Title:
5D Cubic B-Spline Interpolated Compensation of Geometry-Based Errors in Five-Axis Surface Machining

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Introduction:
In computer numerically-controlled (CNC) machining technology, the interpolator is responsible for generation of position commands required to replicate an intended shape of the part with minimum deviation. In the early days of numerically controlled (NC) systems, tool interpolation was performed by means of linear and circular interpolations and this permitted simple computations with a minimal amount of constraints. However, these straightforward interpolation schemes were later found unable to meet the accuracy and productivity expectations associated with the fabrication of mechanical components whose shape is defined by multiple complex or freeform surfaces.

The approximation of 3D tool paths/curves with a set of linear and circular piecewise segments results is often associated with several drawbacks, such as: large number of line segments, discontinuities, feed rate fluctuations as well as heavier data transmission loads. As such, in order to reduce the number of accelerations and decelerations, and to preserve a relatively constant feed rate along the intended tool path, the number of approximating linear piecewise segments should be maximized. On the other hand, long linear segments often mean large deviations from the design surface combined with significant decreases in quality/smoothness of the machined surface.

When it comes to five-axis machining interpolation, two primary options exist: (i) tool path calculation in workpiece coordinate system; and (ii) tool path calculation in machine control coordinate system. Langeron et al. [5] have commented on the relative difficulty associated with prioritization of these tasks in the context of the overall structure of five-axis CNC machining systems, since problem (i) can be solved before or after problem (ii). While most of the prior studies have assumed that problem (i) should be solved before problem (ii), the current study practically advocates for solving (ii) prior to (i). While some prior steps have already been made in this direction [1,9], the current study will provide a direct comparison between the effectiveness of linear and cubic B-Spline interpolations. However, unlike in these past attempts which relied either on conventional B-Spline fitting procedures [9] involving knot removal algorithms [8] or on 5D quadratic interpolations [1], the current work proposes an original B-Spline calculation technique to be implemented directly in the 5D joint space of a five-axis machine tool.

Main idea:
Due to the limitations of the numerical controllers equipping the five-axis machine tools, the cutter is unable to track continuously a 3D tool path located on a sculptured surface. The negative impact of frequent accelerations and decelerations on machining productivity has been experimentally assessed by Vickers and Bradley [10] who have shown more than two decades ago that the programmed feed rate is achieved on average for only 10% of the total machining time. Moreover, sudden changes in

cutter trajectory contribute to the generation of less smooth surfaces and this in turn requires additional polishing operations that will further increase the overall fabrication costs of the mechanical components with shapes delimited by freeform surface geometries.

In five-axis CNC machining, the cutter typically follows the tool path in a sequence of small and discrete motions. The distance between consecutive CC points – that constitutes the core of the discretization algorithm – can be determined from a variety of geometric constraints. Some of the options that were explored in the past are: chordal deviation [4], geometry-based error [6], constant parameter increment [7] or other methods essentially derived from them [11]. Tool path discretization could also be performed in such a way to enforce a constant feed rate along the intended tool path in manner similar to that suggested initially by Farouki and Sakkalis [3]. Regardless of the tool path discretization selected, an array of successive CC points ($\mathbf{P}_{CC_i}$ and $\mathbf{P}_{CC_{i+1}}$) will be generated along the intended tool path (Figure 1).

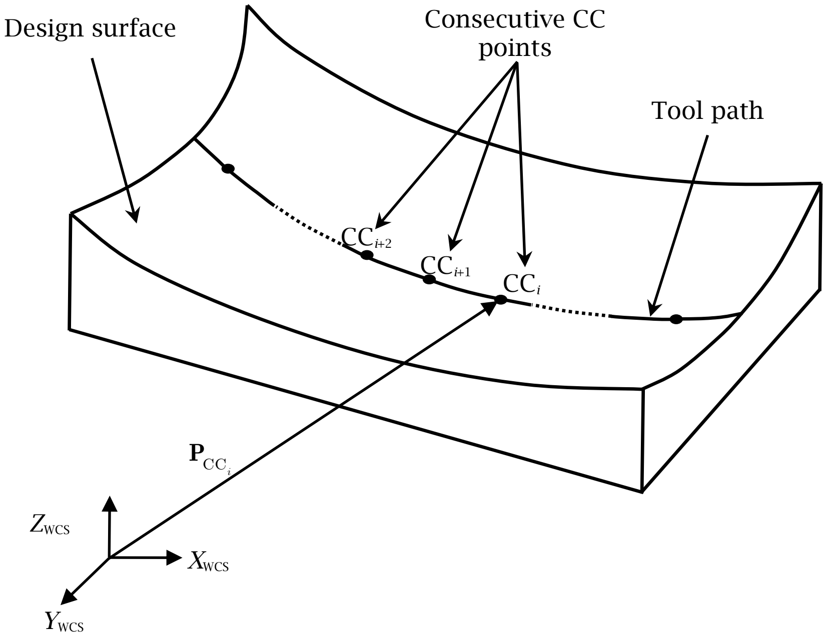


Fig. 1: Discrete CC points along the intended tool path.

