Title:
Generating a Reference Model of the Surface with a Hole for Downstream Process

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Introduction:
3D data designed with 3D CAD systems are becoming vital communication tools between the design process and downstream processes. Quickly distributing 3D data to downstream departments can dramatically enhance their work efficiency. In the downstream department, 3D data received from a design department can be effectively utilized as a reference model for the creation of various process procedures and technical documents, such as creating visual assembly instructions, creating product manuals and product catalogs. In such works, clear visual communication for ease of understanding is important. However, size of 3D CAD data for expressing precise forms tends to big and takes long time to compute, therefore it may interfere with communication among users. In addition, since the internal data structures and tolerance do not coincide in each system, the intended shape model for downstream distribution may not be delivered. If the shape delivery fails in one system, the shape should be modified by using any methods to suit for the system. To overcome the problem, direct modeling which modifies the curve mesh is effective. For example, Fig. 1 shows the gap between two trimmed surfaces which was caused by different tolerance. In direct modeling, modification of a trimmed surface has the restriction where boundary edges must lie on the surface within a certain tolerance. Thus, it is difficult to maintain geometrical consistency of the modified boundary edges and surfaces. Therefore, it is effective to apply a new free-form surface to a closed region enclosed with modified boundary edges because the consistency of the trimmed surface can be maintained. The smoothness is more important than the approximation precision, since downstream process emphasizes surface smoothness rather than the precision of the approximated surface. In a conventional surface fitting method which approximates a surface using sample points derived from the tangent plane, the continuity with an adjacent surface will collapse because the surface was generated individually. In contrast, the surface fitting method [2] in consideration of maintaining G^1-continuity with adjacent surfaces is proposed by Muraki et al. In his method, G^1-continuity is guaranteed on the common boundary edges.

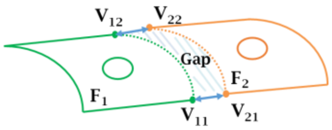

Fig. 1: Modifying a boundary edge in direct modeling for data healing.

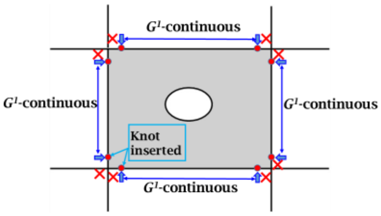

Fig. 2: Problems of the method of Muraki et al. [2].

However, when a surface connects adjacent surfaces with G^1-continuity in two adjoining directions along common boundary edges as shown in Fig. 2, the conditions used as G^1-continuous cannot be fulfilled near the corner portion with a B-spline surface. In Fig. 2, the red x marks indicate the discontinuous portion near the corner of the common boundary edges.

In this paper, we propose a new surface representation to solve the problems of the Muraki's method. In this method, shapes can be approximated in good accuracy as a reference model for downstream processes. Our method generates a trimmed surface that G^1-continuious with adjacent surfaces in all directions.

Related work:

Muraki et al. proposed a reconstruction method of trimmed surfaces for an N-side region and allowed discontinuous portions near the corner of the common boundary edges [2]. His method unites the advantages of the surface interpolation method [1] and the N-side filling method [4]. In a common boundary where two surfaces should be connected with G^1-continuity, input curve mesh is represented by cubic Bezier curves and the cross-boundary derivatives are calculated based on the basis patch method [3]. Among control points of the B-spline surface, the control points that are on the inner side of the boundary curves are calculated by the surface interpolation method. Moreover, other internal control points are calculated by the N-side filling method. However, using conventional a B-spline surface the cross-boundary derivative vectors cannot be specified independently. Concretely, B-spline has no degree of freedom to individually define the connections of two surfaces with G^1-continuity in u and v directions. Moreover, when a surface connects with adjacent surfaces in two adjoining directions along common boundary edges as shown in Fig. 2, knots are inserted near the corners (at parameters 0.05 and 0.95) [2] in order to narrow down the discontinuous section with the adjacent surfaces even if two surfaces have to connect G^1-continuity. However, the method of determining the knot values near the corners are unclear.

In this paper, we focus on the Muraki et al. problems and propose a new surface representation which the cross-boundary derivatives can be specified independently in u and v directions with the B-spline blending functions.

