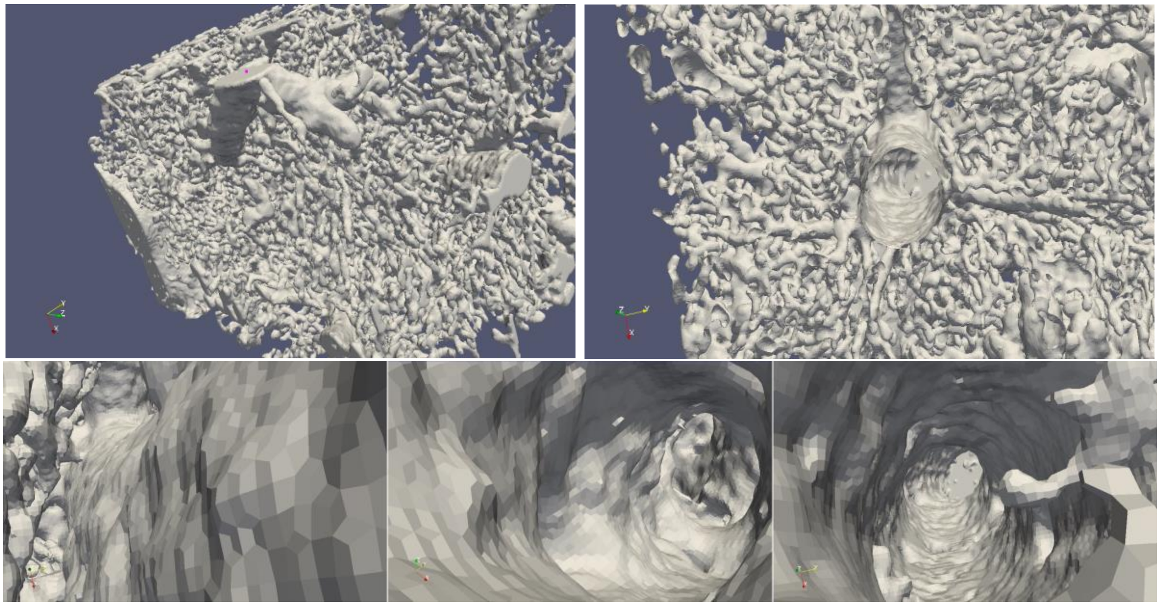


Fig. 3: The exterior and a cross-section of the portal vein system through the liver sample (top). Images of a vein interior (bottom). Note the mesh of quadrilaterals.

Let us remark that the matrix $[\partial_3]$ of the boundary operator $C_3 \rightarrow C_2$, used to compute the boundary of any possible subset of voxels, i.e., of any vector in the linear space C_3 of 3-chains associated with the image, depends *only* on the image shape (n_0, n_1, n_2), and may be computed once for all (choosing a set of standard image shapes), and stored or transmitted accordingly. Since the bottleneck of GPGPU implementations lies in the moving of data from global to local memory, our solution is to store the (sparse) matrix operator $[\partial_3]$ of n^3 voxels, with n {64, 128, 256}, in *Constant Memory*, and move just the (binary) *coordinate vectors* of chains in *Private Memory*. The boundary computation is therefore done by partitioning the image, according to the paradigm *divide et impera*.

Imaging Morphology: In this section we show how to implement *with* LAR the four operators of mathematical morphology, i.e., *dilation*, *erosion*, *opening* and *closing*, by way of matrix operations representing a composition of the linear topological operators of *boundary* and *coboundary* with other incidence relations. We give here just a few hints of these computations. Thanks to its multi-dimensional nature, the LAR implementation of morphological operators is dimension-independent.

Extraction of models from images: Biomedical applications require fast performance with big geometric data, for topological tasks such as model extraction from 3D images. In medical images density values represent scalar fields (cochains) over cubical cellular complexes, and LAR is used to

guarantee topologically correct 3D image segmentation as well as to extract (enumerative) solid models subsequently smoothed out via Laplacian smoothing. This approach has the nice feature that the entire image is partitioned into a set of cochains associated with field values, including the interstitial space, thus providing a well-defined mesh both of the relevant features and of their outer space.

The stored content of any image chain (subset of image elements; either pixels or voxels) shall be seen as a cochain associated to the given chain, and its discrete integrals (e.g. the volume, or surface area, or inertia moments) or other chains to be computed by means of discrete differential operators, shall be computed accordingly, by the proper SpMspV multiplication, taking appropriate benefits by advanced computational hardware, e.g., by GPGPU methods.

Extraction of the liver portal vein system Venous systems are called portal when a capillary bed pools into another capillary bed through veins, without first going through the heart. The hepatic portal system is the system of veins comprising the liver portal vein and its tributaries. The liver is a vital organ of all vertebrates. In turn, hepatic vasculature is essential to the liver function. A good assessment of individual liver vasculature is preliminary to hepatic surgery. While the macroscopic structure of the hepatic vasculature is well studied, the microvasculature is not yet fully understood.

The present work is part of a collaborative effort *with* a Czech research team [2] based at the University of West Bohemia and the Charles University, integrating specialists in biomechanics, biophysics, informatics, liver surgery, radiology, and histology. We aim at increasing both the scope and the resolution of 3D liver imaging —an arduous goal, but crucial to enhance our understanding of liver lobule anatomy and function.

Conclusion:

This paper demonstrated that LAR — a general-purpose framework for solid and geometric modeling — has the capability of generating topologically valid and geometrically accurate boundary models.

Our prototype implementation of LAR is an integral part of a permanent effort to rethink the foundations of solid modeling, aiming at simplifying and generalizing its data representation and disentangling its main algorithms, in order to produce a computational framework well adapted to the “new world” of big geometric data over cloud- and web-based infrastructures. This long-term project has already achieved some tangible results in applications to the extraction of solid models from 3D medical images (as documented in this paper) and to the simplified generation of building models for indoor mapping and the *Internet of Things*.

In conclusion, we would like to remark that any model mesh, either of the interior or the external surface, using either unstructured (triangle, tetrahedra) or structured (quadrilaterals, hexaedra) or more general convex cells, can be stored on computer media, and transmitted on communication networks, using LAR as efficient representation of topology and as support for curved geometry.

We see great opportunities in this project: (i) LAR uses just arrays of signed integers, instead of complicated data structures, to describe 1D/2D/3D/4D/... meshes/images and topologies of any sort and size; (ii) whenever necessary, LAR uses distributed algorithms of *MapReduce* kind; (iii) it is based on the well-established conceptual infrastructure of algebraic and combinatorial topology.

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