In order to address the velocity discontinuities associated with linearly-interpolated motions, a novel interpolation technique based on 5D cubic B-Spline formulation has been developed. Owed to its definition, B-Spline curve is a C^2 continuous composite curve consisting of multiple piecewise curves joined together at knot points. Another major advantage of using 5D B-Spline representation is that no further reparametrization between translational and rotational MCC trajectories is required in order to preserve the desired synchronization between them, since both curves are described by the same parameter and knot vector values. The idea of multi-dimensional B-Spline (NURBS) curves is not absolutely new in five-axis machining, but so far it was only considered from a control-related perspective [2].

The shape of the 5D cubic B-Spline is determined by the (n+1) known data points yielded from tool path discretization algorithm. However, unlike generic B-Spline fitting algorithms based on knot removal technique [8,10] or chord length parametrization [8], the algorithm used in this section is based on a knot vector whose components are synchronized with the set of given data points. This approach will provide a direct comparison basis between 5D linear and B-Spline interpolations, since both schemes approximate the ideal MCC curve with the same number of given data points. While a somewhat similar idea was employed in [1], no particular emphasis was placed at that time on velocity/acceleration discontinuities that inherently occur at the junction between consecutive 5D piecewise quadratic segments, and that constitute one of the focal points of the present work.

In order to demonstrate the utility of the proposed approach, the magnitude and directional velocity discontinuities were assessed while performing 5D linear interpolations along an isoparametric tool path (v = constant) placed on a bicubic Bezier surface patch $\mathbf{P}_S(u,v)$ shown in Figure 2. The approximate dimensions of the surface are 110x140 mm.

The five-axis machine tool assumed for this simulation was a vertical spindle-rotating *BA* type, with *B*-axis defined as primary rotary axis. The rotational arm of this machine – defined as the distance from the intersection of the machine's rotary axes to tool's CL point – had a length of 200 mm. The flat-end cutter assumed for this operation had a diameter of 10 mm. The intended isoparametric tool path was discretized based on a chordal deviation of 0.08 mm. Tool postures at each of the discrete CC points were determined from maximum material removal and gouging avoidance constraints and then converted into machine control coordinates by employing an inverse kinematics algorithm.

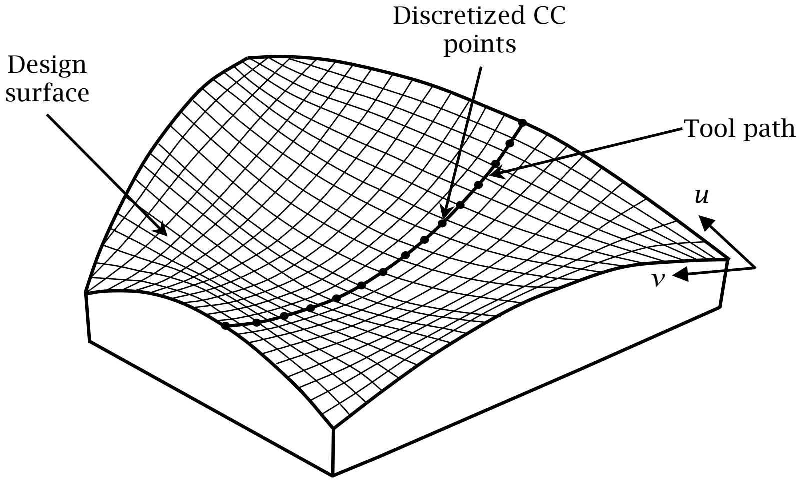


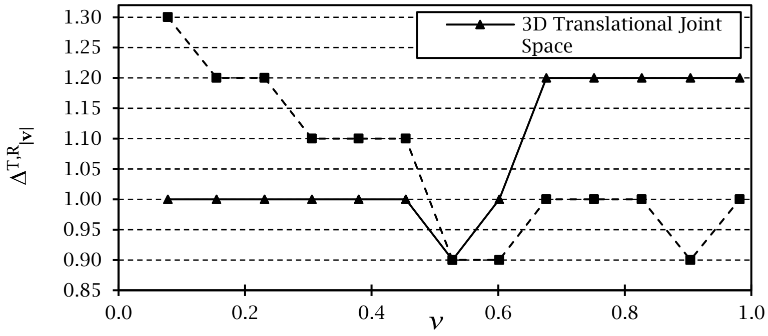
Fig. 2: Tool path and discretized CC points on the Bezier surface.

The angular and magnitude variations of velocity during five-axis linearly-interpolated motions of the cutter along the isoparametric tool path are shown in Figure 3. It can be noticed that while frequent updates in the direction of both translational and rotational velocities are necessary along the tool path, this is not the case for translational velocity magnitude that tends to remain unchanged for multiple consecutive linearly-interpolated segments. The increased rate of changes in angular velocity can be attributed to the highly nonlinear interdependence between tool orientation and rotational machine control coordinates. On the other hand, the reduced rate of changes required in case of translational velocity can be explained based on the quasi-constant arc length distances between successive CC points generated by the discretization technique used.