New surface representation:

In this paper, we integrate the advantages of the B-spline surface and Gregory surface to define a new surface representation. To be more concrete, construction of a new surface representation via the boundary curves and approximation of the inner control points will be studied. We express a fitting surface $\mathrm{S}(u,v)$ using surface control points $\mathrm{P}_{i,j,k}(i=0,\dots,n; j=0,\dots,m; k=0,1)$. Surface $\mathrm{S}(u,v)$ is expressed by Eqn. (1). Where $N_i^3(u)$ and $N_j^3(v)$ are the cubic B-spline basis functions over the knot spans $\mathrm{U}=\left[0,0,0,0,\quad u_0,\dots,u_p,\quad 1,1,1,1\right]$ and $\mathrm{V}=\left[0,0,0,0,\quad v_0,\dots,v_q,\quad 1,1,1,1\right]$.

$$\mathrm{S}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}N_i^3(u)N_j^3(v)\mathrm{Q}_{i,j}(u,v) \tag{1}$$

The rational functions $\mathrm{Q}_{i,j}(u,v)(0\le i\le n; 0\le j\le m)$ are defined by the following relationships:

1. If $(i,j)=(1,1),(1,m-1),(n-1,1),(n-1,m-1)$, then

$$\mathrm{Q}_{1,1}(u,v)=\frac{u\mathrm{P}_{110}+v\mathrm{P}_{111}}{u+v} \qquad (0<u<u_0, 0<v<v_0) \tag{2}$$

$$\mathrm{Q}_{1,m-1}(u,v)=\frac{u\mathrm{P}_{1(m-1)0}+(1-v)\mathrm{P}_{1(m-1)1}}{u+(1-v)} \qquad (0<u<u_0, v_q<v<1) \tag{3}$$

$$\mathrm{Q}_{n-1,1}(u,v)=\frac{(1-u)\mathrm{P}_{(n-1)10}+v\mathrm{P}_{(n-1)11}}{(1-u)+v} \qquad (u_p<u<1, 0<v<v_0) \tag{4}$$

Proceedings of CAD'15, London, UK, June 22-25, 2015, 264-269

$$Q_{n-1,m-1}(u,v)=\frac{(1-u)P_{(n-1)(m-1)0}+(1-v)P_{(n-1)(m-1)1}}{(1-u)+(1-v)}\qquad (u_p<u<1, v_q<v<1) \tag{5}$$

2. If others, then

$$Q_{i,j}=P_{ij0} \tag{6}$$

In this paper, surface is fitted using the Eqn. (1), and the concept of our method is shown in Fig. 3. Fig. 3(a) shows surface *F* that has G^1-continuous adjacent surfaces F_1, F_2, F_3 and F_4 in all directions. In a common boundary where two surfaces are connected with G^1-continuity, each boundary of an input curve mesh is represented by a cubic Bezier curve. The concept of our proposed method explains in the case where knots are inserted at parameters $u_0=0.5$ and $v_0=0.5$ $(p=0, q=0)$ as shown in the Fig. 3(b). The yellow control points are obtained from boundary curves, the red ones are obtained by the joining equations [1] and the blue one is obtained by the least squares approximation method. First, when two surfaces are connected with G^1-continuity, the G^1-continuous control points at the connection section are obtained from joining equations which are describe in the next section. Next, a bi-cubic Gregory patch is constructed by the G^1-continuous control points. Since the constructed Gregory patch is insufficient for representing a trimmed surface, knots are inserted to u and v directions for increasing the degree of freedom. Then, the unknown inner control points are optimized using the least squares approximation method. Finally, a new surface is constructed using Eqn. (1).

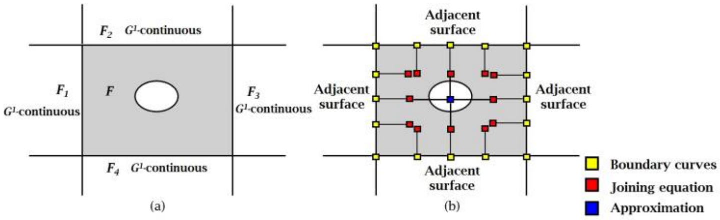

Fig. 3: Concept of our proposed method (b) applied to the gray region of (a).

<u>Joining with the adjacent surface:</u>

In order for two surfaces S^1 and S^2, with a common boundary curve shown in Fig. 4(a) to have a G^1-continuity, the derivative vectors on the boundary curves should satisfy the condition defined by Eqn. (7). Where $k(v)$ and $h(v)$ are scalar functions of v as shown in Eqn. (8). If the vectors are set to $a_i(i=0,...,3), b_i(i=0,...,3), c_i(i=0,...,2)$ as shown in Fig. 4(b). The G^1-continuous control points are calculated by solving Eqn. (9) and (10), where k_0, k_1 are positive real numbers and h_0, h_1 are arbitrary real numbers. The cross boundary derivatives of the two surfaces are calculated by the joining equations [1], and control points which G^1-continuous with adjacent surfaces are obtained. The obtained cross boundary derivatives serve as a condition for connecting two surfaces with G^1-continuity.

$$\frac{\partial S^2(0,v)}{\partial u}=k(v)\frac{\partial S^1(1,v)}{\partial u}+h(v)\frac{\partial S^1(1,v)}{\partial v} \tag{7}$$

$$k(v)=k_0(1-v)+k_1v,\qquad h(v)=h_0(1-v)+h_1v. \tag{8}$$

$$a_0^3=\frac{a_0+b_0}{|a_0+b_0|},\ a_3^3=\frac{a_3+b_3}{|a_3+b_3|},\ a_1^3=\frac{2a_0^3+a_3^3}{3},\ a_2^3=\frac{a_0^3+2a_3^3}{3},\ b_0=k_0a_0+h_0c_0, \tag{9}$$

$$b_3=k_1a_3+h_1c_2,\ b_1=\frac{(k_1-k_0)a_0^3}{3}+k_0a_1^3+\frac{2h_0c_1}{3}+\frac{h_1c_0}{3},\ b_2=k_1a_2^3-\frac{(k_1-k_0)a_3^3}{3}+\frac{h_0c_2}{3}+\frac{2h_1c_1}{3}. \tag{10}$$

Approximation:
In this paper, by the boundary information and sample points on the boundary edges which represent a hole, the inner control point is approximated with the least squares approximation method. A new surface is constructed using the Eqn. (1).

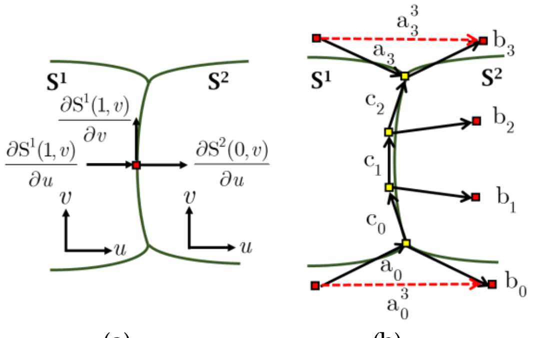

Fig. 4: (a) G^1-continuity condition, (b) connection of two surfaces using the joining equations [1].

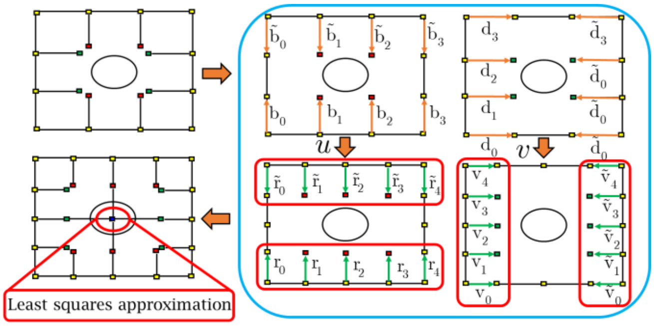

Fig. 5: Inner control point approximation process.

Approximation process:
In Fig. 5, our approximation process of the inner control point is shown. As our concept of proposed method explains, knots are inserted at parameters $u_0 = 0.5$ and $v_0 = 0.5$ $(p = 0, q = 0)$. The process is executed in the following steps:

1. The G^1-continuous control points are obtained from joining equations and a bi-cubic Gregory patch is constructed. Vectors between control vertices $\mathrm{b}_i, \tilde{\mathrm{b}}_i (i = 0,\ldots,3)$ are calculated for u direction. Vectors $\mathrm{d}_j, \tilde{\mathrm{d}}_j (j = 0,\ldots,3)$ are calculated for v direction.
2. Knots are inserted in both u and v directions. After knot insertion, vectors $\mathrm{r}_i = u_0 \mathrm{b}_i,\ (i = 0,\ldots,4)$ are calculated and vectors $\tilde{\mathrm{r}}_i, \mathrm{v}_j, \tilde{\mathrm{v}}_j$ are calculated in the same manner. Additionally, control points are obtained from scaled vectors $\mathrm{r}_i, \tilde{\mathrm{r}}_i$ and $\mathrm{v}_j, \tilde{\mathrm{v}}_j$ as shown in Fig. 5.
3. The points on the boundary edges which represent a hole are calculated as sample points and the center point of the hole is also added to the sample points. It is better to add center point to the sample points because it will improve the approximation accuracy around the hole. In this paper, each boundary edge which represents a hole is equally divided into 10 sections and the number of sample points $t = \textit{the number of boundary edges which represent a hole} \times 10 + 1$ are assumed to be $\mathrm{A}_s (0 \le s \le t)$. The parameters of A_s are assumed to be $\bar{u}_s$ and $\bar{v}_s$, and calculated by projecting A_s on to the Gregory patch constructed in step 1.
4. From the control points generated in step 2 and the sample points generated in step 3, the unknown inner control point is approximated by using Eqn. (11).
5. A new surface is constructed by the control points which were obtained in step 2 and the approximated inner control point which was obtained in step 4.