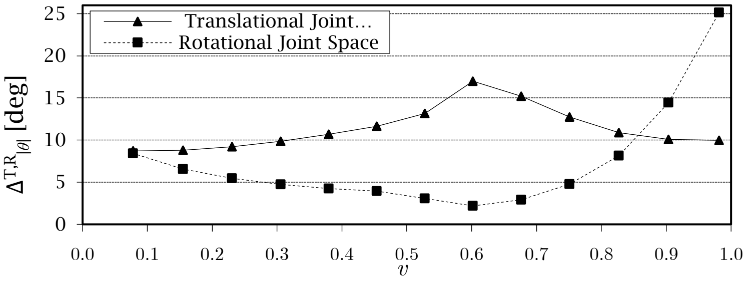
Indeed, since machined surface is characterized by relatively low curvature values, chordal deviation algorithm generates CC points that are quasi-equidistant in space and therefore their associated pivot points will also be located at almost equal distances both in workpiece and machine control coordinate systems. As a result, the changes in translational velocity magnitudes required to pass through these points in 3D translational joint space will be minimal. However, as shown in Figure 3b the direction of the translational velocity has to be tuned continuously, since the selected tool path is a 3D spatial curve that is far from being isoplanar.

All discontinuities outlined by Figure 3 were practically eliminated through the introduction of the proposed 5D cubic B-Spline interpolation scheme. For this particular tool path, the 15 given 5D MCC space points were interpolated with a B-Spline whose shape was determined by 17 control points. The range of variation for curve parameter λ is [0, 14], with a total of 21 knots required. Each distinct knot value is associated with one of the 15 given 5D MCC points.

By comparing the magnitude of geometry-based error values for each of the two analyzed cases of 5D interpolation (Figure 4), it can be noticed that in addition to C^2 continuity, 5D B-Spline interpolation reduces their size by 39.42 times (or with 89.66%) on average. The explanation of this drastic reduction in the magnitude of geometry-based errors along the tool path resides in the superior approximation of the ideal MCC curve enabled by the use of proposed cubic B-Spline interpolation.



a)



b)

Fig. 3: Joint space discontinuities in linearly-interpolated motions along the tool path: velocity magnitude change; and b) velocity direction change.

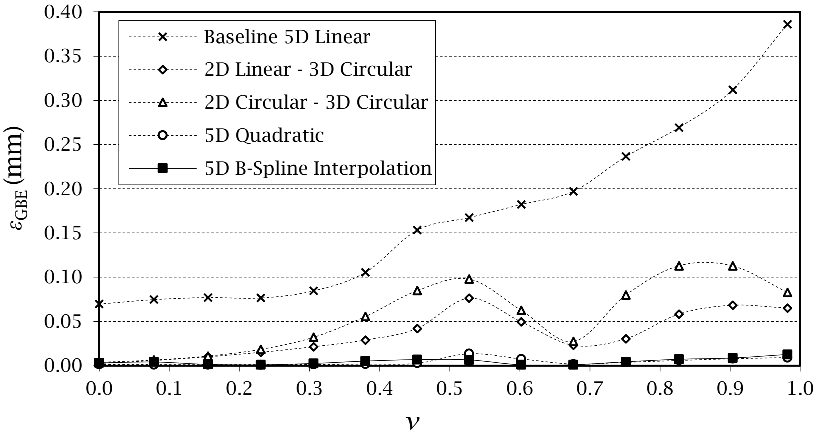


Fig. 4: Comparison between geometry-based errors generated in the context of several different types of 5D interpolations.

The existence of small amounts of geometry-based errors indicates that although the 15 targeted ideal tool postures were reached in a synchronized translational and rotational manner, the developed 5D B-Spline interpolation scheme still causes cutter deviations from the desired tool path between the given MCC points.

Conclusions:

This study proposes an enhanced 5D B-Spline interpolation technique for five-axis sculptured surface machining. Due to the inherent definition of B-Spline curves, a continuous curve is generated in 5D machine control coordinate space, such that no translational or rotational velocity discontinuities are further generated in joint space. In addition to the increased productivity achieved as a result elimination of sudden changes in the magnitude and direction of 5D MCC velocity during linearly-interpolated motions along the tool path, the developed higher-order interpolation scheme enables significant accuracy increases that were quantified through geometry-based error amounts. Indeed, for a certain tool path discretization, 5D B-Spline interpolation introduces significantly lower geometry-based errors along the intended tool path. This notable CAM-originated error reduction/compensation represents in fact a consequence of the improved approximation of 5D ideal MCC trajectory that is enabled by the implementation of the proposed B-Spline interpolation scheme.

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