$$f = \sum_{s=0}^{t-1} \left| \mathrm{A}_s - \mathrm{S}(\bar{u}_s, \bar{v}_s) \right|^2 \tag{11}$$

Evaluation of generated surface:
To verify the accuracy of the generated surface, the distance between the generated surface and the source surface retained by the trimmed surface is measured. The points on the source surface are projected onto the generated surface and the distance between the source surface and generated surface is measured. Moreover, to find the relative error, the ratio of the bounding box size and the maximum distance are calculated [2]. In this paper, when the ratio is smaller than 1% [2], it is assumed that a shape is approximated in good accuracy as a reference model for the downstream process.

Experimental results:

Our method is applied to the shapes with a hole as shown in Fig. 6(a) and (b). Fig. 6(a) shows the control points of generated surface F that has G^1-continuous adjacent surfaces F_1, F_2 and F_3 in three directions. Fig. 6(b) shows the control points of generated surface F that has G^1-continuous adjacent surfaces F_1, F_2, F_3 and F_4 in all directions. The red dots indicate the control points which was generated in step 2 and blue ones indicate the approximated control point in step 4. The error evaluation of the generated surface is shown in Tab. 1, *Avg.* indicates the average error margin value obtained by averaging the distances between the generated surface and the source one. *Max* indicates the maximum error margin value representing the maximum distance between the generated surface and the source one. *Ratio* indicates the ratio of the bounding box size and the maximum distance. The ratio of both objects (a) and (b) are less than 1% as shown in Tab. 1, and we can find that shapes are approximated in good accuracy as reference models for downstream processes. In addition, in order to verify the continuity with adjacent surfaces, the normal vectors on the boundary edges of the generated surface are calculated, shown with blue lines in Fig. 6(c) and (d), and those of the adjacent surfaces are shown with red lines. As shown in Fig. 6 (c) and (d), the normal vectors of the generated surfaces coincide with those of the adjacent surfaces on their boundary edges, and we can find that two surfaces are connected with G^1- continuity.

Conclusion and future works:

In this paper, we have proposed the method of generating a smooth surface with a holes that connects to adjacent surfaces with G^1-continuity by applying our new surface representation to a closed region. The proposed method integrates the advantages of the Gregory surface and B-spline surface. Concretely, the inner control points are obtained based on least squares approximation method, and the G^1-continuous control points on the boundary are obtained from the joining equations.

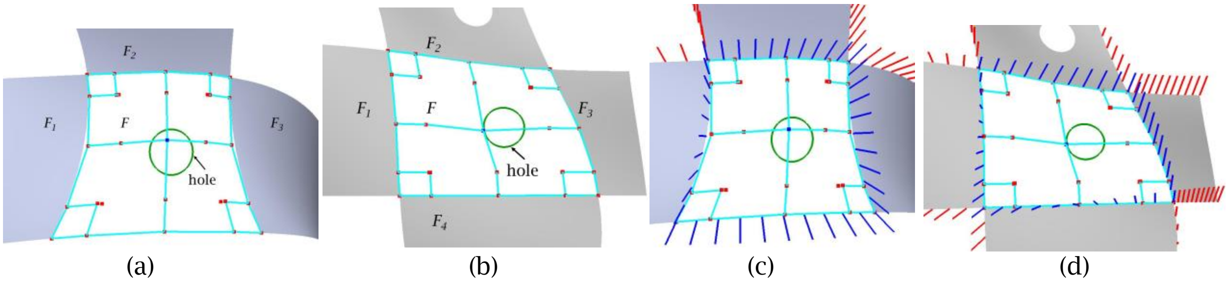

Fig. 6: Control points of generated surfaces (a) and (b). (c) and (d) verification of the continuity with adjacent surfaces: the normal vectors of the generated surfaces coincide with those of the adjacent surfaces on their boundary edges.

Object	*Evaluation object*	*Avg.*	*Max*	*Ratio*
(a)	Trimmed surface	0.352679	1.139214	0.253059 %
(b)	Trimmed surface	0.225614	0.895673	0.182698 %

Tab. 1: Error evaluation.

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